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DYNAMIC ANALYSIS OF FLEXIBLE HOISTING ROPE WITH TIME-VARYING LENGTH

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Abstract. The governing equations of flexible hoisting rope are developed employing Hamilton's principle. Experiments are performed. It is found that the experimental data agree with the theoretical prediction very well. The results of simulation and experiment show that the flexible hoisting system dissipates energy during downward movement but gains energy during upward movement. Further, a passage through resonance in the hoisting system with periodic external excitation is analyzed. Due to the time-varying length of the hoisting rope the natural frequencies of the system vary slowly, and transient resonance may occur when one of frequencies coincides with the frequency of external excitation.

Key words: dynamic analysis; flexible hoisting rope; transient resonance.

1. Introduction.

While rope is employed in hoisting industry such as mine hoists, elevators, cranes etc, it is subject to vibration due to its high flexibility and relatively low internal damping characteristics [1, 2]. Most often these systems are modeled as either an axially moving tensioned beam or string with time-varying length and a rigid body at its lower end [3, 4]. It was shown that the vibration energy of the rope changes in general during elongation and shortening [5, 6]. When the rope length is being shortened, vibration energy increases exponentially with time, causing dynamic instability [7]. The study of rope vibration problems in flexible hoisting systems has attracted wide attention. Chi and Shu [8] calculated the natural frequencies associated with the vertical vibration of a stationary cable coupled with an elevator car. Terumichi and Ohtsuka et al. [9] assumed the velocity of the string is constant and studied the transverse vibrations of a string with time-varying length and a mass-spring system at the lower end with theoretical and experimental methods. Fung and Lin [10] analyzed the transverse vibration of elevator rope with time-varying length and the time-varying mass and inertia of rotor were considered. A variable structure control scheme is proposed to suppress the transient amplitudes of vibrations. Kaczmarczyka and Ostachowiczb [11] studied coupled vibration of deep mine hoisting cable and built a distributed-parameter model. They found that response of the catenary-vertical rope system may feature a number of resonance phenomena. Zhang and Agrawal [12] derived the governing equation of coupled vibration of flexible cable transporter system with arbitrarily varying length. Zhu and Chen [13] investigated the control of elevator cable with theoretical and experimental methods. A novel experimental method is developed to validate the uncontrolled and controlled lateral responses of a moving cable in a high-rise elevator and shown good agreement with the theoretical predictions. Zhang [14] presented a systematic procedure for deriving the model of a cabel transporter system with arbitrarily varying cable length and proposed a Lyapunov controller to dissipate the vibratory energy. Zhang and Zhu et al. [15] derived the governing equation and energy equation of longitudinal vibration of flexible hoisting system with arbitrarily varying length.

While extensive studies focus individually on vibration characteristics of the rope with time-varying length, the dynamic stability of the rope has also been studied by several researchers. Kumaniecka and Niziol [16] investigated the longitudinal-transverse vibration of a hoisting cable with slow variability of the parameters. The cable material non-linearity was taken into account and unstable regions were identified by applying the harmonic balance method. General stability characteristics of horizontally and vertically translating beams and strings with arbitrarily varying length and various boundary conditions were studied in Zhu and Ni [17]. While the amplitude of the displacement can behave in a different manner depending on the boundary conditions, the amplitude of the vibratory energy of a translating medium decrease and increase in general during extension and retraction, respectively. Lee [7] introduces a new technique to analyze free vibration of a string with time-varying length by dealing with traveling waves. When the string length is being shortened, free vibration energy increases exponentially with time, causing dynamic instability.

Extensive research efforts on the flexible hoisting rope with time-varying length have been done in the last few decades as aforementioned, however, the study of most studies were restricted to cases with constant transport speed samples. The dynamic characteristics of flexible hoisting rope with an arbitrarily varying length are the subject of this investigation. The governing equations are developed employing the extended Hamilton's principle. The derived governing equations are shown to be nonlinear partial differential equations(PDEs) with variable coefficients. On choosing proper mode functions that satisfy the boundary conditions, the solution of the governing equations was obtained using the Galerkin's method. In order to evaluate the mathematical model, an experimental set-up is built and some experiments are conducted. Comparing the experimental data to the simulation, a favourable result is obtained, which indicates that the proposed mathematical model is valid for flexible hoisting rope. Further, the phenomenon of passage through resonance in hoisting rope system is studied in this paper. Based on the proposed fundamental dynamic analyses, further vibration control can be adopted for such the flexible hoisting systems in the near future.

2. MODEL OF FLEXIBLE HOISTING SYSTEM.

Flexible hoisting system can be simplified as an axially moving string with time-varying length and a rigid body m at its lower end, as shown in Fig. 1. The rail and the suspension of the rail are assumed to be rigid. The string has Young's modulus E, diameter d and the density per unit length ρ. The origin of coordinate is set at the top end of string and the instantaneous length of string is $l(t)$ at time t. The instantaneous axial velocity, acceleration and jerk of the string are $v(t) = \dot{l}(t)$, $a(t) = \dot{v}(t)$ and $j(t) = \dot{a}(t)$ respectively, where the overdot denotes time differentiation. At any instant t, the transverse displacement of string is described by $y(x,t)$, at a spatial position x, where $0 \le x \le l(t)$. In actual flexible hoisting system, the rotational unbalance of the traction motor or the abnormal off-track of rope possibly causes vibration of the hoisting system. To reproduce this phenomenon, a transverse extrinsic disturbing excitation $e(t)$ is applied at the upper end of the string. In this paper, all the equations and derivations base on the following assumptions.

1. Young's modulus E, diameter d and density ρ of the string are always constants;

2. Only transverse vibration is considered here. The elastic distortion of string arousing from the transverse vibration is much less than the length of the string;

3. The bending stiffness of the string, all the damp and friction, and the influence of air current are ignored.

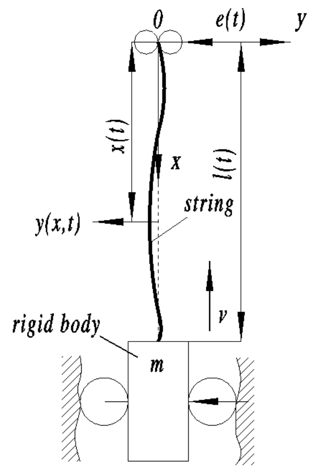

Fig. 1. Schematic of flexible hoisting string with time-varying length.

2.1. Energy of flexible hoisting system. After the string is deformed, the position vector R of a point at x can be written as:

$$\boldsymbol{R} = x(t)\boldsymbol{i} + y(x,t)\boldsymbol{j}\ , \tag{1}$$

where i and j are the unit vectors along the x-axes and y-axes, respectively. The material derivative of R yields the velocity vector

$$\boldsymbol{V} = v(t)\boldsymbol{i} + [y_t + vy_x]\boldsymbol{j}\ , \tag{2}$$

where the subscript t denotes partial differentiation with respect to time, and subscript x denotes partial differentiation with respect to space. Similarly, the position vector R_c and velocity vector V_c of rigid body can be respectively written as:

$$\boldsymbol{R_c} = l(t)\boldsymbol{i} + y(l(t),t)\boldsymbol{j}\ ; \tag{3}$$

$$\boldsymbol{V_c} = v(t)\boldsymbol{i} + y_t(l(t),t)\boldsymbol{j}\ . \tag{4}$$

Then, the kinetic energy of flexible hoisting system is computed by

$$E_k(t) = \frac{1}{2}m\boldsymbol{V_c}\cdot\boldsymbol{V_c}\Big|_{x=l(t)} + \frac{1}{2}\rho\int_0^{l(t)}\boldsymbol{V}\cdot\boldsymbol{V}dx\ . \tag{5}$$

The first term on the right of Eq. (5) represents the kinetic energy of rigid body, the second term represents the kinetic energy of the string. The elastic strain energy of the string is

$$E_e(t) = \int_0^{l(t)}\left(P\varepsilon + \frac{1}{2}EA\varepsilon^2\right)dx\ , \tag{6}$$

where $P(x,t)$ is the quasi-static tension at spatial position x of the string at time t due to gravity. Since the string is acted upon not only by the weight of the concentrated mass at the lowest end but also its own weight, the tension $P(x,t)$ is expressed as

$$P = \left[m + \rho(l(t) - x)\right]g\ . \tag{7}$$

And ε represents the strain measure at spatial position x of the string and can be expressed as

$$\varepsilon = \left(ds - dx\right)/\,dx\ . \tag{8}$$

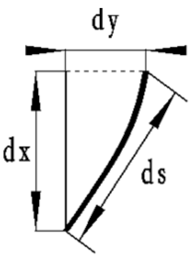

Fig. 2.
A small element of the string in a deformed position.

As shown in Fig. 2, ds can be expressed as

$$ds \approx \sqrt{1+(dy/dx)^2}\,dx \approx \left[1+\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2-\frac{1}{8}\left(\frac{\partial y}{\partial x}\right)^4+\cdots\right]dx \approx \left[1+\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right]dx\,. \quad (9)$$

Substituting Eq. (9) into Eq. (8) yields

$$\varepsilon = \frac{1}{2}y_x^2\,. \quad (10)$$

2.2. Free vibration equations. According to the characteristics of top restriction of the string, the boundary conditions at $x(t) = 0$ are

$$y(0,t)=0\,,\quad y_t(0,t)=0\,. \quad (11)$$

On substitution of Eqs (5) and (6) in the Hamilton's Principle,

$$\int_{t_1}^{t_2}(\delta E_k(t)-\delta E_e(t))dt = 0 \quad (12)$$

and apply the variational operation. Because the length of the string $l(t)$ changes with time, the standard procedure for integration by parts with respect to the temporal variable can't apply. Applying Leibnitz's rule and part integration results in the following expressions

$$\int_0^{l(t)}\rho\left(y_t+vy_x\right)\delta y_t dx =$$

$$=\rho\frac{\partial}{\partial t}\int_0^{l(t)}\left(y_t+vy_x\right)\delta y dx-\rho\left[v\left(y_t+vy_x\right)\delta y\right]\Big|_{l(t)}-\rho\int_0^{l(t)}\frac{\partial}{\partial t}\left(y_t+vy_x\right)\delta y dx\,. \quad (13)$$

Following the standard procedure for integration by parts with respect to the spatial variable and invoking Equation (13), one obtains from Eq. (12),

$$-\int_{t_1}^{t_2}\left[m\frac{\partial}{\partial t}y_t(l,t)+Py_x+\frac{1}{2}EAy_x^3\right]\delta y(l,t)\,d\,t-$$

$$\int_{t_1}^{t_2}\int_0^{l(t)}\left[\rho\frac{\partial}{\partial t}(y_t+vy_x)+\rho v\frac{\partial}{\partial x}\left(y_t+vy_x\right)-\frac{\partial}{\partial x}\left(Py_x\right)-EA\frac{\partial}{\partial x}\left(\frac{1}{2}y_x^3\right)\right]\delta y dx dt = 0\,. \quad (14)$$

Setting the coefficients of δw in Eq. (14) to zero yields the governing equations in the forms

$$\rho(y_{tt}+2vy_{xt}+\dot{v}y_x+v^2y_{xx})-P_xy_x-Py_{xx}-\frac{3}{2}EAy_x^2y_{xx}=0\,;\ 0< x< l(t). \quad (15)$$

The first four terms in Eq. (15) correspond to the local, Coriolis, tangential and centripetal acceleration, respectively. The resulting boundary conditions from Eq. (14) at $x = l(t)$ is

$$my_{tt} + Py_x + \frac{1}{2}EAy_x^3 = 0\ ;\ \ x = l(t). \tag{16}$$

The energy associated with the transverse vibration of the system is

$$E_v(t) = \frac{1}{2}my_t^2(l,t) + \frac{1}{2}\rho\int_0^{l(t)}\left[y_t(x,t) + vy_x(x,t)\right]^2 dx + \frac{1}{2}\int_0^{l(t)}\left(Py_x^2 + \frac{1}{4}EAy_x^4\right)dx\ . \tag{17}$$

2.3. Forced vibration equations. When the external excitation occurs at the upper end of string, the governing Eq. (15) must be adjusted. Compared with Eq. (12), the corresponding boundary conditions at are changed into

$$y(0,t) = e(t)\ ;\quad y(l,t) = 0\ . \tag{18}$$

Obviously, the boundary conditions are non-homogeneous and difficult to be applied directly. Here, the procedure described in Reference [5] is used to transfer the governing Eq. (15) with non-homogeneous boundary conditions into equation of motion with homogeneous boundary conditions. The transverse displacement is expressed in the form

$$y(x,t) = w(x,t) + h(x,t), \tag{19}$$

where $w(x,t)$ is the part that satisfies the homogeneous boundary conditions and $h(x,t)$ is the part that does not satisfy the homogeneous boundary conditions. Substitute Eq. (19) into Eq. (15) yields

$$\rho(w_{tt} + 2vw_{xt} + \dot{v}w_x + v^2w_{xx}) - P_xw_x - Pw_{xx} - \frac{3}{2}EAw_x^2w_{xx} + \rho(h_{tt} + 2vh_{xt} + \dot{v}h_x + v^2h_{xx});$$

$$-P_xh_x - Ph_{xx} - EA(3w_xw_{xx}h_x + \frac{3}{2}w_{xx}h_x^2 + \frac{3}{2}w_x^2h_{xx} + 3w_xh_xh_{xx} + \frac{3}{2}h_x^2h_{xx}) = 0\ ;\ 0 < x < l(t), \tag{20}$$

where $w(x,t)$ is the state variables. Equation (20) describes the transverse vibration of the flexible hoisting system under extrinsic disturbing excitation. The corresponding boundary condition is

$$mw_{tt} + Pw_x + \frac{1}{2}EAw_x^3 + mh_{tt} + Ph_x + \frac{1}{2}EA\left(3w_x^2h_x + 3w_xh_x^2 + h_x^3\right) = 0\ ;\quad x = l(t). \tag{21}$$

Set the function $h(x,t)$ to first-order polynomial,

$$h(x,t) = a_0(t) + a_1(t)\frac{x}{l(t)}\ . \tag{22}$$

Then, when $x(t) = 0$ and $x(t) = l(t)$,

$$h(0,t) = e(t)\ ;\ \ h(l(t),t) = 0 \tag{23}$$

Substituting Eq. (23) into Eq. (22), the coefficients $a_0(t)$ and $a_1(t)$ can be obtained as,

$$a_0(t) = e(t)\ ;\ \ a_1(t) = -e(t)\ . \tag{24}$$

Therefore,

$$h(x,t) = e(t) - e(t)\frac{x}{l(t)}\ . \tag{25}$$

Once $h(x,t)$ is known, the solutions for $w(x,t)$ is sought from Eq. (20). $y(x,t)$ is obtained subsequently from Eq. (19). Equation (20) is a partial differential equation which describes the dynamics of the flexible hoisting string. The equation defined over time-dependent spatial domain rendering the problem non-stationary. Hence, the exact solution to this problem is not available, and recourse must be made to an approximate analysis. In what follows, numerical techniques are employed to obtain approximate solution for the governing equation.

3. Discretization of the governing equation.

Equation (20) is a partial differential equation with infinite dimensions and many parameters are time-variant. It is impossible to obtain an exact analytical solution from Eq. (20). In this section, Galerkin's method is applied to truncate the infinite-dimensional partial differential equation into a nonlinear finite-dimensional ordinary differential equation with time-variant coefficients. Then, solve them with numerical methods. In order to map Eq. (20) onto the fixed domain, a new independent variable $\zeta=x/[l(t)]$ is introduced and the time-variant domain [0, $l(t)$] for x is converted to a fixed domain [0, 1] for ζ. According to the characteristic of taut translating string, the solution of the transverse vibration $w(x,t)$ is assumed in the form [11, 12]

$$w(x,t)=\sum_{i=1}^{n}\varphi_i(\zeta)q_i(t)=\sum_{i=1}^{n}\varphi_i(x/l)q_i(t)\,, \tag{26}$$

where $q_i(t)$ (i = 1,2,3,…,n) is the generalized coordinates respect to $w(x,t)$, n is the number of included modes; $\varphi_i(\zeta)$ is trial function [11,12],

$$\varphi_i(\zeta)=\sqrt{2}\sin i\pi\zeta\,. \tag{27}$$

Consequently, expansion Eq. (26) results in the expressions for partial derivatives of the transverse displacement function:

$$w_x(x,t)=\frac{1}{l}\sum_{i=1}^{n}\varphi_i^{'}(\zeta)q_i(t)\,;\quad w_{xx}(x,t)=\frac{1}{l^2}\sum_{i=1}^{n}\varphi_i^{''}(\zeta)q_i(t)\,;$$

$$w_{xt}(x,t)=\sum_{i=1}^{n}\left[\frac{1}{l}\varphi_i^{'}(\zeta)\dot{q}_i(t)-\frac{\zeta v}{l^2}\varphi_i^{''}(\zeta)q_i(t)-\frac{v}{l^2}\varphi_i^{'}(\zeta)q_i(t)\right];$$

$$w_{tt}(x,t)=\sum_{i=1}^{n}\varphi_i(\zeta)\ddot{q}_i(t)-\frac{2\zeta v}{l}\sum_{i=1}^{n}\varphi_i^{'}(\zeta)\dot{q}_i(t)+$$

$$+\left[\frac{2\zeta v^2}{l^2}\sum_{i=1}^{n}\varphi_i^{'}(\zeta)-\frac{\zeta a}{l}\sum_{i=1}^{n}\varphi_i^{'}(\zeta)+\frac{\zeta^2 v^2}{l^2}\sum_{i=1}^{n}\varphi_i^{''}(\zeta)\right]q_i(t)\,. \tag{28}$$

Substituting Eq. (28) into Eq. (20), multiplying the governing equation by $\varphi_j(\zeta)$ (j = = 1, 2, 3, …, n), integrating it from ζ = 0 to 1, and using the boundary conditions and the orthonormality relation for $\varphi_i(\zeta)$ yield the discretized equation of transverse vibration for the flexible hoisting rope with time-variant coefficients

$$\boldsymbol{M}\ddot{\boldsymbol{Q}}+\boldsymbol{C}\dot{\boldsymbol{Q}}+\boldsymbol{K}\boldsymbol{Q}+\boldsymbol{S}(\boldsymbol{Q})=\boldsymbol{F}\,, \tag{29}$$

where $\boldsymbol{Q}=[q_1(t),q_2(t),\cdots,q_n(t)]^T$ is vector of generalized coordinate, **M**, **C**, **K** and **F** are matrixes of mass, damp, stiffness and generalized force respect to **Q**, respectively. **S**(***Q***) is higher order item of generalized coordinate. The matrices are expressed as follows:

$$M_{ij}=\rho\delta_{ij}\,;\quad C_{ij}=\int_0^1\frac{2v}{l}(1-\zeta)\varphi_i^{'}(\zeta)\varphi_j(\zeta)\,d\zeta\,;$$

$$K_{ij}(t)=\frac{\rho a}{l}\int_0^1(1-\zeta)\varphi_i^{'}(\zeta)\varphi_j(\zeta)d\zeta-\frac{\rho v^2}{l^2}\int_0^1(1-\zeta)^2\varphi_i^{'}(\zeta)\varphi_j^{'}(\zeta)d\zeta+$$

$$+\frac{\rho g}{l}\int_0^1(1-\zeta)\varphi_i^{'}(\zeta)\varphi_j^{'}(\zeta)d\zeta-$$

$$-\frac{mg}{l^2}\int_0^1\varphi_i^{''}(\zeta)\varphi_j(\zeta)d\zeta-\frac{3EA}{2l^4}e^2(t)\int_0^1\varphi_i^{''}(\zeta)\varphi_j(\zeta)d\zeta\ ;$$

$$S_{\boldsymbol{j}}(\boldsymbol{Q})=-\frac{3EA}{2l^4}\int_0^1\left(\sum_{i=1}^n\varphi_i^{'}(\zeta)q_i(t)\right)^2\sum_{i=1}^n\varphi_i^{''}(\zeta)q_i(t)\varphi\varphi_j(\zeta)d\zeta-$$

$$-\frac{3EA}{l^4}e(t)\int_0^1\sum_{i=1}^n\varphi_i^{'}(\zeta)q_i(t)\sum_{i=1}^n\varphi_i^{''}(\zeta)q_i(t)\varphi_j(\zeta)d\zeta\ ;$$

$$\boldsymbol{F}_j=-\rho\left[\ddot{e}(t)+\frac{2v}{l}\dot{e}(t)+\frac{a}{l}e(t)-\frac{2v^2}{l^2}e(t)\right]\int_0^1(1-\zeta)\varphi_j(\zeta)d\zeta-\frac{\rho g}{l}e(t)\int_0^1\varphi_j(\zeta)d\zeta\ , \quad (30)$$

where the superscript " ' " denotes partial differentiation for normalized variable ζ, δ_{ij} is the Kronecker delta defined by $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i\neq j$ (i=1,2,3,…,n, j=1,2,3,…,n). Solving the ordinary differential Eq. (29) with numerical methods may yield the instantaneous values of Q. Substituting these values into Eq. (26) may yield the instantaneous values of the transverse vibration of the string $w(x, t)$. The mathematical model defined by Eq. (29) illustrates the true dynamic nature of the flexible hoisting string, and can be used to predict and analyze the dynamic characteristics of flexible hoisting string.

4. Experiment.

4.1. Experiment set-up.

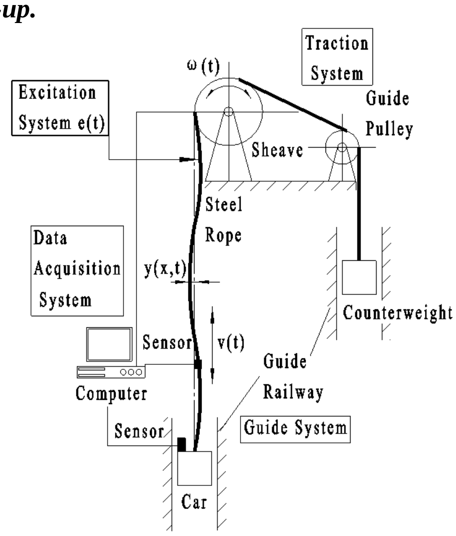

Fig. 3.
Schematic diagram of experimental set-up for flexible hoisting system.

To validate the mathematical model, an experimental set-up of flexible hoisting system is designed and built as Fig. 3. The set-up, simulating the hoisting system of traction elevator, consists of traction system, guide system, excitation system and data acquisition system. A frequency conversion motor is applied in flexible hoisting system. The rotation speed of motor may be controlled by adjusting the output of transducer to obtain the anticipant motion curve of hoisting system. A thin steel rope with a diameter of 3.2mm is chosen as the hoisting rope. The model car and counterweight are made up of many weights. The mass of car and counterweight is changeable by adding or reducing the number of weight.

The hoisting rope at the car side is the main research object, whose dynamic behavior will be studied, in this set-up. To simulate the extrinsic disturbing excitation in actual hoisting system, a transverse vibration exciter is appied at the top of the objective rope. The output of the exciter is decided by an adjustable signal generator. A micro-sensor with a mass of 4g is attached at a certain position of the objective rope to acquire the transverse vibration acceleration of rope. The signals from micro-sensor are transmitted to a computer and saved. Fig. 4 gives the actual picture of the experimental set-up. The main parameters of test are shown in Table 1.

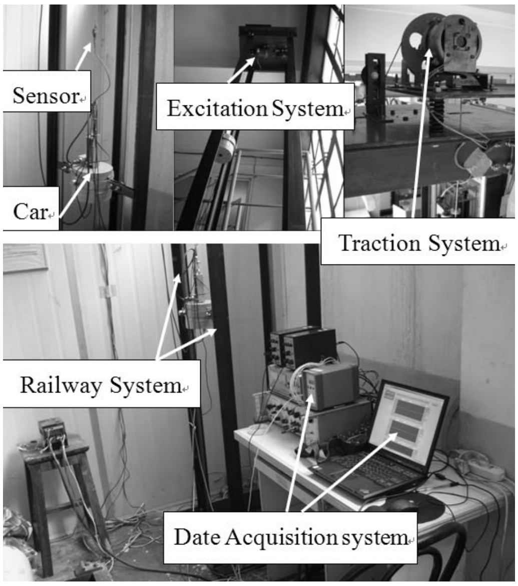

Fig. 4.
Actual picture of the experimental set-up for flexible hoisting system.

Table 1. Parameters of experimental set-up of flexible hoisting system.

Items	Data values
Density per unit length ρ(kg/m)	0.042
Young's modulus E (N/m^2)	1×10^{12}
Rope diameter d(m)	3.2×10^{3}
Hoisting mass m(kg)	15
Excitation $e(t)$(m)	$5\times10^{4}\sin(\pi t)$
Minimum length of the string $l_{\min}(t)$(m)	0.8
Maximum length of the string $l_{max}(t)$(m)	4.8
Maximum velocity v_{max}(m/s)	0.55
Maximum acceleration a_{max}(m/s^2)	0.4
Total travel time t(s)	8
Number of transverse modes n	4

4.2. Experiment procedure. Now the transverse vibration of flexible hoisting system will be calculated with theoretical equation and tested with experimental set-up, respectively. And the results will be compared. All the parameters, using in calculation and test, are the same. A downward or upward movement of car is prescribed to be the input of theoretical equations and experimental set-up. At the beginning, the car starts up at the top of flexible hoisting system and goes down. When arriving at the bottom, the car pauses for a moment and turns back to the start. Fig. 5 gives the prescribed displacement, velocity, acceleration curves of flexible hoisting system, where the processes of acceleration, deceleration and uniform speed downwards and upwards are included.

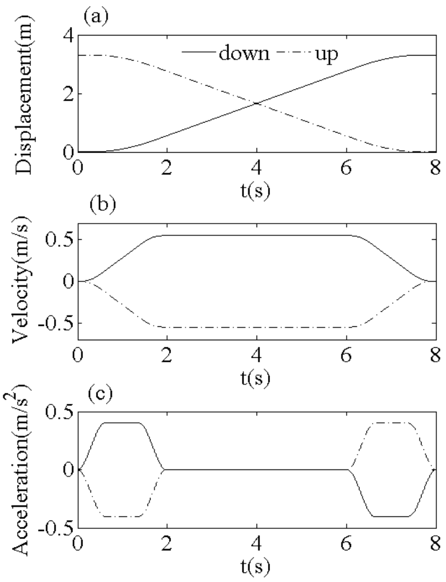

Fig. 5.
Movement profile of flexible hoisting system: (a) $l(t)$ *; (b)* $v(t)$ *; (c)* $\dot{v}(t)$.

In actual flexible hoisting system, the turning disbalance of rotor or the abnormal off-track of rope is the main origin of extrinsic disturbing excitation for flexible hoisting system. It is supposed that a transverse extrinsic disturbing excitation e(t) is applied at the top end of flexible hoisting system to reproduce this phenomenon. The extrinsic disturbing excitation disturbs the dynamic behavior of flexible hoisting rope only when the rope is moving. It is applied to the theoretical equations and experimental set-up.

During following calculation, the number of included modes in w(x,t) n is set to 4, which was already proved to be a proper value with a great deal calculating results and comparisons. When $n = 4$, the less calculating time and the satisfying veracity of results may be simultaneously obtained.

4.3. Free vibraion responses. The numerical simulations with the exact experiment parameters are conducted in order to compare with the experiments and the experimental results are favourably compared with the simulations, which can be seen in Figs. 6 (downward movement) and 7 (upward movement). Comparing the results of test and calculation in Figs. 6 and 7, the extent and trend of vibration curves are similar. Therefore, it can be concluded that the theoretical equations, proposed in this paper, may be used to evaluate the vibration of flexible hoisting rope.

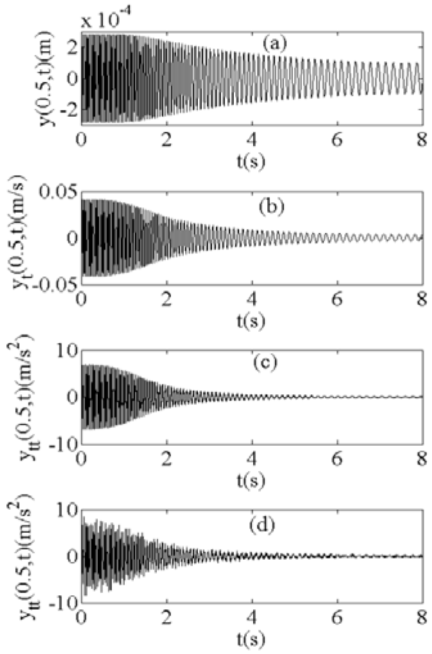

Fig. 6.
Free vibration responses of the flexible hoisting rope at 0.5m above the car during downward movement: (a) Displacement curve; (b) Velocity curve; (c) Acceleration curve(simulation); (d) Acceleration curve(experiment).

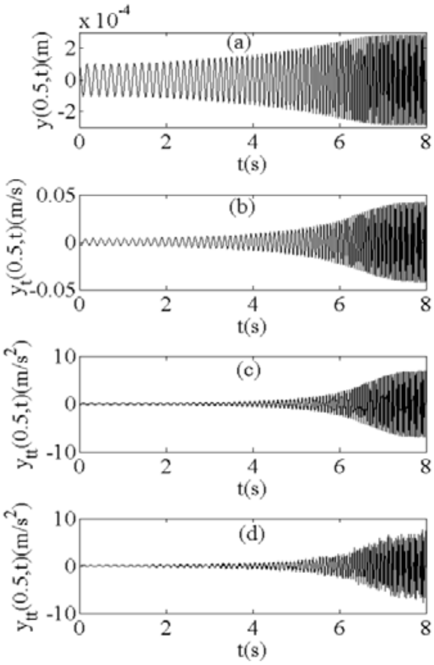

Fig. 7.
Free vibration responses of the flexible hoisting rope at 0.5m above the car during upward movement: (a) Displacement curve; (b) Velocity curve; (c) Acceleration curve(simulation); (d) Acceleration curve(experiment).

Fig. 6 displays reducing vibration amplitudes with increasing length of the rope during downward movement. This is due to the energy of flexible hoisting system transfers from the transverse vibration to the axial motion by bringing some mass into the domain of effective length, i.e., the axially hoisting rope is dissipative during downward movement, thus leading to a stabilized transverse dynamic reponse, as shown in Fig. 8(a). A possible physical interpretation of the result is as follows: during downward movement negative external work is required to maintain the prescribed axial motion which, in turn, brings about a convection of mass in the domain of effective length. At the same time, frequencies of the transverse vibration reduce with increasing length of the rope. This is due to the fact that the mass of the rope increase and the stiffness of the rope decrease, i.e., the rope becomes somewhat "softer".

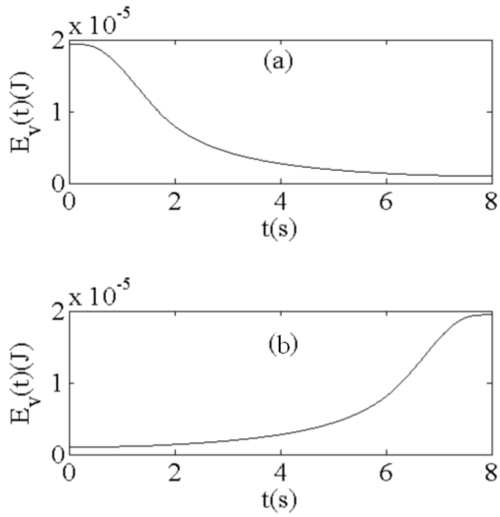

Fig. 8.
Total energy associated with the transverse vibration of the system during movement: (a) Downward movement; (b) Upward movement.

By contrast, in Fig. 7, we observe that vibration amplitudes of the rope increase with decreasing length of the rope during upward movement. This is due to the energy of flexible hoisting system transfers from the axial motion to the transverse vibration by leaving some mass out of the domain of effective length, i.e., the axially hoisting rope gains energy during upward movement, thus leading to an unstabilized transverse dynamic reponse, as shown in Fig. 8(b).

A possible physical interpretation of the result is as follows: during upward movement positive external work is required to maintain the prescribed axial motion which, in turn, brings about a convection of mass out of the domain of effective length. In the mean time, frequencies of the transverse vibration increase with decreasing length of the rope. This is due to the fact that the mass of the rope decrease and the stiffness of the rope is increase, i.e., the rope becomes somewhat "stiffer".

4.4. Forced vibraion responses. During movement of hoisting system, the flexible hoisting system is subjected to vibration caused by various sources of excitation. They include excitations due to the irregularities of the guiding system and rotational unbalance of the traction motor as well as environmental phenomena such as air current.

The system parameters are changing due to the time-varying length of the rope. The rate of variation of the length is, however, slow, and the vibrations represent waves in a slowly varying domain. Hence, the hoisting rope is essentially a nonstationary vibration system with slowly varying frequencies. Therefore, a passage through resonance may occur when one of the slowly varying frequencies coincides with the frequency of the extrinsic disturbing excitation at some critical time instant.

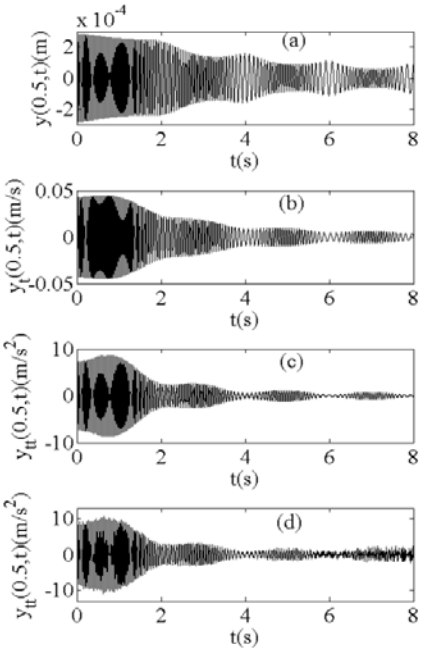

Fig. 9.
Forced vibration responses of the flexible hoisting rope at 0.5m above the car during downward movement: (a) Displacement curve; (b) Velocity curve; (c) Acceleration curve(simulation); (d) Acceleration curve(experiment).

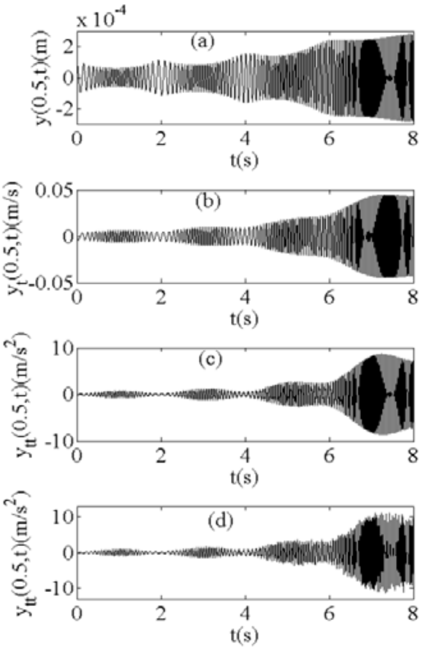

Fig. 10.
Forced vibration responses of the flexible hoisting rope at 0.5m above the car during downward movement: (a) Displacement curve; (b) Velocity curve; (c) Acceleration curve(simulation); (d) Acceleration curve(experiment).

Forced vibration responses for hoisting rope with extrinsic disturbing excitation are illustrated in Figs. 9(downward movement) and 10(upward movement). From Figs. 9 and 10, it can be seen that transient resonance occurs during movement of hoisting system. The amplitudes exhibit oscillatory behavior before the resonance, and near the resonance the amplitudes increase rapidly and decline afterwards due to damping, developing damped beat phenomena. This is due to one of time-varying frequencies of the hoisting rope coincides with the frequency of the extrinsic disturbing excitation during movement of hoisting system. It should be noted that the adverse dynamic response in hoisting system promote large oscillations in rope tension. The phenomenon cannot be ignored, as the high amplitude in the tension contributes directly to fatigue of rope. Fatigue often results in the hoisting ropes being discarded after lower working cycles. Therefore, suitable strategy can be sought to minimize the effects of adverse dynamic response of the system.

5. Conclusions.

The nonlinear dynamic characteristics for a flexible hoist rope with time-varying length considering coupling of axial movement and flexural deformation are analyzed in this paper. The flexible hoisting system is modeled as an axially moving string with time-varying length and a rigid body at its lower end. The governing equations are derived by using Leibnitz's rule and the Hamilton's principle. The Galerkin's method is used to truncate the infinite-dimensional partial differential equations into a set of nonlinear finite-dimensional ordinary differential equations with time-variant coefficients.

To validate the theoretical model, an experimental set-up of flexible hoisting system is built and some experiments are performed. By comparing the experimental results to the numerical simulation, a good agreement between the simulation and experiment is obtained, thus validating the mathematical model of flexible hoisting system. Based on the simulation and experiment, the following conclusions can be obtained:

1. The flexible hoisting rope with time-varying length experiences instability during upward movement, the natural frequences are increasing because of the reducing mass and the increasing stiffness of the rope, and the energy transforms from the axial movement into the flexible deformation.

By contrast, it is stable during downward movement, the natural frequences are decreasing because of the increasing mass and the reducing stiffness of the rope, and the energy coverts from the flexible deformation into the axial movement.

2. The flexible hoisting rope is a nonstationary oscillatory system with slowly varying frequencies. The transient resonance may occur when one of time-varying frequencies of the hoisting rope coincides with the frequency of the extrinsic disturbing excitation.

3. The proposed the theoretical model and analyses about the dynamic characteristics of flexible hoisting system in this paper will be helpful for the researchers to comprehend its dynamic behavior and develop the proper method to suppress the vibration in practice.

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Р Е З Ю М Е . Основні рівняння гнучкого підіймального тросу отримано застосуванням принципу Гамільтона. Проведено експерименти, результати яких добре узгоджуються з теоретичним передбаченням. Результати моделювання і експеримент показують, що гнучка підіймальна система розсіює енергію при спуску і накопичує енергію при підйомі. Далі досліджувався перехід гнучкої підіймальної системи через резонанс за умови періодичного зовнішнього збудження. Якщо довжина гнучкої підіймальної системи змінюється з часом, то власні частоти системи слабо змінюються і можуть спостерігатися перехідні резонанси, коли одна з частот співпадає з частотою зовнішнього збудження.

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