

## Dynamic transformation of domain wall structure in ortho-ferrites

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The stationary dynamics of domain walls in rare-earth ortho-ferrites has been studied theoretically taking into account a possibility of its structure transformation near the sound speed. The study proceeds from the fact that the total energy of the moving domain wall includes a magnetic component and a magnetostrictive one.

Теоретически исследована стационарная динамика доменной границы (ДГ) в редкоземельных ортоферритах с учетом возможности перестройки ее структуры вблизи скорости звука. Изучение основывается на том, что полная энергия движущейся ДГ складывается из магнитной и магнитострикционной составляющих.

Rare-earth ortho-ferrites (REOF)  $RFeO_3$  (where R is a rare-earth ion) of the crystal symmetry group  $D_{2h}^{16}$  are non-collinear antiferromagnetics with a weak ferromagnetism (WFM) [1]. The great interest in investigation of that WFM class is due, along with other circumstances, to high motion speeds of domain walls (DW) therein (exceeding by several times the sound wave propagation speed), that are record among magnetics studied to date [2]. The DW structures can be subdivided into two types, namely, those with and without ferromagnetism vector  $\mathbf{m}$  turning [1, 3]. It is just the structure with simultaneous turn of the ferro- and antiferromagnetism vectors  $\mathbf{m}$  and  $\mathbf{l}$ , respectively (the (ac) type) that answers to the 1st type, while that where the  $\mathbf{l}$  vector is turned and the  $\mathbf{m}$  one changes only in its module (the (ab) type), to the 2nd one. The realization of one DW type or another as well as the turn planes of the  $\mathbf{m}$  and  $\mathbf{l}$  vectors therein are defined by the signs of the ortho-ferrite anisotropy constants and by relations between those [3]. Under certain conditions, one DW type may transform itself into another one, as is observed in dysprosium ortho-ferrite  $DyFeO_3$  at  $T = 150^\circ K$  [4].

Experimental studies of dynamics of homogeneous domain walls in REOF has revealed singularities in the dependence of

the DW stationary motion near the sound speed [5–9]. This is associated with a sharp increasing of the crystal strain near the sound speed. Two possible situations have been well studied theoretically [10]. First, a speed band gap may appear due to renormalization of the magnetic anisotropy constant. Second, an indefinite speed dependence on the field as well as the sections of negative differential mobility may arise. The theory describes qualitatively the observed supersonic dynamics of DW in REOF. A substantial drawback of the theory, however, consists in that the dynamic changes in the DW structure are neglected, although this assumption may be valid enough only under a strong dissipation in the elastic subsystem. In this case, the dynamic renormalization of the magnetic anisotropy constants is small enough to provide the DW structure transformation. It has been shown in [11] that there is another opportunity which may give rise to that phenomenon. This opportunity is associated with the fact that a DW has not only a magnetic mass but also a magnetostrictive one. This work is aimed just at theoretical study of the DW stationary dynamics in a REOF taking into account the possibility of its structure transformation near the sound speed. The study is based on the fact that

the total energy of a moving DW includes a magnetic component and a magnetostrictive one.

For simplicity sake, the REOF will be considered in the frame of two-sublattice model. Its state is convenient to describe by normalized vectors  $\mathbf{m}$  and  $\mathbf{l}$ :

$$\mathbf{m} = \frac{1}{2M_0}(\mathbf{M}_1 + \mathbf{M}_2), \quad \mathbf{l} = \frac{1}{2M_0}(\mathbf{M}_1 - \mathbf{M}_2), \quad (1)$$

where  $\mathbf{M}_1$ ,  $\mathbf{M}_2$  are magnetization vectors of the sublattices;  $M_0$ , the sublattice equilibrium magnetic moment. The Cartesian coordinate axes  $x$ ,  $y$ ,  $z$  are assumed to be directed along the crystallographic axes  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ , respectively. The crystal elastic strain will be characterized by the displacement vector  $\mathbf{u}$ . Minimizing the system thermodynamic potential  $\Phi(\mathbf{m}, \mathbf{l}, \mathbf{u})$  with respect to  $\mathbf{m}$  and taking into account  $\mathbf{ml} = 0$ ,  $l^2 = 1 - m^2 \approx 1$ ,  $\Phi$  can be presented as

$$\Phi = \Phi_m + \Phi_y + \Phi_{my}, \quad (2)$$

$$\Phi_m = \frac{1}{2}A(\nabla\mathbf{l})^2 - \frac{1}{2}\chi_{\perp}[H^2 - (\mathbf{H}\mathbf{l})^2] - m_z^0 H_z l_z - m_x^0 H_x l_x + \frac{1}{2}K_{ac}l_z^2 + \frac{1}{2}K_{ab}l_y^2, \quad (2a)$$

$$\Phi_{MY} = \delta_{XX}\epsilon_{XX} + \delta_{YY}\epsilon_{YY} + \delta_{ZZ}\epsilon_{ZZ} + 2\delta_{XY}\epsilon_{XY} + 2\delta_{XZ}\epsilon_{XZ} + 2\delta_{YZ}\epsilon_{YZ}, \quad (2b)$$

$$\Phi_y = \frac{1}{2}c_1\epsilon_{XX}^2 + \frac{1}{2}c_2\epsilon_{YY}^2 + \frac{1}{2}c_3\epsilon_{ZZ}^2 + c_4\epsilon_{XX}\epsilon_{YY} + c_5\epsilon_{XX}\epsilon_{ZZ} + c_6\epsilon_{ZZ}\epsilon_{YY} + 2c_7\epsilon_{XY}^2 + 2c_8\epsilon_{XZ}^2 + 2c_9\epsilon_{YZ}^2, \quad (2c)$$

where

$$\begin{aligned} \delta_{XX} &= \delta_1(l_X^2 - l_Z^2) + \delta_2(l_Y^2 - l_Z^2), \\ \delta_{YY} &= \delta_3(l_X^2 - l_Z^2) + \delta_4(l_Y^2 - l_Z^2), \\ \delta_{ZZ} &= \delta_5(l_X^2 - l_Z^2) + \delta_6(l_Y^2 - l_Z^2), \\ \delta_{XY} &= \delta_7 l_X l_Y, \quad \delta_{XZ} = \delta_8 l_X l_Z, \quad \delta_{YZ} = \delta_9 l_Z l_Y, \\ \epsilon_{ik} &= \frac{1}{2} \left( \frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i} \right), \end{aligned} \quad (3)$$

$A$  is the inhomogeneous exchange constant;  $\chi_{\perp} = M_0/2H_E$ , transversal susceptibility;  $m_z^0$  and  $m_x^0$ , the weak ferromagnetic

moment components;  $K_{ac}$  and  $K_{ab}$ , anisotropy constants;  $c_i$  and  $\delta_i$ , elastic and magnetoelastic constants, respectively.

To describe the system dynamics, the following densities of Lagrangian function  $L$  and dissipative Rayleigh function  $R$  depending only on the antiferromagnetism vector  $\mathbf{l}$  and the displacement one  $\mathbf{u}$  [10]:

$$L = \frac{1}{2}\rho\dot{\mathbf{u}}^2 + \frac{1}{2}(\chi_{\perp}/\gamma)\dot{\mathbf{l}}^2 - (\chi_{\perp}/\gamma)\mathbf{H}[\mathbf{l}] - \Phi, \quad (4)$$

$$R = \frac{1}{2}\alpha(M_0/\gamma)\dot{\mathbf{l}}^2 + \frac{1}{2}\sum\eta_{iklm}\dot{\epsilon}_{ik}\dot{\epsilon}_{lm}, \quad (5)$$

where  $\gamma$  is the gyromagnetic ratio;  $\alpha$ , damping constant for the magnetic subsystem;  $\eta_{iklm}$ , the elastic subsystem viscosity tensor having the same symmetry properties as the elastic module tensor in (2c);  $\rho$ , the crystal density. From (4) and (5), a corresponding motion equation can be derived for angles  $\theta$  and  $\varphi$  defining the orientation of antiferromagnetism vector  $\mathbf{l}(\sin\theta\cos\varphi, \sin\theta\cos\theta, \cos\theta)$ . The equations defining the displacement of elastic medium elements (displacement vector  $\mathbf{u}$ ) have the form

$$\rho\ddot{\mathbf{u}} = -\frac{\partial\Phi}{\partial\mathbf{u}} - \frac{\partial R}{\partial\mathbf{u}}. \quad (6)$$

Let the antiferromagnetism vector  $\mathbf{l}$  in the homogeneous equilibrium crystal state be oriented along the  $a$  axis (that is, let the high-temperature magnetic phase be considered):  $K_{ac}, K_{ab} > 0$ . Let a planar DW moving along the  $a$  axis ( $x$  one) be considered. The solution of Eq. (6) will be searched for as a sole wave describing the stationary motion of the DW, i.e.

$$\theta = \theta(x - vt), \quad u_l = u_l(x - vt), \quad \varphi = const. \quad (7)$$

Then the elastic strain equation has the form

$$\begin{aligned} (\rho v^2 - c_1)u'_x &= \delta_l(\varphi)\sin^2\theta - \eta_l v u''_x, \\ (\rho v^2 - c_7)u'_y &= \delta_7(\varphi)\sin\theta\cos\theta - \eta_7 v u''_y, \\ (\rho v^2 - c_8)u'_z &= \delta_8(\varphi)\sin\theta\cos\theta - \eta_8 v u''_z, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \delta_7(\varphi) &= \delta_7\sin\varphi, \quad \delta_8(\varphi) = \delta_8\cos\varphi, \\ \delta_l(\varphi) &= -[(\delta_1 - \delta_2) + (\delta_1 + 2\delta_2)\cos^2\varphi]. \end{aligned} \quad (9)$$

If the magnetization distribution in the DW is assumed to be known ( $\sin\theta = \text{th}\xi$  where  $\xi = (x - vt)/\Delta$ ), then, according to the method proposed in [10], the solution for displacement vector can be obtained in the form

$$u'_x = \frac{1}{2\pi\rho} \int_{-\infty}^{+\infty} \frac{e^{iq\varphi} \delta_l \varphi_l}{q[s_1^2 - v^2 - iq\tilde{\eta}_1 v]} dq, \quad (10)$$

$$u'_y = \frac{1}{2\pi\rho} \int_{-\infty}^{+\infty} \frac{e^{iq\varphi} i \delta_7(\varphi) \varphi_t}{q[s_7^2 - v^2 - iq\tilde{\eta}_7]} dq,$$

$$u'_z = \frac{1}{2\pi\rho} \int_{-\infty}^{+\infty} \frac{e^{iq\varphi} i \delta_8(\varphi) \varphi_t}{q[s_8^2 - v^2 - iq\tilde{\eta}_8]} dq,$$

$$\varphi_t = \frac{\pi q^2 \Delta^2}{\text{ch}(\pi \Delta q / 2)}, \quad \varphi_l = \frac{\pi q^2 \Delta^2}{\text{sh}(\pi \Delta q / 2)},$$

$$\Delta = [A / (K_{ab} - K_{cb} \cos^2 \varphi)]^{1/2} (1 - v^2 / c^2)^{1/2} \quad (11)$$

where  $c = \gamma(A / \chi_{\perp})^{1/2}$  is the limiting DW speed coincident with that of spin waves,

$$s_i^2 = \frac{c_i}{\rho}, \quad \tilde{\eta}_i = \frac{\eta_i}{\rho}.$$

Then, using Eqs. (2) and (10), the total DW energy can be calculated as

$$E_{tot} = E_m \left\{ 1 + \frac{2(1 - v^2 / c^2)}{\pi^2 (K_{ab} - K_{cb} \cos^2 \varphi)} \left[ B_8(\varphi, v) \cos^2 \varphi + B_7(\varphi, v) \sin^2 \varphi + B_l(\varphi, v) \left( 1 + \frac{\delta_1 + 2\delta_2}{\delta_1 - \delta_2} \cos^2 \varphi \right)^2 \right] \right\},$$

$$E_m = \frac{2\sqrt{A(K_{ab} - K_{cb} \cos^2 \varphi)}}{\sqrt{1 - v^2 / c^2}},$$

$$B_7(\varphi, v) = \frac{\delta_7^2}{c_7} \left( 3 \left( \frac{v}{s_7} \right)^2 - 1 \right) J_y(\varphi, v), \quad (12)$$

$$B_8(\varphi, v) = \frac{\delta_8^2}{c_8} \left( 3 \left( \frac{v}{s_8} \right)^2 - 1 \right) J_z(\varphi, v),$$

$$B_l(\varphi, v) = \frac{(\delta_1 - \delta_2)^2}{c_l} \left( 3 \left( \frac{v}{s_l} \right)^2 - 1 \right) J_x(\varphi, v) = B_1(\varphi, v),$$

$$J_z(\varphi, v) =$$

$$= \int_0^{\infty} \frac{k^2 dk}{\left[ 1 - \left( \frac{v^2}{s_8^2} \right)^2 + \left( \frac{v\tilde{\eta}_8}{s_8^2} \right)^2 \frac{4k^2}{\pi^2 \Delta^2(\varphi)} \right]} \frac{1}{\text{ch}^2 k},$$

$$J_y(\varphi, v) =$$

$$= \int_0^{\infty} \frac{k^2 dk}{\left[ 1 - \left( \frac{v^2}{s_7^2} \right)^2 + \left( \frac{v\tilde{\eta}_7}{s_7^2} \right)^2 \frac{4k^2}{\pi^2 \Delta^2(\varphi)} \right]} \frac{1}{\text{ch}^2 k},$$

$$J_x(\varphi, v) =$$

$$= \int_0^{\infty} \frac{k^2 dk}{\left[ 1 - \left( \frac{v^2}{s_l^2} \right)^2 + \left( \frac{v\tilde{\eta}_l}{s_l^2} \right)^2 \frac{4k^2}{\pi^2 \Delta^2(\varphi)} \right]} \frac{1}{\text{sh}^2 k}.$$

$$\tilde{\eta}_{7,8,l} = \frac{\eta_{7,8,l}}{\rho}.$$

Fig. 1 presents the dependence of total energy on the motion speed for DW of (ac) ( $\varphi = 0$ ) and (ab) ( $\varphi = \pi/2$ ) types. In a certain speed range close to the transversal sound speed, the (ab) type wall is seen to have a lower energy than the (ac) one (Fig. 1a). This means that at  $v_1$ , the (ac) type wall should be transformed into the (ab) one while at  $v_2$ , the reverse transition will take place. Thus, there is a dynamic transformation of DW structure. Note that such transformation is not always possible (Fig. 1b). At low  $\delta_i$  values and large  $\eta_i$  ones, that transformation does not occur.

It is known from the theory [3] that it is sign of  $K_{cb}$  that defines which of two possible DW types is realized in an ortho-ferrite. Near  $v_1, v_2$ , however, the magnetoelastic energy increases so considerably that the effective anisotropy constant in the (bc) plane having the form

$$\tilde{K}_{cb} = K_{cb} - \frac{4}{\pi^2} (1 - v^2 / c^2) (B_8 - B_7 + 2B_1 r_1), \quad (13)$$

$$r_1 = \frac{\delta_1 + 2\delta_2}{\delta_1 - \delta_2}$$

may change its sign. It is just the cause that results in the change of DW structure.

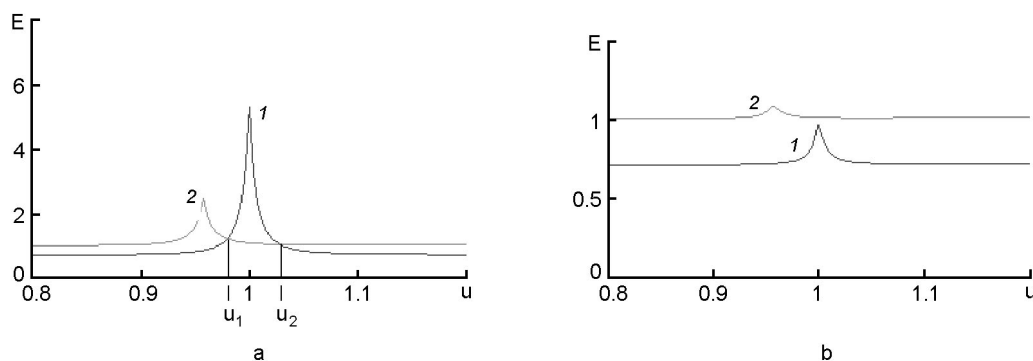


Fig. 1. Dependence of a DW total energy  $E = E_{tot}(AK_{ab})^{-1/2}/2$  on the motion speed at  $u = v/s_8$ ,  $s_8 > s_7$ ,  $K_{ab}/K_{ac} = 2$ ,  $\eta_1 = 0.5 \text{ Erg}\cdot\text{s}/\text{cm}^2$ ,  $\eta_7 = 0.009 \text{ Erg}\cdot\text{s}/\text{cm}^2$ ,  $\eta_8 = 0.1 \text{ Erg}\cdot\text{s}/\text{cm}^2$  (1, ac type DW; 2, ab type DW) at  $(\delta_1 + 2\delta_2)/(\delta_2 - \delta_1) = 0.5$ ,  $\delta_7 = 4 \cdot 10^7 \text{ Erg}/\text{cm}^3$ ,  $\delta_8 = 4 \cdot 10^7 \text{ Erg}/\text{cm}^3$  (a);  $\delta_7 = 10^7 \text{ Erg}/\text{cm}^3$ ,  $\delta_8 = 10^7 \text{ Erg}/\text{cm}^3$  (b).

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## Динамічна перебудова структури доменних меж у ортоферитах

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Теоретично досліджено стаціонарну динаміку доменної межі (ДМ) у рідкісно-земельних ортоферитах з урахуванням можливості перебудови її структури поблизу швидкості звуку. Дослідження базується на тому, що повна енергія ДМ, що рухається, складається з магнітної та магнітострикційної компонент.