# Diffraction efficiency of H-PDLC film with comb-shaped electrodes

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A theoretical model of holographic polymer-dispersed liquid crystal film with comb-shaped electrodes has been constructed. The model combines anisotropic coupled-wave theory, Monte-Carlo simulations for director profile within a liquid crystal droplet and statistical averaging with the orientational distribution function for droplet symmetry axes. Influence of the electric field applied along the grating vector on the diffraction efficiency has been studied. The results obtained indicate a way to improve the grating characteristics.

Построена теоретическая модель голографической полимер-диспергированной жид-кокристаллической (Г-ПДЖК) пленки с гребенчатыми электродами. В модели объединяется анизотропная теория связанных волн, компьютерное моделирование распределения директора внутри жидкокристаллической капли методом Монте-Карло и статистическое усреднение с функцией распределения осей капель. Исследовано влияние электрического поля, приложенного вдоль вектора решетки, на дифракционную эффективность. Полученные результаты указывают способ улучшения характеристик решетки.

Holographic polymer-dispersed liquid crystals (H-PDLCs) are of interest due to a number of potential applications thereof, include reflecting flat-panel displays, optical connectors, diffraction lenses with tunable focal length, optical data storage, and image capture devices. H-PDLC materials are formed by exposing a light-sensitive homogeneous monomer and LC mixture by an interference pattern. The monomer amount in the illuminated areas becomes reduced due to polymerization and monomers from the dark areas diffuse into the bright ones. The result of the writing process is a grating formed due to a periodic distribution of liquid crystal (LC) nano sized domains embedded in a solid polymer matrix. The main optical properties defined by this periodic distribution can be manipulated easily using an external electric field. General requirements for H-PDLC transmission gratings are high diffraction efficiency, high angular

selectivity, low driving voltages, and fast switching. Many research groups are engaged in optimization and improving of those characteristics. Embedding of comb electrodes into the film is one way to control the refractive index modulation and diffraction efficiency for both light polarization types: *p*-polarization and *s*-polarization. The analysis performed in this work is based on direct calculation of the director profile within the LC droplets. Below, the model studied is presented.

To develop the model, some assumptions are to be made on the LC/polymer grating composite. The grating is assumed to contain regions of high liquid-crystal droplet concentration separated by solid polymer regions. In this work, a transmission H-PDLC grating with comb-shaped electrodes is considered. Fig. 1 presents the problem geometry. The semi-transparent surfaces are the comb electrodes. The relative permittivity

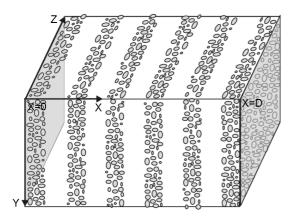


Fig. 1. Schematic representation of H-PDLC grating. The comb electrodes ares shown as semi-transparent surfaces at x=0 and x=D.

tensor is assumed to vary along the x axis according to cosine law:

$$\hat{\varepsilon} = \hat{\varepsilon}^0 + \hat{\varepsilon}^1 \cos(Kx), \tag{1}$$

where  $\hat{\epsilon}^0$  is the average relative permittivity tensor; K, the grating wave vector;  $\hat{\epsilon}^1$ , the relative permittivity modulation tensor having the form

$$\varepsilon_{\perp,\parallel}' = \frac{2f_c}{\pi} \sin(\alpha \pi) (\varepsilon_{\perp,\parallel}^{LC} - \varepsilon_{Pol}), \qquad (2)$$

where  $f_c$  is the volume fraction of separated LC phase in a channel;  $\epsilon_{Pol}$ , the polymer dielectric permittivity;  $\alpha$ , the fraction of a grating period  $\Lambda$  occupied by the PDLC channel [1]. The droplet dielectric permittivity  $\epsilon_L^{LC}(\epsilon_L^{LC})$  for light polarized parallel

(perpendicular) to the grating vector is a function of  $n_i$ ,  $n_i$  director configuration:

$$\langle \hat{\varepsilon}^{LC} \rangle = \frac{1}{V} \int \hat{\varepsilon}^{LC}(\mathbf{r}, \mathbf{E}) d\mathbf{r},$$
 (3)

where  $\varepsilon_{ij}(\mathbf{E}) = \varepsilon_{\perp} \delta_{ij} + (\varepsilon_{\parallel} - \varepsilon_{\perp}) n_i(\mathbf{E}) n_j(\mathbf{E})$ . The

LC director profile in each LC droplet is inhomogeneous and controlled by the balance of elastic energy, cohesion energy at the droplet surface, and the external electric field strength.

LC droplets form nanoscale domains in H-PDLC. Detailed studies by scanning electron microscopy have revealed that these domains can be roughly ellipsoidal but are irregularly shaped often enough [2]. Sutherland et al. [1] developed a droplet model based on Kogelnik's [3] coupled wave theory for Bragg gratings and Wu [4] model of droplet axis reorientation to explain the

switching curves of H-PDLC gratings. Instead of treating the LC droplets as an inhomogeneous medium, they assume a uniform droplet with refractive index dependent on the angle between the LC symmetry axis and the applied field.

We have simulated numerically the director distribution inside an ellipsoidal LC droplet to find the relative dielectric permittivity tensor of LC. The model used is the simplest model of nematic LC, which describes orientational interaction of molecules is the Lebwohl-Lasher lattice model [5], where the particles are treated as interaction sites ("spins") with continuously varying orientation but fixed positions in the grating. That model has been described in detail in our previous works [6]. There are several other models that describe the LC properties more realistically, but those are much more complicated.

There are several theoretical approaches to consideration of the diffraction from a birefringent grating. Recently, Montemezzani and Zgonik [7] have modified the Kogelnik coupled-wave theory to involve the optical anisotropy effects in the bulk diffraction gratings prepared using birefringent materials. We have used the results of this theory to calculate the diffraction efficiency for the transmission gratings. The diffraction efficiency of an unslanted bulk grating prepared using a birefringent optical material is determines as

$$\eta = \frac{\sin^2 \sqrt{v^2 + \xi^2}}{1 + v^2 / \xi^2} \tag{4}$$

with coupling and detuning parameters defined respectively as

$$v^{2} = \frac{k_{0}^{2} A_{r}^{2} d^{2}}{16n_{1}n_{2}g_{1}g_{2}\cos\theta_{1}\cos\theta_{2}},$$
 (5)

$$\xi^2 = \frac{\Delta k_r^2 d^2}{4}.\tag{6}$$

In these expressions,  $k_0$  is the incident wave vector in vacuum;  $A_r$ , a coupling constant dependent on polarization; d, the sample thickness;  $n_1$  and  $n_2$ , the unperturbed refractive indices for the incident and diffracted wave, respectively. The walk-off parameters  $g_{1,2}$  are cosines of angles between the energy propagation direction and wave vector directions in the incident and diffracted beams, and  $\theta_1$ ,  $\theta_2$  are the angles between the normal to the H-PDLC surface and the Poynting vectors for incident and

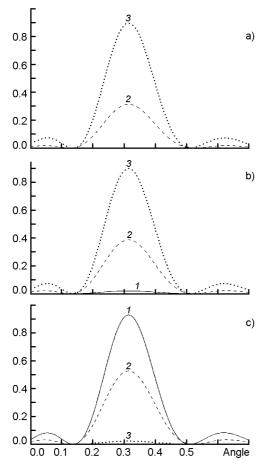


Fig. 2. Diffraction efficiency for Gaussian distribution of the prolate LC droplets. Fig. 2a), 2c) show angular selectivity of diffraction efficiency at different electric field strengths for dispersion  $\sigma=0.1$ , for p-polarized reading beam and s-polarized reading beam, respectively. Fig. 2b) shows the same but for  $\sigma=0.3$ . Curves I – means critical electric field E=1; 2 – means zero electric field E=0.5; 3 – means zero electric field E=0.

diffracted waves. Detailed expressions for the parameters to calculate the diffraction efficiency can be found in [7].

In a previous work, we have studied the diffraction properties of H-PDLC films containing ellipsoidal droplets under electric field perpendicular to the sample surfaces (z-axis). Here, we consider a similar film, the difference being that the electric field is applied by comb electrodes along vector grating (x-axis in Fig. 1), and the axes of rotation symmetry in the droplets are statistically skewed perpendicular to the grating vector. The form of distribution function selected in this work is the Gaussian one (7)

$$p(u) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{u}{2\sigma^2}],\tag{7}$$

where u is the cosine of the angle between vector perpendicular to the grating vector (along y in laboratory system of coordinates) and the droplet rotational symmetry axis,  $\sigma$  is the dispersion.

Let an H-PDLC cell be considered containing bipolar liquid crystal droplets. The shape of the droplets can vary from oblate spheroid to prolate one. The LC director profile in each LC droplet depends on the balance of elastic energy associated with director inhomogeneity within the droplet, the cohesion energy at the droplet surface, the droplet size and shape, and the magnitude of the external electric field. We also took into account that the electric field inside an ellipsoidal droplet depends strongly on droplet shape. To find the electric field inside the droplet, we have used the solution of a classical problem of electric field determination inside a dielectric ellipsoid placed in a uniform electric field [8]. We calculated the droplet dielectric tensor in "droplet" system of coordinates (axis y corresponds to the rotational symmetry one), then we have returned to the laboratory system of coordinates and calculate the average tensor elements for the droplet, using Gaussian distribution (7). After averaging over the orientational distribution function, we have obtained a diagonal tensor for relative dielectric permittivity. Then, making use of the Montmezzani theory [7] described above, we have obtained dependences of diffraction efficiency on the angle of incidence for different values of the applied field.

For numerical calculations, the following material parameters have been selected: the polymer dielectric permittivity  $\epsilon_{Pol}=2.34$ , the  $\epsilon_{\parallel}^{LC}=2.82$  dielectric permittivity of LC and  $\epsilon_{\parallel}^{LC}=2.36$ , the

grating period  $\Lambda=1~\mu m$ , the film thickness  $d=8~\mu m$ . All the parameters are important variables in determining the performance of H-PDLC materials. Diffraction properties were studied for the probing light wavelength 0.633  $\mu m$ . Sutherland [1] used the same parameters to study theoretically the transmission H-PDLC grating.

We have considered the behavior of the film diffraction efficiency under applied field in the case of increasing dispersion  $\sigma$ . The E value is measured in dimensionless units. E=1 corresponds to applied electric field value of about 25 V/ $\mu$ m. Calculation of

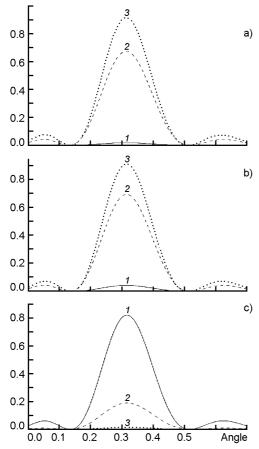


Fig. 3. The same as Fig. 2 for oblate droplet.

the diffraction efficiency has been calculated in the case of prolate LC droplets for *p*-polarized and *s*-polarized incident beams (see Fig. 2).

Similar results for oblate LC droplets are shown in Fig. 3.

The diffraction efficiency of s-polarized beam is defined by modulation of  $\epsilon_{\perp}^{1}$  component of relative permittivity tensor which depends

on  $\varepsilon_{\perp}^{LC}$  (Eq. 2). As to p-polarized beam, the diffraction efficiency is defined by modulation of  $\varepsilon_{\parallel}^{1}$  component of relative-permittivity tensor which depends on  $\varepsilon_{\perp}^{LC}$  (Eq. 2). Components of relative permittivity tensor  $\varepsilon_{\perp}^{LC}$  and  $\varepsilon_{\parallel}^{LC}$  are different, this explains different behavior of s-polarized and p-polarized beams.

Thus, the polarization and switching properties of transmission H-PDLC gratings have been studied theoretically. By applying of electric field along grating vector, the balance between diffraction efficiencies of p-polarized and s-polarized reading beams can be regulated. Increasing dispersion  $\sigma$ results in an increase of the diffraction efficiency. The external electric field of critical strength reorients the LC molecules inside the droplets along the field, so the difbetween director orientations disappears that, in turn, results in equal diffraction efficiency values for films containing prolate and oblate droplets. These effects can be used to improve the electrooptical properties of H-PDLC films.

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## Дифракційна ефективність голографічної полімер-диспергованої рідкокристалічної плівки з гребінчастими електродами

#### В.Кубицький, В.Решетняк, Т.Галстян

Побудовано теоретичну модель голографічної полімер-диспергованої рідкокристалічної (Г-ПДРК) плівки з гребінчастими електродами. У моделі поєднується анізотропна теорія зв'язаних хвиль, комп'ютерне моделювання розподілу директора всередині рідкокристалічної краплі методом Монте-Карло та статистичне усереднення з функцією розподілу вісей симетрії крапель. Досліджено вплив електричного поля, прикладеного вздовж вектора гратки, на дифракційну ефективність. Отримані результати вказують шлях покращення характеристик гратки.