 

# Polymorphous billiard with chaotic beams dynamics

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A new type of billiard is introduced – the one with chaotic beams dynamics. The dynamics of dumbbell-like polymorphous billiards family is studied. The chaotic behaviour of the beams and their uniform stationary distribution are proved. The chaotic properties found let one use polymorphous billiard in physical applications.

Предложен новый тип биллиарда с хаотической динамикой лучей. Изучена динамика семейства полиморфных биллиардов в форме гантели. Доказано хаотическое поведение лучей и их равномерное стационарное распределение. Обнаруженные хаотические свойства позволяют использовать полиморфный биллиард в различных физических приложениях.

Billiards, i.e. systems with elastic or mirror reflections, occupy the central position in the deterministic chaos theory and have numerous physical applications [1]. Chaotic billiards, in which beams dynamics is everywhere chaotic, have won a special popularity. Close trajectories in such billiards exponentially quickly diverge in the phase space and mix up. The most well-known among the chaotic billiards are dispersing Sinai billiards [2] and defocusing Bunimovich billiards [3]. The paper introduces a new type of chaotic billiard – polymorphous billiard. Unlike the known ones, it contains dispersing as well as focusing regions of the boundary and has no neutral components. Its characteristic phase dynamics (at control parameter changes) is studied and Lyapunov exponent as well as invariant reflections density on the boundary are calculated by the example of the simplest specimen of this family - dumbbell-like billiard.

Let us smoothly join (so that the tangent has no discontonuities) an even number of arcs taken from one circle to get a closed curve. We shall call the billiard limited by such a curve polymorphous. Its boundary is formed by the arcs of the same circle and has everywhere constant curvature to sign. Some examples of such billiards one can see in Fig. 1. The simplest polymorphous billiard is a dumbbell-like billiard, the boundary of which is formed by arcs of four circles. A smaller number of arcs is impossible, otherwise the smoothness of the boundary obtained would have been broken.

To study the "dumbbell" dynamics, let us use geometro-dynamical approach [4, 5], in which beams dynamics is described in a special symmetric phase space. Let us chose angle $\chi$ between the axis, connecting the centres of the convex components of the border and the beam, drawn to the point of contact between the convex and concave components (Fig. 1) as the control parameter of the dynamic system. This angle $\chi$ corresponds to the width of the middle of the dumbbell. It is changed from $\pi/2$ to $\pi/6$. At there is no narrow middle and we have a billiard in a circle instead of the dumbbell one. At $\chi=\pi/3$ the circles corre-

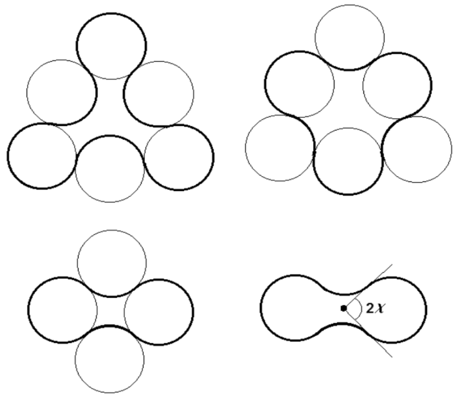


Fig.1. Examples of polymorphous billiards. Above – two dual (relative to the change of all the convex components for concave and vice versa) polymorphous billiards of 3 order, formed due to 6 disks joining. Below – "dumbbell" billiard and its control parameter – angle $\chi$.

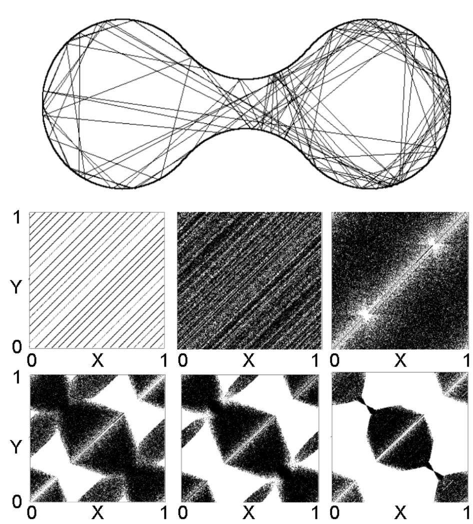


Fig. 2. Geometrical (above) and phase (below) portrait of "dumbbell" billiard with its middle increase at $\chi = \pi/2$; $\chi = 1.569$; $\chi = 1.552$; $\chi = \pi/3$; $\chi = \pi/2$; $\chi = \pi/6$.

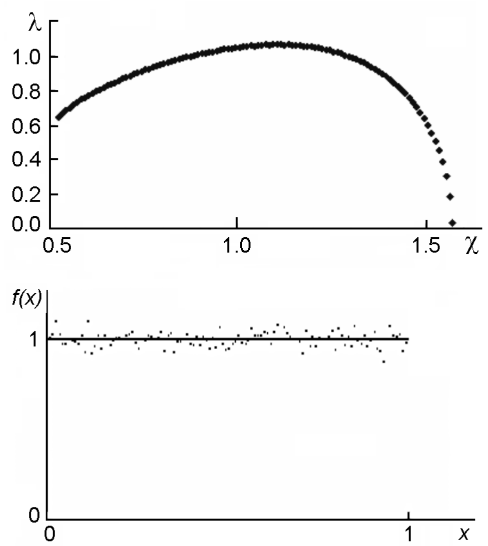


Fig. 3. The dependence of Lyapunov exponent of the "dumbbell" on the control parameter (above). Stationary density of reflections $f(x)$ on the billiard boundary (below). The full length of the billiard boundary is normalized to one.

sponding to the convex parts of dumbbell boundary, contact (inside of the billiard). At $\chi = \pi/2$ we have the most symmetric configuration. At $\chi = \pi/6$ the middle reaches its maximum and billiard falls into two ones.

Fig. 2 shows the phase portrait of the "dumbbell", the control parameter changing. When the narrow middle appears, the beams dynamics in the billiard is always chaotic. This is also confirmed by Lyapunov exponent dependence, shown in Fig. 3. At all the values of the control parameter (except the integrable case of a billiard in a circle at $\chi = \pi/2$) Lyapunov exponent is strictly positive. Angle $\chi$ increasing, in the phase space hollow zones — lacunas — appear. They correspond to the classically forbidden beams in the region of geometrical shadow of the billiard. When the middle gets larger, the volume of lacunas increases. Lacunas play the part of topological obstacles in the phase space. Near them the phase cascade has discontinuities. As a result, its chaoticity increases with the increase of lacunas volume. This can be seen on the graph of the Lyapunov exponent (Fig. 3). At $\chi = \pi/3$ the exponent reaches its maximum, because total length of dispersing components (the reflection from which causes trajectories dispersing) of the boundary is maximal at this value. With further extending of the middle the lacunas volume increase is accompanied by the reducing of scattering components relative length. So Lyapunov exponent decreases. When the middle is about to reach its maximum at $\chi = \pi/6$ it tends to a finite value. Lacunas overlapping in the phase space corresponds to intermediate value $\chi = \pi/3$ (equidistant from points $\chi = \pi/2$ and $\chi = \pi/6$). In the phase space there appears one common region of forbidden movement instead of two isolated lacunas.

It is convenient to describe the statistic properties of the "dumbbell" with stationary density of reflection about the billiard boundary [5]. Numeric calculations have shown that here it remains constant (Fig. 3), as well as in the case of billiard in a circle. This is obviously connected with the constancy of absolute curvature for the given billiard. Beams visit the billiard's boundary with approximately equal frequency. So, in the asymptotic limit they are uniformly distributed in the phase space as well.

One should expect analogous behaviour of the beams in polymorphous billiard of arbitrary form. The chaotic properties found let one use polymorphous billiard in applications. In particular, "dumbbell" form can be used for atomic [6], microwave [7] or semiconductor billiards [8]. Chaos peculiarities in polymorphous billiards can also influence the character of light pass in optic nanoceramics microclusters, formed due to coagulating of ball-like nanoparticels etc.

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# Поліморфний більярд з хаотичною динамікою променів

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Запропоновано новий тип більярду з хаотичною динамікою променів. Вивчено динаміку ряду поліморфних більярдів у формі гантелі. Доведено хаотичну поведінку променів та рівномірність їхнього стаціонарного розподілу. Винайдені хаотичні властивості дозволяють використовувати поліморфний більярд у різних фізичних системах.