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Spectrum of nonlinear magnetoelastic waves in a cubic ferromagnet

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Peculiarities of the nonlinear magnetoelastic (ME) wave spectrum in a cubic ferromagnet with induced uniaxial anisotropy along [111] have been studied theoretically. It has been shown that in the case of resonance, as the wave attains a certain velocity in the magnet along the [111] axis, a possibility appears for a movement of the 60-degree domain wall without magnetization deviation from the spin rotation plane. The presence of ME interaction has been found to result in a violation of conditions for that resonance type appearance and to give rise to new solutions.

Теоретически исследуются особенности спектра нелинейных магнитоупругих (МУ) волн в кубическом ферромагнетике с наведенной вдоль [111] одноосной анизотропией. Показано, что в резонансном случае при достижении волной определенной скорости в магнетике вдоль оси [111] возможно движение 60-градусной доменной границы без выхода намагниченности из плоскости вращения спинов. Установлено, что наличие МУ взаимодействия приводит к нарушению условий возникновения такого типа резонанса и появлению новых решений.

It is known that nonlinear magnetoelastic (ME) waves may be induced in magnetics due to magnetostriction. The nature of those waves is defined by a considerable nonlinearity that is introduced into the elastic subsystem by the magnetic one [1-3]. When studying the nonlinear ME vibrations in various media, two approaches are used. The first one is based on consideration of the medium dynamics ME equations that are solved in a slightly linear approximation for a specific magnetic [4–7]. The other approach proceeds from solution of nonlinear differential equations (the Landau-Lifshits ones, etc.) describing the waves with arbitrary amplitudes; to that solution, approximated [3] or qualitative methods [8] are applied. In this connection, the work [9] is worth to be noted where the domain wall dynamics in weak ferromagnets is studied taking into account the interaction with the crystal acoustic subsystem as well as the dissipative processes. A particular attention is given there to the near-sonic speed range of the domain walls and solitary waves. It has been shown, in particular, that at the wave speed close to the sound one, the contribution from the ME energy increases considerably and this may result in the sign reversal of the total anisotropy energy. The latter, in turn, changes the domain wall structure and size, thus influencing its dynamic behavior.

Within the framework of the second approach, the possible types of nonlinear ME waves propagating along the anisotropy axis were found for a number of easy-plane magnetics. In particular, it has been shown [3] that in tetragonal ferromagnets (including cubic ferromagnets with induced uniaxial anisotropy (IUA) along [001]), the following kinds of stationary nonlinear ME waves may propagate: nonlinear periodic, solitary, and spiral ones, as well as waves with a non-uniform precession of the magnetization vector [10]. In crystals with complex anisotropy (e.g., in ferrites-garnets), a wider spectrum of nonlinear ME wave types is possible under the same approximation. So in a (011) plate, the appearance of a new (as compared to those studied in [3, 10]) wave type (e.g., solitons) is explained mainly by taking into account the second constant of the cubical anisotropy (CA), K_2 [11], the contribution thereof being increased significantly as the temperature is lowered and becoming comparable to the contribution from the 1st constant K_1 (it is even possible the case when $K_2 > K_1$ [12, 13]). In this connection, of interest is to study the possible types of nonlinear ME waves in a cubic crystal shaped as a (111) oriented plate.

Let the nonlinear ME dynamics be considered in a (111) crystal plate with complex anisotropy taking into account the ME interaction. The energy density of such a magnet can be presented as

$$
\begin{aligned}
E = {} & A[(\theta')^2 + \sin^2\theta(\varphi')^2] + K_u\sin^2\theta + \\
& + K_1[\tfrac{1}{4}\sin^4\theta + \tfrac{1}{3}\cos^4\theta + \tfrac{\sqrt{2}}{3}\sin^3\theta\cos\theta\cos3\varphi] + \\
& + \frac{K_2}{54}[\sin^3\theta\cos3\varphi - \frac{\sqrt{2}}{2}\cos\theta(3\sin^2\theta - 2\cos^2\theta)]^2 + \\
& + \frac{1}{3}B_1[u'_x\frac{\sqrt{2}}{2}(-\sin^2\theta\cos2\varphi + \sqrt{2}\sin2\theta\cos\varphi) + \\
& + u'_y\sqrt{2}\sin\theta\sin\varphi \times (\sin\theta\cos\varphi + \\
& + u'_y\sin\theta\sin\varphi(\cos\theta - \sqrt{2}\sin\theta\cos\varphi) + \\
& + u'_z(3\cos^2\theta - 1) + \sqrt{2}\cos\theta) + u'_z] + \\
& + \frac{1}{3}B_2[u'_x\frac{\sqrt{2}}{2}(\sin^2\theta\cos2\varphi + \frac{\sqrt{2}}{2}\sin2\theta\cos\varphi) + \\
& + \frac{1}{6}((u'_x)^2 + (u'_y)^2)(C_{11} - C_{12} + C_{44}) + \\
& + \frac{1}{6}(u'_z)^2(C_{11} + 2C_{12} + 4C_{44})],
\end{aligned}
\qquad (1)
$$

where θ, φ are the polar and azimuth angles of the magnetization vector $\mathbf{M}$ in the $OX\|[11\bar{2}]$, $OY\|[\bar{1}10]$, $OZ\|[111]$ coordinate system; A, the exchange parameter; K_u, the IUA constant; K_1, K_2, the 1st and 2nd CA constants, respectively; u_i, components of the shift vector $\mathbf{u}$; B_i, C_{ij}, ME and elastic constants, respectively; M_s, the saturation magnetization.

The system dynamics is described by the Landau-Lifshits equations for a dissipation-free medium and by elasticity equations having the form

$$\dot{\theta} = -\frac{\gamma}{M_s\sin\theta}\frac{\delta E}{\delta\varphi},\ \dot{\varphi}\sin\theta = \frac{\gamma}{M_s}\frac{\delta E}{\delta\theta},\ \rho\ddot{u}_i = \frac{\delta E}{\delta u_i}, \qquad (2)$$

where γ is the gyromagnetic ratio; ρ, the crystal density.

The solutions of system (2) will be searched for in the form $\theta = \theta(z - Vt)$, $\varphi = \varphi(z - Vt)$, $\mathbf{u} = \mathbf{u}(z - Vt)$, ($V$ being the ME wave speed). Then the system of equations describing the ME waves will take the form

$$
\begin{aligned}
& \theta' V\sin\theta = \\
& = \frac{\gamma}{M_s}\{2A(\varphi'\sin^2\theta)' + \sqrt{2}K_1\sin^3\theta\cos\theta\sin3\varphi + \\
& + \frac{K_2}{9}\sin^3\theta\sin3\varphi[\sin^3\theta\cos3\varphi + \\
& + \frac{\sqrt{2}}{2}\cos\theta(3\sin^2\theta - 2\cos^2\theta)] + \\
& + u_x'(b_1\sin2\theta\sin\varphi + 2b_2\sin^2\theta\sin2\varphi) + \\
& + u_y'(2b_2\sin^2\theta\cos2\varphi - b_1\sin2\theta\cos\varphi)\}; \\
& \varphi' V = \frac{\gamma}{M_s}\{A(2\theta'' - (\varphi')^2\sin2\theta) + \\
& + K_1[\sin^3\theta\cos\theta - \frac{4}{3}\cos^3\theta\sin\theta + \\
& + \sqrt{2}\sin^2\theta\cos^2\theta\cos3\varphi - \frac{\sqrt{2}}{3}\sin^4\theta\cos3\varphi] - \\
& - \frac{K_2}{27}[3\sin^2\theta\cos\theta\cos3\varphi - \frac{3\sqrt{2}}{2}\sin^3\theta + \\
& + \sqrt{2}\sin\theta\cos^2\theta + 6\sqrt{2}\sin\theta\cos^2\theta] \times \\
& \times [\sin^3\theta\cos3\varphi + \frac{\sqrt{2}}{2}\cos\theta(3\sin^2\theta - 2\cos^2\theta)] - \\
& - u_x'(2b_1\cos2\theta\cos\varphi + b_2\sin2\theta\cos2\varphi) - \\
& - u_y'(2b_1\cos2\theta\sin\varphi - 2b_2\sin2\theta\sin2\varphi) + B_2u_z'\}.
\end{aligned}
\qquad (3)
$$

$$
\begin{aligned}
& \frac{\partial^2 u_x}{\partial z^2} = \frac{1}{\zeta_1}\frac{\partial(b_1\sin2\theta\cos\varphi + b_2\sin^2\theta\cos2\varphi)}{\partial z}; \\
& \frac{\partial^2 u_y}{\partial z^2} = \frac{1}{\zeta_1}\frac{\partial(b_1\sin2\theta\sin\varphi + b_2\sin^2\theta\sin2\varphi)}{\partial z}; \\
& \frac{\partial^2 u_z}{\partial z^2} = \frac{1}{\zeta_2}\frac{\partial(B_2\cos2\theta + (B_1 - B_2)/3 + \sigma)}{\partial z};
\end{aligned}
\qquad (4)
$$

where

$$b_1 = (2B_1 + B_2)/3,$$
$$b_2 = \sqrt{2}(B_1 - B_2)/3,$$

$$\zeta_1 = \frac{2}{3}(V^2/s_{t1}^2 - 1)(C_{11} - C_{12} + C_{44}),$$
$$\zeta_2 = \frac{1}{3}(V^2/s_{l2}^2 - 1)(C_{11} + 2C_{12} + 4C_{44}),$$
$$s_{l2} = \sqrt{(C_{11} + 2C_{12} + 4C_{44})/3\rho},$$

$s_{t1} = \sqrt{(C_{11} - C_{12} + C_{44})/3\rho}$, are the longitudinal and transversal sound speed, respectively; the prime means here the differentiation with respect to z.

Substituting (4) into (3), we obtain equations where the strain tensor is excluded. Those are nonlinear differential equations which cannot be reduced to the known integrable ones, because they are too complicated and cumbersome. But in the case when $K_u < 0$, $|K_u^*| >> K_1^*$, K_2, $|K_1^* - K_1|$, where $K_1^* = K_1 + 3\sqrt{2}b_1b_2/2\zeta_1$,

$K_u^* = K_u - 2\pi M_s^2 - B_2(B_1 + 2B_2)/3\zeta_2$, are renormalized CA and IUA constants taking into account the ME interaction and magnetostatic one, the solutions of the system so obtained can be searched for in the form $\theta = \pi/2 - \chi$ where $\chi << 1$. Then the second equation of the system (3) will have the form

$$\cos\theta \approx \chi = \tag{5}$$
$$= \frac{1}{2\left|K_u^*\right|}\left[\frac{M_sV}{\gamma}\varphi' - \frac{\sqrt{2}}{3}\left(K_1^* + \frac{1}{6}K_2\right)\cos3\varphi\right].$$

It follows from (5) that the magnetization exit out of the (111) plane is defined by contributions from rotating moments caused by the dynamic demagnetizing field (the 1st item in square brackets) and the CA fields (2nd item), all the items being reduced to the IUA field. Those moments may either amplify each other or weaken each other down to complete mutual compensation.

Let the possible solutions of Eqs.(3), (4) be considered for easy-plane ferromagnets.

(i) If the rotating moments are fully compensated, then $\chi = 0$ and, in the case when there is no ME interaction, we obtain

$$\frac{M_sV}{\gamma}\varphi' = \frac{\sqrt{2}}{3}(K_1 + \frac{1}{6}K_2)\cos3\varphi. \tag{6}$$

Thus, the 1st equation of the system (4) will take the form

$$(V^2/s^2 - 1)\varphi'' + \frac{1}{\Delta_0^2}\sin6\varphi = 0, \tag{7}$$

where $\Delta_0 = 6\sqrt{A/K_2}$, $s = 2\gamma\sqrt{A|K_u|}/M_s$ is a certain characteristic speed corresponding to the minimum speed of a spin wave. The system of equations (6), (7) has a unique solution which, at $V < s$ and $K_2 > 0$, corresponds to the rotation wave of the magnetic moment which describes the motion of a 60-degree domain wall (DW) separating two domains, one of those having $\mathbf{M}\|[10\bar{1}]$ ($\varphi = -\pi/6$) while the other $\mathbf{M}\|[01\bar{1}]$ ($\varphi = \pi/6$). At $V > s$ ($K_2 > 0$), the solution corresponds to a moving 60-deg DW with $\mathbf{M}\|[11\bar{2}]$ ($\varphi = 0$) in one domain and $\mathbf{M}\|[\bar{2}11]$ ($\varphi = \pi/3$) in the other one. As the sign at K_2 reverses, those solutions answering to different relations between V and s become interchanged. Such DW are possible to be excited only at low temperatures, that is, when $K_2 \neq 0$. The DW motion under consideration is of resonance character, because it may be realized only if the condition (6) is met. It follows from the consistency condition of Eqs.(6) and (7) that the ME wave speed should met the following relationship:

$$V = \sqrt{3}s(\kappa_1 + \kappa_2/6)/\sqrt{\kappa_2}, \tag{8}$$

where $\kappa_1 = K_1/|K_u|$, $\kappa_2 = K_2/|K_u|$.

(ii) If there is no complete compensation of rotating moments ($\chi \neq 0$), then χ is defined by Eq.(5) and the system (3), (4) is reduced to sin-Gordon equation. Its solutions are known, and for nonlinear ME waves, those are presented in [3]. If the ME interaction cannot be neglected, the situation becomes much more complicated. When the stress matching in neighboring domains [14] is taken into account, the equilibrium positions at the infinity for $\mathbf{M}$ are set by the conditions

$$\varphi\Big|_{z\to\pm\infty} = n\pi/2, \text{ where } n = \pm1, \pm3, \ldots,,$$

$$\theta\Big|_{z\to\pm\infty} = \pi/2, \quad \chi\Big|_{z\to\pm\infty} = 0. \tag{9}$$

Then, the system (3), (4) under account for (9) is reduced to the following equation:

$$(V^2/\tilde{s}^2 - 1)\varphi'' = \tag{10}$$
$$= d_1\sin6\varphi - (2/3)d_2\sin4\varphi - (d_3 + d_2/3)\sin2\varphi,$$

where $d_1 = 1 - \beta(K_1^* + K_2/6)/2\left|K_u^*\right|$,

$d_2 = 3\sqrt{2}\beta b_1b_2/A\zeta_1\left|K_u^*\right|$,

$d_3 = (\Delta^2{}_0/A)[b_2^2/\zeta_1 - (b_1b_2)^2/\left|K_u^*\right|\zeta_1^2]$,

$\beta = 1/[6K_1^*/K_2 + 1]$,

$\Delta = \Delta_0/\beta^{1/2}$ is the DW width, $\tilde{s} = 2\gamma\sqrt{A|K^*{}_u|}/M_s$; here, the prime means the derivative with respect to a new variable $\zeta = z/\Delta_0$.

The expression (10) is a ternary sin-Gordon equation that has a wide spectrum of solutions depending on sign of d_1, d_2, d_3 parameters defined by V, s_{t1}, s_{l2} values. It is seen from consideration of (10), the relations between V and $\tilde{s}$ influence also the form of its possible solutions. The quantity $\tilde{s}$ itself depends on V. Thus, solving those relations for V, we obtain that the condition $V > \tilde{s}$ is realized in the regions $V > \max(V_1, V_2)$ and

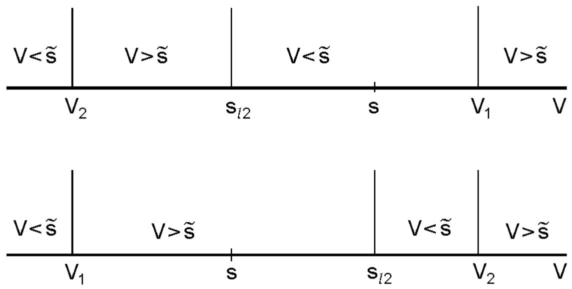

Fig. 1. ME wave speed ranges answering to conditions $V > \tilde{s}$, $V < \tilde{s}$ for $s_{l2} < s$ and $s_{l2} > s$.

$\min(V_1,V_2) < V < s_{l2}$ while $V < \tilde{s} - s_{l2} < V < \max(V_1, V_2)$ and $V < \min(V_1,V_2)$ (Fig. 1) ($\max(\min)(V_1,V_2) = \{V_1(V_2)$ at $s_{l2} < s$ and $V_2(V_1)$ at $s_{l2} > s\}$).

Here,

$$V_{1,2}^2 = \left\{s_{l2}^2 + s^2 \pm [(s_{l2}^2 + s^2)^2 - 4s_{l2}^2(s^2 + \tilde{D})]^{1/2}\right\}/2, \qquad (11)$$

where

$\tilde{D} = 4\gamma^2 A B_2(B_1 + 2B_2)/M_s^2(C_{11} + 2C_{12} + 4C_{44})$. In this case, the condition $(s_{l2}^2 + s^2)^2 > 4s_{l2}^2(s^2 + \tilde{D})$ should be met. In the absence of ME interaction, $\tilde{s} = s$ (the solutions for that case are presented above). Taking the ME interaction into account, we obtain a certain speed range where the sign between V and $\tilde{s}$ becomes reversed.

For ferrites-garnets, in particular, for iron-yttrium garnet ($Y_3Fe_5O_{12}$), at $T = 300$ K and K_1, $K_2 < 0$, the relation $s < s_{l2}$ is typical ($s_{t1} = 3.52 \cdot 10^3$ m/s, $s_{l2} = 6.55 \cdot 10^3$ m/s, $s = 6.26 \cdot 10^3$ m/s). In such conditions, the solution satisfying (9) appears in the following speed range:

$$s_{t1} < V < s_{t1}\left[1 + 3b_1^2/2\left|K_u\right|(C_{11} - C_{12} + C_{44})\right]^{1/2}, \qquad (12)$$

and it describes a moving 180-deg DW with "bottlenecks" (inflection points in the $\varphi(\xi)$ plot, Fig. 2, appearing in the static case when there is a metastable axis in the spin rotation region). Outside the range (12), there are no solutions answering to the rotation solitary wave of the vector **M** (with preset equilibrium positions at infinity). This seems to be associated with appearance of more complex structure waves with $\varphi(z)$, $\theta(z)$, for example, of the ME waves with non-uniform precession [10].

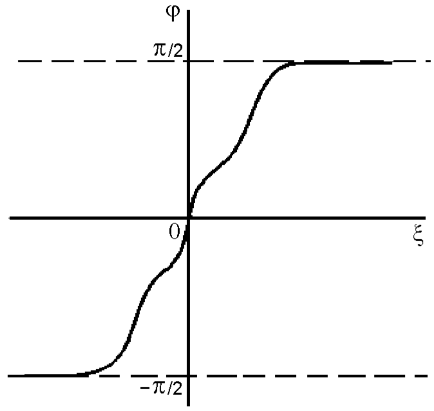

Fig. 2. Plot of moving DW with "bottlenecks".

Thus, in a ferrite-garnet (111) plate where the ME interaction in negligible at low temperature, a new resonance type is possible that is associated with compensation of dynamic moments caused by the dynamic demagnetizing field and CA. In this case, the 60-deg DW may propagate along [111] without magnetization exit out of the spin rotation plane. The presence of ME interaction that is not negligible as compared to CA and IUA results in lowered crystal symmetry and a violation of conditions for appearance of the above-mentioned resonance. In this case, the ME dynamics of the magnet is described by a ternary sin-Gordon equation that allows a solution in the form of a moving 180-deg DW with "bottlenecks". Topologically, it is a correlated state of three 60-deg DW and exists within a certain speed range. The latter statement is consistent with conclusions made in [8, 9] and has a general character.

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Спектр нелінійних магнітопружних хвиль у кубічному феромагнетику

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Теоретично досліджуються особливості спектра нелінійних магнітопружних хвиль у кубічному феромагнетику з наведеною вздовж [111] одновісною анізотропією. Показано, що у випадку резонансу при досягненні хвилею певної швидкості в магнетику вздовж осі [111] можливий рух 60-градусної доменної межі без виходу намагніченості з площини обертання спінів. Встановлено, що наявність магнітопружної взаємодії призводить до порушення умов виникнення такого типу резонансу та до появи нових рішень.