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FIRST BAIRE CLASS FUNCTIONS IN THE PLURI-FINE TOPOLOGY

Let $B_1(\Omega, \mathbb{R})$ be the first Baire class of real functions in the pluri-fine topology on an open set $\Omega \subseteq \mathbb{C}^n$ and let $H_1^*(\Omega, \mathbb{R})$ be the first functional Lebesgue class of real functions in the same topology. We prove the equality $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ and show that for every $f \in B_1(\Omega, \mathbb{R})$ there is a separately continuous function $g : \Omega^2 \rightarrow \mathbb{R}$ in the pluri-fine topology on Ω^2 such that f is the diagonal of g .

Keywords: plurisubharmonic function, first Baire class, separately continuous function, pluri-fine topology, first functional Lebesgue class.

*This paper is dedicated to Professor Vladimir Gutlyanskii
on the occasion of his 75-th anniversary.*

1. Introduction.

The first Baire class functions is a classical object for the studies in Real Analysis, General Topology and Descriptive Set Theory. There exist many interesting characterizations of these functions. Let us denote by I the closed interval $[0, 1]$.

Theorem 1.1. *The following conditions are equivalent for every $f : I \rightarrow I$.*

1. *The function f is a Baire one function.*
2. *There is a separately continuous function $g : I \times I \rightarrow I$ such that f is the diagonal of g .*
3. *Each nonvoid closed set $F \subseteq I$ contains a point x such that the restriction $f|_F$ is continuous at x .*
4. *The sets $f^{-1}(a, 1]$ and $f^{-1}[0, a)$ are F_σ for every $a \in I$.*
5. *For all $a, b \in I$ with $a < b$ and for every non-void subset $F \subseteq I$, the sets $f^{-1}[0, a]$ and $f^{-1}[b, 1]$ cannot be simultaneously dense in F .*

It is a classical result in the real function theory that the diagonals of separately continuous functions of n variables are exactly the $(n - 1)$ Baire class functions. See R. Baire [1] for the original proof in the case where $n = 2$, and H. Lebesgue [9, 10] and H. Hahn [6] for arbitrary $n \geq 2$. A proof of the equivalence of (1), (3), (4) and (5) in the situation of a metrizable strong Baire space can be found, for example, in [11, Theorem 2.12, p. 55]. The goal of our paper is to find similar characterizations of the first Baire class functions on the topological space (Ω, τ) , where Ω is an open subset of \mathbb{C}^n and τ is the pluri-fine topology on Ω . The pluri-fine topology τ is the coarser topology on Ω such that all plurisubharmonic functions on Ω are continuous. The topology τ was introduced by B. Fuglede in [5] as a basis for a fine analytic structure in \mathbb{C}^n . E. Bedford

and B. A. Taylor note in [2] that the pluri-fine topology is Baire and has the quasi-Lindelöf property. S. El. Marzguioui and J. Wiegierinck proved in [14] that τ is locally connected and, consequently, the connected components of open sets are open in τ (see also [15]). It should be noted that τ is not metrizable (see Corollary 1.8 below). Thus, it is not clear whether the above formulated characterizations of the first Baire class functions are valid for (Ω, τ) .

Let us recall some definitions.

Let X be an arbitrary nonvoid set. For integer $m \geq 2$ the set Δ_m of all m -tuples (x, \dots, x) , $x \in X$, is by definition, the *diagonal* of X^m . The mapping $d_m : X \rightarrow X^m$, $d_m(x) = (x, \dots, x)$, is called the *diagonal mapping* and, for every function $f : X^m \rightarrow Y$, the composition $f \circ d_m$,

$$X \ni x \longmapsto f(x, \dots, x) \in Y$$

is, by definition, the *diagonal* of f .

Let X and Y be topological spaces. A function $f : X \rightarrow Y$ is a *first Baire class function* if there exists a sequence $(f_n)_{n \in \mathbb{N}}$ of continuous functions $f_n : X \rightarrow Y$ such that the limit relation

$$f(x) = \lim_{n \rightarrow \infty} f_n(x) \tag{1}$$

holds for every $x \in X$. Similarly, for an integer number $m \geq 2$, a function $f : X \rightarrow Y$ belongs to the *m -Baire class functions*, if (1) holds with a sequence $(f_n)_{n \in \mathbb{N}}$ such that each of f_n is in a Baire class less than m . A function $f : X \rightarrow Y$ is a *first functional Lebesgue class function*, if for every open subset G of the space Y , the inverse image $f^{-1}(G)$ is a countable union of functionally closed subsets of X . We will denote by $B_1(X, Y)$ (by $H_1^*(X, Y)$) the set of first Baire (first functional Lebesgue) class functions from X to Y and by F_σ^* (G_δ^*) the set of all countable unions (countable intersections) of functionally closed (functionally open) subsets of X . Thus

$$\begin{aligned} (f \in H_1^*(X, Y)) &\Leftrightarrow (f^{-1}(G) \in F_\sigma^* \text{ for all open } G \subseteq Y) \\ &\Leftrightarrow (f^{-1}(F) \in G_\delta^* \text{ for all closed } F \subseteq Y). \end{aligned} \tag{2}$$

Recall that a subset A of a topological space X is *functionally closed*, if there is a continuous function $f : X \rightarrow I$ such that $A = f^{-1}(0)$.

Definition 1.2. Let μ be a topology on the Cartesian product $X = \prod_{i=1}^m X_i$ of nonvoid sets X_1, \dots, X_m , $m \geq 2$, and let Y be a topological space. A function $f : X \rightarrow Y$ is called *separately continuous* if, for each m -tuple $(x_1, \dots, x_m) \in X$, the restriction of f to any of the sets

$$\{(x, x_2, \dots, x_m) : x \in X_1\}, \{(x_1, x, \dots, x_m) : x \in X_2\}, \dots, \{(x_1, \dots, x_{m-1}, x) : x \in X_m\}$$

is continuous in the subspace topology generated by μ .

If $a = (a_1, a_2, \dots, a_n) \in \mathbb{C}^n$, $b = (b_1, b_2, \dots, b_n) \in \mathbb{C}^n$ and $z \in \mathbb{C}$, then we shall write $a + bz$ for the n -tuple $(a_1 + b_1z, a_2 + b_2z, \dots, a_n + b_nz)$.

Definition 1.3. Let \mathbb{C}^n , \mathbb{C} and $[-\infty, \infty)$ have the Euclidean topologies, $n \geq 1$ and let $\Omega \subseteq \mathbb{C}^n$ be a non-void open set. A function $f : \Omega \rightarrow [-\infty, \infty)$ is plurisubharmonic (psh) if f is upper semicontinuous and, for all $a, b \in \mathbb{C}^n$, the function

$$\mathbb{C} \ni z \mapsto f(a + bz) \in [-\infty, \infty)$$

is subharmonic or identically $-\infty$ on every component of the set

$$\{z \in \mathbb{C} : a + bz \in \Omega\}.$$

In what follows, τ denotes the pluri-fine topology on Ω , i.e., the coarsest topology in which all psh functions are continuous.

As was noted in [14], many results related to the classical fine topology which were introduced by H. Cartan are valid for the pluri-fine topology. For example, τ is Hausdorff and completely regular. It is well known that Cartan's fine topology is not metrizable and all compact sets are finite in this topology. The topology τ also has these properties.

Let $\pi_j : \Omega^m \rightarrow \Omega$ be the j -th projection of Ω^m on Ω , $j \in \{1, \dots, m\}$. We identify Ω^m with the corresponding subset of \mathbb{C}^{mn} and denote by τ_m the pluri-fine topology on Ω^m .

Lemma 1.4. All projections $\pi_j : (\Omega^m, \tau_m) \rightarrow (\Omega, \tau)$, $j = 1, \dots, m$, are continuous.

Proof. Let Y be a topological space. It follows from a general result on the continuity of the mappings to a topological space X with a topology generated by a family \mathfrak{F} of functions f on X (see [4, p. 31]), that $\psi : Y \rightarrow X$ is continuous if and only if the composition $f \circ \psi$ is continuous for every $f \in \mathfrak{F}$. Hence, we need to show that the functions

$$\Omega^m \xrightarrow{\pi_j} \Omega \xrightarrow{f} [-\infty, \infty) \quad (3)$$

are continuous in the topology τ_m for every psh function f . Note that all projections π_j are analytic. Consequently, in (3) we have a composition of an analytic function with a psh function. Since such compositions are psh (see, for example, [7, p. 228]), they are continuous by the definition of pluri-fine topology. \square

Substituting \mathbb{C} instead of Ω and n instead of m we obtain the following

Corollary 1.5. All projections $\pi_j : \mathbb{C}^n \rightarrow \mathbb{C}$, $j = 1, \dots, n$, are continuous mappings with respect to the pluri-fine topologies on \mathbb{C}^n and \mathbb{C} .

Proposition 1.6. Let Ω be a non-void open subset of \mathbb{C}^n and let A be a compact set in (Ω, τ) . Then A is finite, $|A| < \infty$.

Proof. If f is a psh function on \mathbb{C}^n , then the restriction $f|_{\Omega}$ is psh on Ω . Hence it is sufficient to show that $|A| < \infty$ for the case $\Omega = \mathbb{C}^n$. By Corollary 1.5 every projection π_j is continuous. Hence the sets $A_j = \pi_j(A)$, $j = 1, \dots, n$, are compact. As was mentioned above, every compact set in (\mathbb{C}, τ) is finite. Consequently, we have $|A_j| < \infty$, $j = 1, \dots, n$. These inequalities and $|A| \leq \prod_{j=1}^n |A_j|$ imply that A is finite. \square

Proposition 1.7. *Let Ω be a non-void open subset of \mathbb{C}^n . The pluri-fine topology τ is not first-countable for any $n \geq 1$.*

Proof. Suppose, contrary to our claim, that τ is first-countable. The topology τ is Hausdorff. Since (Ω, τ) is not discrete, Ω contains an accumulation point a which is the limit of a non-constant sequence $(a_k)_{k \in \mathbb{N}}$ of points of Ω . It is clear that the set

$$A = \{a\} \cup \left(\bigcup_{k=1}^{\infty} \{a_k\} \right)$$

is an infinite compact subset of Ω . The last statement contradicts Proposition 1.6. \square

Corollary 1.8. *The pluri-fine topology τ on a non-void open set $\Omega \subseteq \mathbb{C}^n$ is not metrizable for any integer $n \geq 1$.*

Proof. Since every metrizable topological space is first countable, the corollary follows from Proposition 1.7. \square

M. Brelot in [3] considers a fine topology generated by a cone of lower-semicontinuous functions of the form $f : X \rightarrow (-\infty, \infty]$. Every plurisuperharmonic function satisfies these conditions and such functions are just the negative of plurisubharmonic functions. Thus, the pluri-fine topology τ is an example of fine topologies studied in [3].

2. Separately continuous functions and the first Baire functions in the pluri-fine topology.

The following is a result from Mykhaylyuk's paper [17] (see also [16]).

Lemma 2.1. *Let X be a topological space and let X^m be a Cartesian product of $m \geq 2$ copies of X with the usual product topology. Then for every $(m - 1)$ -Baire class function $g : X \rightarrow \mathbb{R}$ there is a separately continuous function $f : X^m \rightarrow \mathbb{R}$ such that $f(x, \dots, x) = g(x)$ holds for every $x \in X$.*

Let us denote by t^m the Tychonoff topology (= product topology) on the product of m copies of the topological space (Ω, τ) . The topology t^m is the coarsest topology on Ω^m making all projections $\pi_j : \Omega^m \rightarrow \Omega$, $j = 1, \dots, m$, continuous. Lemma 2.1 directly implies the following.

Lemma 2.2. *Let $m \geq 2$ be an integer. For every $(m - 1)$ -Baire class function $g : \Omega \rightarrow \mathbb{R}$ in the pluri-fine topology τ there is a separately continuous function $f : \Omega^m \rightarrow \mathbb{R}$ in the Tychonoff topology t^m such that g is the diagonal of f .*

The following theorem gives a “pluri-fine” analog of the first implication from Theorem 1.1.

Theorem 2.3. *Let Ω be a non-void open subset of \mathbb{C}^n , $n \geq 1$ and let $m \geq 2$ be an integer. For every $(m - 1)$ Baire class function $g : \Omega \rightarrow \mathbb{R}$, in the pluri-fine topology τ , there is a separately continuous function $f : \Omega^m \rightarrow \mathbb{R}$, in the pluri-fine topology τ_m , such that*

$$g = f \circ d_m, \tag{4}$$

where d_m is the corresponding diagonal mapping.

Proof. By Lemma 2.2, it is sufficient to show that t^m is weaker than τ_m . From the definition of Tychonoff topology it follows at once that t^m is weaker than τ_m if and only

if all projections $\pi_j: \Omega^m \rightarrow \Omega$, $j \in \{1, \dots, m\}$, are continuous mappings on (Ω^m, τ_m) . The continuity of these projections follows from Lemma 1.4. \square

Proposition 2.4. *The equality*

$$H_1^*(X, \mathbb{R}) = B_1(X, \mathbb{R}) \quad (5)$$

holds for every topological space X .

Proof. Let X be a topological space and let Y be an arcwise connected, locally arcwise connected, metrizable space. Then every $f \in H_1^*(X, Y)$, with separable $f(X)$, belongs to $B_1(X, Y)$ (see [8]). Hence $H_1^*(X, \mathbb{R}) \subseteq B_1(X, \mathbb{R})$ holds.

It still remains to make sure that

$$H_1^*(X, \mathbb{R}) \supseteq B_1(X, \mathbb{R}) \quad (6)$$

is valid for every topological space X . The following is a simple modification of well known arguments.

Let $f \in B_1(X, \mathbb{R})$. Consider a sequence $(f_n)_{n \in \mathbb{N}}$ of continuous real valued functions on X such that the limit relation $f(x) = \lim_{n \rightarrow \infty} f_n(x)$ holds for every $x \in X$. Let $(\varepsilon_m)_{m \in \mathbb{N}}$ be a strictly decreasing sequence of positive real numbers with

$$\lim_{m \rightarrow \infty} \varepsilon_m = 0. \quad (7)$$

Let us prove the equality

$$f^{-1}(-\infty, a) = \bigcup_{m=1}^{\infty} \bigcup_{p=1}^{\infty} \left(\bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m) \right) \quad (8)$$

for every $a \in \mathbb{R}$. It is sufficient to show that for every $x \in f^{-1}(-\infty, a)$ there are $m, p \in \mathbb{N}$ such that

$$x \in \bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m). \quad (9)$$

Let $x \in f^{-1}(-\infty, a)$. Then we have $\lim_{n \rightarrow \infty} f_n(x) < a$. The last inequality and (8) imply $\lim_{n \rightarrow \infty} f_n(x) < a - \varepsilon_{m_1}$ for some m_1 . Consequently, there is $p \in \mathbb{N}$ such that $f_n(x) < a - \varepsilon_{m_1}$ for all $n \geq p$, that is

$$x \in \bigcap_{k=p}^{\infty} f_k^{-1}(-\infty, a - \varepsilon_m).$$

Since the sequence $(\varepsilon_m)_{m \in \mathbb{N}}$ is strictly decreasing, the inclusion

$$(-\infty, a - \varepsilon_m) \subseteq (-\infty, a - \varepsilon_{m+1}]$$

follows for every m . Hence (9) holds with $m = m_1 + 1$.

Note now that $f_k^{-1}(-\infty, a - \varepsilon_m]$ is functionally closed as a zero-set of the continuous function

$$g_{k,m,a}(x) := \min(\max(f(x) - f(a - \varepsilon_m); 0); 1).$$

Since each countable intersection of functionally closed sets is functionally closed [4, p. 42–43], equality (8) implies $f^{-1}(-\infty, a) \in F_\sigma^*$. Moreover, if $g = -f$ and $b = -a$, then $f^{-1}(a, \infty) = g^{-1}(-\infty, b)$ holds. Hence, the set $f^{-1}(a, \infty)$ belongs to F_σ^* .

We can now easily prove (6). Indeed, it is sufficient to show that $\{x : a < f(x) < b\}$ is a countable union of functionally closed sets for every $f \in B_1(X, \mathbb{R})$ and every interval $(a, b) \subseteq \mathbb{R}$. Using (8), we obtain

$$f^{-1}(a, b) = \left(\bigcup_{i=1}^{\infty} H_i \right) \cap \left(\bigcup_{i=1}^{\infty} F_i \right) = \bigcup_{i,j=1}^{\infty} (H_i \cap F_j), \quad (10)$$

where all H_i and F_j are functionally closed. It was mentioned above that the countable intersection of functionally closed sets is functionally closed. Hence, by (10), $f^{-1}(a, b) \in F_\sigma^*$, so that (6) follows. \square

Corollary 2.5. *The equality $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ holds if the non-void open set $\Omega \subseteq \mathbb{C}^n$ is endowed by the pluri-fine topology τ .*

This corollary and Theorem 2.3 imply the following result.

Theorem 2.6. *Let Ω be a non-void open subset of \mathbb{C}^n , $n \geq 1$, and let $g : \Omega \rightarrow \mathbb{R}$ be a first functional Lebesgue class function on (Ω, τ) . Then there is a separately continuous function $f : \Omega^2 \rightarrow \mathbb{R}$ in the pluri-fine topology τ_2 on Ω^2 such that g is the diagonal of f .*

The proof of the next proposition is a variant of Lukeš-Zajiček’s method from [12, 13].

Proposition 2.7. *Let X be a topological space. Then, for every $f : X \rightarrow \mathbb{R}$, the following conditions are equivalent.*

1. *The function f belongs to $B_1(X, \mathbb{R})$.*
2. *For each couple of real numbers a, b with $a < b$ there are $H_1, H_2 \in F_\sigma^*$ such that*

$$f^{-1}(a, +\infty) \supseteq H_1 \supseteq f^{-1}(b, +\infty), \quad (11)$$

$$f^{-1}(-\infty, b) \supseteq H_2 \supseteq f^{-1}(-\infty, a). \quad (12)$$

Proof. It suffices to show that $f \in B_1(X, \mathbb{R})$ if (11) and (12) hold. (The converse implication follows from (5) and (2).)

Using (6) and (10), it is easy to see that we need only to make sure the statements

$$f^{-1}(a, +\infty) \in F_\sigma^* \quad \text{and} \quad f^{-1}(-\infty, a) \in F_\sigma^*$$

for every $a \in \mathbb{R}$. Suppose (11) holds. Then, for every $m \in \mathbb{N}$, there is $H^m \in F_\sigma^*$ such that

$$f^{-1}\left(a + \frac{1}{m}, +\infty\right) \subseteq H^m \subseteq f^{-1}\left(a + \frac{1}{m+1}, +\infty\right).$$

Consequently,

$$\begin{aligned} f^{-1}(a, +\infty) &= \bigcup_{m=1}^{\infty} f^{-1}\left(a + \frac{1}{m}, +\infty\right) \subseteq \bigcup_{m=1}^{\infty} H^m \\ &\subseteq \bigcup_{m=1}^{\infty} f^{-1}\left(a + \frac{1}{m+1}, +\infty\right) = f^{-1}(a, +\infty). \end{aligned}$$

Thus,

$$f^{-1}(a, +\infty) = \bigcup_{m=1}^{\infty} H^m.$$

It implies $f^{-1}(a, +\infty) \in F_{\sigma}^*$, because every countable union of sets from F_{σ}^* belongs to F_{σ}^* .

Similarly, using (12), we can prove that $f^{-1}(-\infty, a) \in F_{\sigma}^*$. \square

In the following corollary we consider the classes $B_1(\Omega, \mathbb{R})$ and F_{σ}^* with respect to the pluri-fine topology τ on Ω .

Corollary 2.8. *Let Ω be a non-void open subset of \mathbb{C}^n , $n \geq 1$, and let f be a real valued function on Ω . Then f belongs to $B_1(\Omega, \mathbb{R})$ if and only if, for each couple $a, b \in \mathbb{R}$, with $a < b$, double inclusions (11) and (12) hold for some $H_1, H_2 \in F_{\sigma}^*$.*

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Функции первого класса Бэра в топологии, порожденной плорисубгармоническими функциями.

Пусть $B_1(\Omega, \mathbb{R})$ – множество функций первого класса Бэра в топологии, порожденной плорисубгармоническими функциями на открытом множестве $\Omega \subseteq \mathbb{C}^n$, и пусть $H_1^*(\Omega, \mathbb{R})$ – первый функциональный класс Лебега вещественнозначных функций в той же топологии. Мы доказываем равенство $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ и показываем, что для всякой $f \in B_1(\Omega, \mathbb{R})$ существует раздельно непрерывная функция $g : \Omega^2 \rightarrow \mathbb{R}$ в топологии, порожденной плорисубгармоническими функциями и такая, что f является диагональю g .

Ключевые слова: плорисубгармоническая функция, первый класс Бэра, раздельно непрерывная функция, порожденная плорисубгармоническими функциями топология, первый функциональный класс Лебега.

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Функції першого класу Бера у топології, що породжена плорисубгармонійними функціями.

Нехай $B_1(\Omega, \mathbb{R})$ – множина функцій першого класу Бера у топології, породженій плорисубгармонійними функціями на відкритій множині $\Omega \subseteq \mathbb{C}^n$ та нехай $H_1^*(\Omega, \mathbb{R})$ – перший функціональний клас Лебега дійсних функцій у тій же топології. Ми доводимо рівність $B_1(\Omega, \mathbb{R}) = H_1^*(\Omega, \mathbb{R})$ та показуємо, що для кожної $f \in B_1(\Omega, \mathbb{R})$ існує нарізно неперервна функція $g : \Omega^2 \rightarrow \mathbb{R}$ у топології, що породжена плорисубгармонійними функціями та така, що f є діагоналлю g .

Ключові слова: плорисубгармонійна функція, перший клас Бера, нарізно неперервна функція, породжена плорисубгармонійними функціями топологія, перший функціональний клас Лебега.

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