Functional Materials 11, No.3 (2004)

Normal modes for the vortex state magnetic dots

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Submicron permalloy magnetic dots have a vortex ground state, and application of an in-plane magnetic pulse will result in precession of the vortex about the dot axis at a frequency in the sub GHz range. The precession frequency of this mode is calculated using a perturbation technique based on vortex-magnon scattering including the magnetostatic interaction. These calculations show that the frequencies vary between 0.2 and 0.8 GHz for 60 nm disks or radii between 250 and 1000 nm, which agrees with recent experimental data. There is also a higher frequency mode between 5 and 7 GHz.

Субмикронные пермаллоевые магнитные точки имеют вихревое основное состояние, и приложение магнитного импульса в плоскости точки приводит к прецессии вихря вокруг оси точки с частотой в субгигагерцевом диапазоне. Частота этой прецессии вычислена с использованием теории возмущений, базирующейся на теории рассеяния магнонов на вихре, включающей магнитостатическое взаимодействие. Расчет показывает, что частоты изменяются в пределах от 0.2 до 0.8 GHz для 60 нм дисков с радиусом между 250 и 1000 нм, что согласуется с последними экспериментальными данными. Существует также мода с более высокой частотой, лежащей между 5 и 7 GHz.

Arrays of magnetic dots constructed from soft magnetic materials such as permalloy have applications in the area of high-density magnetic storage. For this reason, many experimental investigations have been done investigating both the static and dynamic properties of isolated dots [1, 2] as well as arrays [3, 4] of dots. One of the most interesting properties of the cylindrical dot from a theoretical perspective is the presence of a vortex ground state when the dot radius is in the submicron range owing to competition between the exchange and the magnetostatic interactions. In this article the dynamic properties of a single dot in the vortex state are investigated by calculating the magnon normal mode frequencies with the magnetostatic interaction as a perturbation.

The particular case that we are interested in is vortex precession that arises from an initial displacement of the vortex core from the dot center by a magnetic field pulse. It is known that an in-plane magnetic field pulse will move the vortex in a direction perpendicular to the applied field, and when the field is switched off vortex precession results. This effect has been observed [5] by making time-resolved Kerr microscopy measurements on single permalloy dots of different radii to obtain the precession frequency. Previous theoretical calculations [6, 7] of this precession frequency have been based on three different models that have included both the exchange interaction as well as the magnetostatic or dipolar interaction. The magnetostatic interaction in the vortex-state dot arises from effective magnetostatic volume and surface charges, which are from the divergence of the magnetization and the components of the magnetization normal to the dot surfaces, respectively. In the vortex ground state the only contribution to the effective magnetostatic charge comes from the small out-of-plane magnetization at the vortex core, which is equivalent to easy-plane anisotropy. Therefore, the dot in this ground state is equivalent to the two-dimensional (2D) easy-plane ferromagnet without the magnetostatic interaction, and the consideration of vortex dynamics will be a perturbation on this system. Previous models include the rigid vortex model and the pole-free model. For the rigid vortex model it is clear that vortex motion will result in an effective magnetostatic surface charge at the dot edge, which will increase the system energy resulting in oscillation. On the other hand, the pole-free model has no additional magnetostatic charge from motion of the vortex core, but an increase in exchange energy leads to a vortex core oscillation. It has recently been shown using micromagnetic simulations that the pole-free model gives a better estimate of the magnon frequency. A more recent technique uses the vortex-magnon interaction [7] with the magnetostatic interaction arising from surface charges only with the frequency obtained from an effective boundary condition at the dot edge. In the following the precessional frequency of the vortex-state dot is calculated the full magnetostatic interaction using all of the contributions to the magnetostatic charge.

Considering small oscillations of the magnetization about the vortex ground state, where the magnetization is expressed as $\mathbf{M} = M_s$ (sinθ cosφ, sinθ sinφ, cosθ) for polar and azimuthal angles θ and φ, and these can be written in terms of polar coordinates (r, χ) in the dot plane as

$$\theta = \theta_0(r), \quad \varphi = \chi + \pi/2. \tag{1}$$

For 2D easy-plane ferromagnets this magnetization has the general form such that $\theta_0(r) \to \pi/2$ exponentially for large r and $\sin\theta_0 \to 0$ at the origin eliminating the singularity and lowering the energy. First it is assumed that the aspect ratio, L/R is small, where L and R are the dot thickness and radius. Then the magnetization is uniform along the dot axis, and in the continuum approximation the two contributions to the energy can be written as

$$W = \frac{L}{2}\int\int\left[(A/M_s^2)(\nabla\mathbf{M})^2 - \mathbf{M}\mathbf{H}_m\right]d^2x. \tag{2}$$

The first term is the contribution from the exchange interaction, which is short-range and local in nature. The second term contains the nonlocal magnetostatic field, $\mathbf{H}_m$ obtained from the potential, $\mathbf{H}_m = -\nabla\Phi$. The sources of this field are both volume charges arising from $\nabla\cdot\mathbf{M}$ and surface charges from the normal component of $\mathbf{M}$ at the surface. For the ground state magnetization given by Eq.(1) the only source of magnetostatic charge is the surface M_z at the two dot faces close to the dot axis giving the energy density, $-\mathbf{M}\cdot\mathbf{H}_m = 2\pi M_z^2$ corresponding to the easy-plane ferromagnet, which is local in nature. Then, for the considerations of excitations for $L << R$ the only other contributions to the magnetostatic potential are from the volume and edge contributions,

$$\Phi_v = \int\frac{\nabla\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}r'dr'd\chi'dz', \tag{3}$$
$$\Phi_s = -R\int\frac{\mathbf{r}\cdot\mathbf{M}(R)}{|\mathbf{r}-R\hat{r}|}d\chi'dz',$$

where $(\mathbf{r}-\mathbf{r}')^2 = r^2 + r'^2 - 2rr'\cos(\chi-\chi') + (z-z')^2$, and the z direction is along the dot axis.

Now consider small deviations from the static vortex solutions having the symmetry of translations of the form

$$\theta = \theta_0(r) + f(r)\cos(\chi + \omega t) \text{ and} \tag{4}$$
$$\varphi = \chi + \pi/2 + \frac{1}{\sin\theta_0}g(r)\sin(\chi + \omega t),$$

where $\theta_0(r)$ and $\chi + \pi/2$ are the polar and azimuthal angles of the static vortex magnetization, and the last terms are small, time-dependent corrections. Linearizing the Landau-Lifshitz equation using Eq.(1) results in the set of coupled equations

$$\Omega f = \hat{H}_1 g + \hat{V}f + \hat{U}_1, \tag{5a}$$

$$\Omega g = \hat{H}_2 f + \hat{V}g + U_2, \tag{5b}$$

where $\Omega = \omega/4\pi\gamma M_s$, and the operators are

$$\hat{H}_1 = -l_0^2\nabla^2 + l_0^2\frac{\nabla^2(\sin^2\theta_0)}{\sin\theta_0}, \tag{6}$$
$$\hat{H}_2 = -l_0^2\nabla^2 + \left(\frac{l_0^2}{r^2} - 1\right)\cos 2\theta_0,\ \hat{V} = \frac{2l_0^2\cos\theta_0}{r^2}$$

with the exchange length given by $l_0 = \sqrt{A/4\pi M_s^2}$. The local part of the magnetostatic field is contained in the Schroedinger-like operators, $\hat{H}_1$ and $\hat{H}_2$ with the edge and volume magnetostatic effects included in the integral operators,

$$\hat{U}_1 = -\frac{1}{4\pi M_s}\frac{\partial\Phi}{\partial r} \tag{7}$$

$$\hat{U}_2 = \frac{\cos\theta_0}{4\pi M_s r}\Phi.$$

For this mode the solutions [8] of Eqs. (5a,b) without the magnetostatic term are known for any boundary condition. In general, far from the vortex core the magnon mode solutions are of the form $g(k,r) = J_1(kr) + \sigma_1(kl_0)\cdot Y_{1m}(kr)$, where k is the wavenumber and $\sigma_1(kr) = \pi k l_0/4$ is the scattering amplitude. It has previously been shown [7] that the edge contribution to the magnetostatic energy results in a concentration of magnetostatic charge at the dot edge leading to an effective boundary condition

$$R\frac{dg}{dr}\bigg|_{r=R} + \Lambda g(R) = 0, \tag{8}$$

where $\Lambda = RL\ln(4R/L)/(2\pi l_0^2)$. Therefore, the only remaining nonlocal part of the magnetostatic interaction is from the volume contribution. Without the volume contribution the scattering problem has already been solved, the solutions g_0 and f_0 are known, and the wavenumber is determined by the effective boundary condition. Next a perturbation technique is developed to obtain the frequency in terms of the nonlocal magnetostatic operators with $\Phi_v/4\pi M_s \approx L/R$ being a small parameter. Then combining Eqs.(5a,5b) the following expression for the frequency can easily be obtained

$$2\Omega\langle f_0 g_0\rangle = \langle f_0\hat{U}_1 + g_0\hat{U}_2\rangle, \tag{9}$$

where $\hat{U}_1$ and $\hat{U}_2$ contain only the volume contribution, and the brackets indicate the integration, $\langle\ldots\rangle = 2\pi\int_0^R \ldots r dr$. Integration by parts gives the simple expression for the frequency

$$2\Omega\int_0^R f_0 g_0 r dr = \frac{1}{4\pi}\int_0^R dr\int_0^R dr'\rho(r)S(r,r')\rho(r'), \tag{10}$$

where $\rho(r) = \cos\theta_0 f_0 + \frac{d}{dr}(rg_0)$. The function S is obtained from the magnetostatic potential, which after integration over z' for the case when $L < R$ is approximately

$$S(r,r') = L\int_0^{2\pi}\frac{\cos\alpha}{\sqrt{r^2 + r'^2 + L^2/4 - 2rr'\cos\alpha}}d\alpha, \tag{11}$$

which is just a combination of elliptic integrals or modulus $\kappa = 4rr'/[(r + r')^2 + L^2/4]$. Next it is necessary to know the form of the zeroth order functions to use in Eq.(10) and obtain the frequencies for any modes. We begin with the solution [8] of Eqs.(5a,b) without the magnetostatic interaction which is

$$g_0 = \left(\frac{2}{l_0}J_1(kr) + \frac{\pi k}{2}Y_1(kr)\right)\sin\theta_0 \tag{12}$$
$$\text{and } f_0 = \theta_{0r} + \Omega g_0.$$

Also the function S has the form

$$S(r,r') = \frac{4L}{\sqrt{(r + r')^2 + L^2/4}}\left[\frac{2}{\kappa^2}(K(\kappa) - E(\kappa)) - K(\kappa)\right], \tag{13}$$

where K and E are elliptic integrals of the first and second kind.

Using the effective boundary condition given by Eq.(8) the smallest solution is $k_1 \approx l_0/R^2$, which is used in the numerical integration of Eq.(10) to determine the frequencies of this mode at various dot radii. It is also remarked that the condition $kr << 1$ is valid for all r resulting in $f_0 \approx \theta_{0r}$ and $g_0 \approx -\sin\theta_0/r$, and since $\Omega << 1$ it is sufficient to include only the term that is linear in Ω from Eq.(10) giving the following expression for the frequency

$$2\Omega = \frac{1}{4\pi}\int_0^R dr\int_0^R \{dr'\rho(r)S(r,r')\rho(r'). \tag{14}$$

Numerical integration of Eqs.(13,14) with the k values from Eq.(12) now give the R dependence of the frequency for permalloy dots with $l_0 = 4.8$ nm and $4\pi\gamma M_s = 30$ GHz. These calculated frequencies for $L = 60$ nm dots are indicated by the solid curve in Fig. 1, and experimental data from Ref.[5] for of radii 250, 500, and 1000 nm are also included. Notice also that the measured radial dependence of the frequency agrees very well with the calculated frequencies.

There are also higher frequency $m = 1$ modes corresponding to the next zero of Eq.(12), that is approximately $k_2 \approx j_2/R$, where j_2 is the first root of the $J_1(kR)$ Bessel function. Numerical solution of the ef-

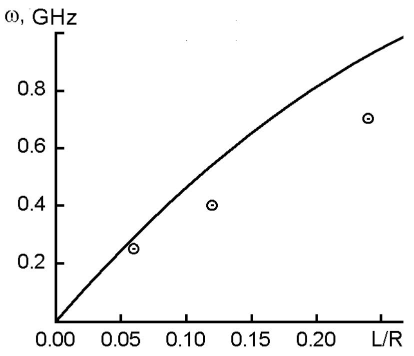

Fig. 1. Frequency of the translation mode versus L/R for $L = 60$ nm calculated from Eq.(14), solid curve. Experimental data from Ref.[5], symbols.

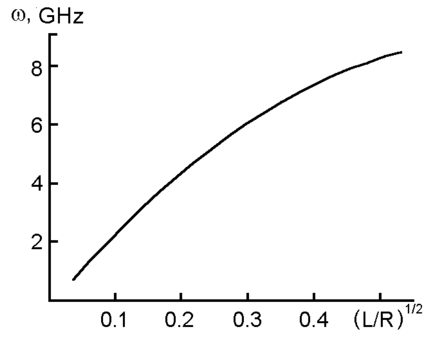

Fig. 2. Frequency of the higher translational mode versus $\sqrt{L/R}$ calculated from Eq.(10).

fective boundary condition gives the wavenumber for this mode, and these values of k are used in Eq.(10) to obtain the frequency of this mode. In this frequency range $f_0 \approx \Omega g_0$ so the quadratic term on the left hand side is dominant and the linear term is neglected. In this case the frequency is approximately proportional to $\sqrt{L/R}$, so the frequency is plotted in Fig. 2 as a function of this quantity. Both of these $m = 1$ modes correspond to vortex displacement with this higher frequency superimposed on the lowest frequency precession mode.

In conclusion, vortex-magnon dynamics including the full magnetostatic interaction without additional model assumptions gives dot radius-dependent frequencies that agree very well with experimental observations. Moreover, the generality of this method will allow the determination of frequencies of other modes.

Acknowledgement. This research was supported by the National Science Foundation Grants number DMR-9972507 and DMR-9974273.

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Нормальні моди для магнітних точок у вихровому стані

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Субмікронні пермаллоєві магнітні точки мають вихровий основний стан, і додаток магнітного імпульсу в площині точки приводить до прецесії вихря навколо вісі точки з частотою у субгігагерцевому діапазоні. Частоту цієї прецесії обчислено з використанням теорії збурювань, що базується на теорії розсіювання магнонов на вихрі, що включає магнітостатичну взаємодію. Розрахунок показує, що частоти змінюються у межах від 0.2 до 0.8 GHz для 60 нм дисків з радіусом між 250 і 1000 нм, що узгоджується з останніми експериментальними даними. Існує також мода з більш високою частотою, що лежить між 5 і 7 GHz.