Fizika Nizkikh Temperatur, 1999, v. 25, No 8/9, p. 936–943

The ground state of the «frozen» electron phase in two-dimensional narrow-band conductors with a long-range interelectron repulsion. Stripe formation and effective lowering of dimension

A. A. Slutskin, V. V. Slavin, and H. A. Kovtun

B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 47, Lenin Ave., 310164, Kharkov, Ukraine
E-mail: slutskin@theor.kharkov.ua

Received March 29, 1999

In narrow-band conductors a weakly screened Coulomb interelectron repulsion can supress narrow-band electrons' hopping, resulting in formation of a «frozen» electron phase which differs principally from any known macroscopic self-localized electron state including the Wigner crystal. In a zero-bandwidth limit the «frozen» electron phase is a classical lattice system with a long-range interparticle repulsion. The ground state of such systems has been considered in the case of *two* dimensions for an *isotropic* pair potential of the mutual particle repulsion. It has been shown that particle ordering into stripes and effective lowering of dimension resides universally in the ground state for any physically reasonable pair potential and for any geometry of the conductor lattice. On the basis of this fact a rigorous general procedure to describe the ground state fully has been formulated. Arguments have been adduced that charge ordering into stripes in high-T_c superconductors testifies to the presence of a «frozen» electron phase in these systems.

PACS: 64.60.Cn, **64.60.–i**

1. Introduction

High-T_c superconductors studies have caused a surge of interest in the properties of narrow-band layered and two-dimensional ($2D$) conductors. An important consequence of the layerness is substantial weakening of the screening of a Coulomb interaction between the charge carriers. (The screening radius cannot, under any circumstances, be less than the interlayer distance). In addition, in layered conductors it is possible to well separate the charge carriers (for definiteness, we consider them electrons) from the donors, so that the mean energy, u_{ee} , of the long-range interelectron repulsion prevails over the energy of an electron attraction to the donors. Under these conditions it is the mutual repulsion of narrow-band electrons that can suppress their tunneling between the equivalent orbits of the conductor lattice, resulting in formation of a «frozen» electron phase (FEP) which differs principally from any known macroscopic self-localized electron state including the Wigner crystal [1]. The FEP occurs when the electron bandwidth, t, is less than $\delta u = (a/r_{ee})u_{ee}$, where δu is the typical change in the narrow-band electron Coulomb energy in electron hopping, a is the range of hopping, r_{ee} is the mean electron separation. The high-T_c cuprates, grain boundaries of polycrystal electroceramic materials [2], as well as some art $2D$ conductors [3–5] appear to be most favorable for $2D$ FEP coming to existence.

The macroscopic behavior of the $2D$ FEP is rather unconventional. Its distinctive features are rooted in properties of its ground state (GS) at $t << \delta u$. In the limit $t/\delta u \to 0$ the GS of the $2D$ FEP is much the same as that of other $2D$ lattice systems with a long-range interparticle repulsion. (An example is an ensemble of adsorbed atoms strongly interacting with their substrate and mutually repelling each other [6].) As far as we know, neither the thermodynamics nor the GS of such systems have been studied adequately. Here we offer a unified approach to the description of the GS of the $2D$ zero-bandwidth FEP (and similar lattice

systems) with an *isotropic* pair potential of the interelectron repulsion, $v(r)$ (r is the distance between interacting electrons). The key point of our consideration is a new phenomenon — a *zero-temperature effective lowering of dimension* (LOD) — which we have revealed to underlie (*despite the pair potential isotropy*) the main GS properties of the $2D$ FEP for: i) arbitrary arrangement of the sites which can be occupied by electrons provided that the sites constitute a primitive lattice (it is called host lattice below); ii) any filling factor, $\rho = N/\mathcal{N}$ ($N \to \infty$ and $\mathcal{N} \to \infty$ are the total numbers of the electrons and the host-lattice sites, respectively); iii) any physically reasonable $v(r) > 0$. We take the term LOD to mean that the GS of the $2D$ FEP is a set of different effective $1D$ FEP whose «particles» are periodic stripes on the lattice of the $2D$ conductor. For each $1D$ system of the set there is its own ρ interval where this $1D$ FEP represents the $2D$ one, the whole range, $0 \le \rho \le 1$, comprising all the intervals. The LOD enables us to offer a rigorous analytical procedure for the $2D$ FEP GS description, using the exact results of the general theory of the $1D$ lattice systems with a long-range interparticle repulsion [7–9].

2. Hamiltonian. Simple crystals

The Hamiltonian, $\mathcal{H}$, of the system under consideration has the form

$$\mathcal{H}\{n(\mathbf{r})\} = \frac{1}{2} \sum_{\mathbf{r} \neq \mathbf{r}'} v(|\mathbf{r} - \mathbf{r}'|)\, n(\mathbf{r}) n(\mathbf{r}') \,, \qquad (1)$$

where $\mathbf{r} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2$ are radius vectors of the host-lattice sites, $m_{1,2}$ are integers, $\mathbf{a}_{1,2}$ are host-lattice primitive translation vectors (PTVs); the occupation numbers of the host-latice sites, $n(\mathbf{r}) = 0$ or 1, are microscopic variables*; the sum is taken over the whole host lattice. The pair potential is assumed to be an everywhere convex function of the form $v(r) = \tilde{v}(r)/r$, where the function $\tilde{v}(r)$ depends on the character of the screening medium and its position with respect to the $2D$ FEP. In any case $\tilde{v}(r)$ tends to zero as r^{-2} or faster when $r \to \infty$; $\tilde{v}(0) = e^2/\kappa$ (e is the electron charge, κ is the dielectric permittivity). Otherwise, $\tilde{v}(r)$ can be reckoned as arbitrary: as will be shown below, its specific form is immaterial to our approach.

Among the GS configurations $\{n(\mathbf{r})\}$ with different ρ the *simplest* ones are $2D$ crystals with one electron per cell («S-crystals»)**. Their inverse ρ values make up an infinite set of integers $Q_j = |\det (m^j_{\kappa\lambda}(\mathbf{a}_1, \mathbf{a}_2))|$, where j indexes S-crystals, the integers $m^j_{\kappa\lambda}$ ($\kappa, \lambda = 1, 2$) are components of S-crystal PTVs in the $\mathbf{a}_\lambda$ basis; Q_j is the j-th S-crystal elementary-cell area measured in units of that of the host lattice, $\sigma_0 = |\mathbf{a}_1 \times \mathbf{a}_2|$.

Our strategy is to derive the full description of the GS for any $\mathbf{a}_{1,2}$, starting with consideration of small vicinities of $\rho = 1/Q_j$. Since specific $m^j_{\kappa\lambda}$ values are irrelevant to this reasoning, we drop the index j at Q and at other characteristics of the S-crystals for a while.

Due to the discreteness of the system with the Hamiltonian (1) a macroscopically small change, $\delta\rho$, in ρ ($\delta\rho \to 0$, $N^{1/2}|\delta\rho| \to \infty$ when $N \to \infty$) produces only *isolated defects* in an S-crystal, the space structure of the defects essentially depending on whether they result from an increase or a decrease in ρ. This fact is expressed by the identity

$$E_g(N \pm |\delta N|,\ \mathcal{N} \pm |\delta\mathcal{N}|) - E_g(N,\ \mathcal{N}) =$$

$$= \pm \mu_\pm |\delta N| \mp P_\pm |\delta\mathcal{N}| \,, \qquad (2)$$

where E_g is the GS energy, δN and $\delta\mathcal{N}$ are changes in N and $\mathcal{N}$ producing $\delta\rho$. The proportionality coefficients, $\mu_- < \mu_+$, $P_- < P_+$, are the values of the chemical potential, μ, and the pressure, P, which are the endpoints of the μ and P intervals of S-crystal existence. They are determined by the energies of formation of corresponding defects. Thus, in some vicinity of $\rho = 1/Q$ the GS is bound to be a superstructure of the defects. Our next step is to find them.

3. Zero-dimensional defects and their coalescence

Adding one electron to or removing from an S-crystal results in the formation of a zero-dimensional defect, «+defecton» or «–defecton», respectively. One can be inclined to think that δN should be identified exactly with the total number of $\pm$defectons spatially separated, $\pm\mu_\pm$ being simply the energy of $\pm$defecton formation, $\epsilon_\pm$. However, this seemingly evident statement is actually incorrect

* The ρ values are assumed to be not too close to $\rho = 1$, so that we can neglect the configurations with $n(\mathbf{r}) = 2$ in (1) and take no account of the spin variables.

** As was shown in Ref. [10], if $v(r)$ were box-like, it would be not the case due to a clusterization of the particles.

due to a *coalescence* of defectons of the same «sign». In other words, if the number, $|\nu|$, of S-crystal electrons removed ($\nu < 0$) or added ($\nu > 0$) is more than 1, a *bound state* of $|\nu|$ ±defectons arises whose energy is less than $|\nu|\epsilon_{\pm}$. We have revealed the coalescence by computation, using a «dipole» description of the GS with $\nu = \pm 1, \pm 2, \ldots$, which we have specially worked out for this purpose. The dipole approach offers a clear view of how the defectons' bound state arises despite the fact that the defectons of the same sign repel each other, being widely spaced.

At $\nu \neq 0$ a perturbed S-crystal is formed where, beside electrons placed at host-lattice sites in the interstices of the S-crystal ($\nu > 0$) or empty S-crystal sites, «holes», ($\nu < 0$), there are generally a certain number of S-crystal electrons shifted from their native S-crystal sites. The latter can be considered as «antiparticles» whose charge is equal to the electron one in magnitude but is opposite in sign, a pair «an electron shifted by a vector $\boldsymbol{\xi}$ + its antiparticle located at an S-crystal site $\mathbf{r}$» being the «$\mathbf{r}$, $\boldsymbol{\xi}$-dipole». Thus, the perturbation of the S-crystal can be envisioned as an ensemble consisting of several dipoles and $|\nu|$ interstitial particles/holes (IP/Hs). The dipoles interact with the IP/Hs and with each other. The energy of interaction between an IP/H (at $\mathbf{r} = 0$) and $\mathbf{r}$, $\boldsymbol{\xi}$-dipole is $u_{\boldsymbol{\xi}}(\mathbf{r}) = \text{sign}\ \nu(v(|\mathbf{r} - \boldsymbol{\xi}|) - v(|\mathbf{r}|)) \equiv \text{sign}\ \nu\hat{\Delta}_{\boldsymbol{\xi}} v(|\mathbf{r}|)$; the energy of interaction between $\mathbf{r}$, $\boldsymbol{\xi}$- and $\mathbf{r}'$, $\boldsymbol{\xi}'$-dipole is $u_{\boldsymbol{\xi},\boldsymbol{\xi}'}(\mathbf{r} - \mathbf{r}') = \hat{\Delta}_{\boldsymbol{\xi}}\hat{\Delta}_{\boldsymbol{\xi}'} v(|\mathbf{r} - \mathbf{r}'|)$. The IP/Hs, in turn, undergo a mutual repulsion and are exposed to an «external» field, $u(\mathbf{r})$, which is equal to $-2u_0$ for holes (here and further on u_{ee} of the S-crystal is denoted by u_0), and for IPs it is the field produced at a point $\mathbf{r}$ by all electrons of the S-crystal. In these terms the change in the GS energy at a given ν, $U_{GS}(\nu)$, takes the form

$$U_{GS}(\nu) = \min\,(V_{\text{rep}} + U_d + U_{\text{exc}} + U)\ . \qquad (3)$$

Here $V_{\text{rep}} = \sum_{\alpha<\beta} v(|\mathbf{r}_{\alpha\beta}|)$ is the energy of the mutual repulsion of the IP/Hs; $U_d = \sum_{\alpha,i} u_{\boldsymbol{\xi}_i}(\mathbf{r}_{\alpha i})$ is the energy of their interaction with the dipoles; $U_{\text{exc}} = \sum_i \delta u_{\boldsymbol{\xi}_i} + \sum_{i<k} u_{\boldsymbol{\xi}_i,\boldsymbol{\xi}_k}(\mathbf{r}_{ik}) > 0$ is the excitation energy of an S-crystal with n_d dipoles at $\nu = 0$; $\delta u_{\boldsymbol{\xi}} \sim u_0|\boldsymbol{\xi}|^2/r_{ee}^2 > 0$ is the energy of formation of one dipole; $U = \sum_{\alpha} u(\mathbf{r}_{\alpha})$ is the energy of the IP/Hs in the external field mentioned; the indices $\alpha = 1, \ldots, |\nu|$ and $i = 1, \ldots, n_d$ enumerate the IP/H raduis-vectors and dipoles, respectively, n_d is the total number of the dipoles; $\mathbf{r}_{ab} \equiv \mathbf{r}_a - \mathbf{r}_b$. The minimum is taken with respect to n_d, the dipole variables, $\mathbf{r}_i$, $\boldsymbol{\xi}_i$, and $\mathbf{r}_{\alpha}$. Therefore, the dipole approach allows to work with only a few discrete variables. This facilitates considerably the Monte-Carlo computer simulation of the ±defectons ($U_{GS}(\pm 1) = \pm\,\epsilon_{\pm}$) and their coalescence at $|\nu| > 1$.

The mechanism of the coalescence can be elucidated by the following heuristic arguments. The GS total dipole energy, $E_d(\nu) = U_{\text{exc}}(\nu) + U_d(\nu)$, is negative, so that for any $|\nu|$ the GS space structure is determined by an interplay between negative U_d and positive U_{exc}, V_{rep}. The IP/H — dipole interaction gives the maximal gain in energy when *each* IP/H is embedded in a «shell» of four dipoles that are attracted to it, the dipoles' antiparticles forming a parallelogram of a size $\sim r_{ee} \sim Q^{1/2}$ (Fig. 1). The shells of neighboring IP/Hs are bound to share some of their dipoles for U_{exc} (and hence n_d) to be as small as possible. This requirement can be fulfilled only when all IP/Hs are *aligned in a row*, the near-neighbor IP/Hs being shifted relative to one another by the same S-crystal PTV with the modulus $\sim r_{ee}$ (Fig. 1). In such a case $|E_d(\nu)|$ is more than the magnitude of the dipole energy of $|\nu|$ infinitely separated defectons, $E_d^{\infty} = |\nu|E_d(\pm 1)$. The coalescence arises when the energy gain, $\Delta = |E_d(\nu)| - |E_d^{\infty}|$, exceeds V_{rep} of the IP/Hs aligned in the row. Since $\Delta \sim |\nu| v(r_{ee})$ this condition is met if $v(r)$ decreases not too slowly, or, more exactly, if

$$\gamma = \int_{r_{ee}}^{\infty} v(r)dr/r_{ee}v(r_{ee}) \lesssim 1\ . \qquad (4)$$

The computer simulation carried out with the model potential $v(r) \propto r^{-\beta} \exp(-r/R)$ over a wide range of the parameters, β, R, has confirmed that the condition (4) is really the criterion of the coalescence for any $|\nu|$ (and any $\mathbf{a}_{1,2}$).

Criterion (4) is for the most part fulfilled. It holds for any $\tilde{v}(r)$ (Sec. 2) such that $\tilde{v}(0) - \tilde{v}(r_{ee}) \sim \tilde{v}(0)$. This case will be the focus of our attention from here on. The parameter γ becomes $>> 1$ if $\tilde{v}(r)$ decreases substantially only for r which are exponentially large in γ. In this limit the mutual repulsion of the IP/Hs disrupts their row, and there is no coalescence, at least for sufficiently large $|\nu|$.

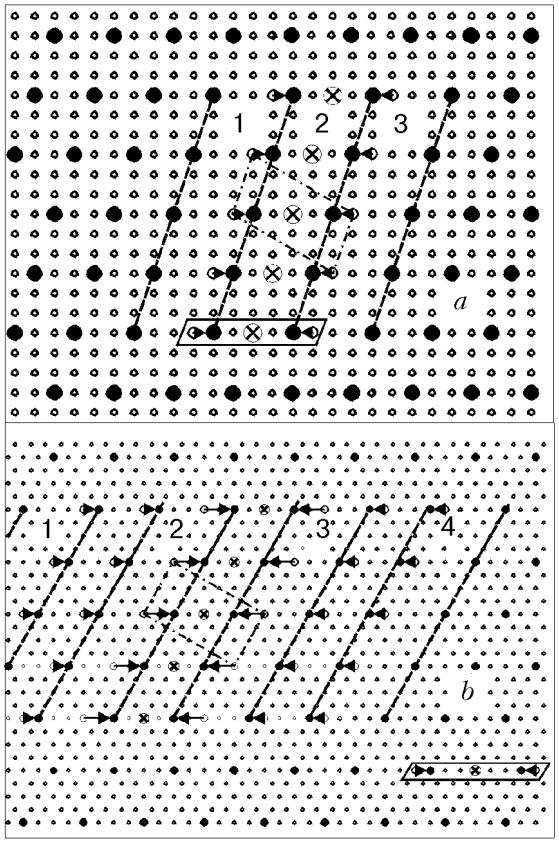

Fig. 1. The coalescence for square (*a*, $Q = 9$) and triangular (*b*, $Q = 16$) host lattices ($\nu = -5$). Here ○ denotes host-lattice sites, ● – particles; ⊙ – antiparticles; ⊗ – holes; → – dipoles; solid boxes mark off a single defecton. The shells of dipoles surrounding holes are marked off by the dash-dotted parallelograms. The dotted lines show nucleation of the elementary stripes enumerated by 1, 2, ... In the case (*a*) boundary effects dominate the mutual repulsion of the unfinished –stripes; in the case (*b*) the tendency to –stripes divergence is seen. Both configurations refer to the pair potentials which meet the condition $\gamma \sim 1$.

However, in Sec. 8 it is outlined that the LOD governs the GS in this rather special case, too.

4. The lowering of dimension

The elementary stripes in the 2D «frosen» electron phase

As follows from the aforesaid, the bound state of $|\nu|$ defectons is transformed into a periodic *stripe-like* structure with an infinite increase in $|\nu|$ (Fig. 1). It consists of elementary 1*D* defects which, as will be shown below, repel each other. Therefore, it is *the simplest* 1*D defects* that are expected to form the GS superstructure. An arbitrary 1*D* defect of such a type is a stripe of rarefaction or compression that arises when an S-crystal part adjacent to a line of electrons with some PTV, $\mathbf{d}$, is shifted as a whole relative to the other one by a host-lattice translation vector, ξ. Formation of one stripe of length L_s changes $\mathcal{N}$ by $\delta\mathcal{N} = \pm\sigma L_s$ ($\sigma = |\mathbf{d}\times\xi|$, L_s is measured in units of $|\mathbf{d}|$). The corresponding change in energy, δE, is proportional to $\delta\mathcal{N}$:

$$\delta E/|\delta\mathcal{N}| = \varepsilon(\mathbf{d}, \xi) = \sigma^{-1} \sum_{n=1}^{\infty} {\sum_{\mathbf{r}}}' u_{\xi}(\mathbf{r} - n\mathbf{f}) \ . \quad (5)$$

Here $\sum'_{\mathbf{r}}$ means summation over the S-crystal semiplane $\mathbf{r} = k\mathbf{d} + l\mathbf{f}$ ($-\infty < k < \infty$, $-\infty < l \le 0$); $\mathbf{f}$ is any S-crystal PTV other than $\mathbf{d}$. The GS is realized by the stripes with $\mathbf{d} = \mathbf{d}_{\pm}$ and $\xi = \xi_{\pm}$ which minimize $\varepsilon(\mathbf{d}, \xi)$ at a given sign of $-\delta\mathcal{N}$ (– or + symbolizes rarefaction or compression respectively). We will call these stripes «–stripes» or «+stripes».

The energies $\varepsilon_{\pm} = |\varepsilon(\mathbf{d}_{\pm}, \xi_{\pm})|$ are the quantities $P_{\pm}$ (see (2)) associated with ±stripes formation. The corresponding $\mu_{\pm}$, as follows from general thermodynamic considerations, are

$$\tilde{\varepsilon}_{\pm} = u_0 + Q\varepsilon_{\pm} \ . \quad (6)$$

Lest there be no contradiction with the fact of the coalescence, energies $\tilde{\varepsilon}_{\pm}$ and $\epsilon_{\pm}$ are bound to satisfy inequalities

$$\epsilon_{-} < \tilde{\varepsilon}_{-} < \tilde{\varepsilon}_{+} < \epsilon_{+} \ . \quad (7)$$

When $Q >> 1$ and $v(r)$ goes to zero over distances $R << r_{ee} \sim Q^{1/2}$, they follow from simple estimates. Taking into account that $|\xi_{\pm}| \sim a_0$, and, correspondingly, $|\mathbf{d}_{\pm}\times\xi_{\pm}| \sim Q^{1/2}\sigma_0$, from Eq. (5) we obtain: $\varepsilon_{\pm} \sim (a_0 Q^{1/2}/R)u_0$. On the other hand, $|\epsilon_{-}| \sim u_0 \sim v(r_{ee})$, and hence, $\tilde{\varepsilon}_{-} >> |\epsilon_{-}|$. In the case under consideration $\epsilon_{+} \sim v(r_{\min})$, where $r_{\min}$ is the least of the distances between the IP and the S-crystal sites. This energy is much more than ε_{+} as $R << r_{ee}$.

To make sure that the inequalities (7) hold for other $v(r)$ and R/r_{ee} we have computed $\varepsilon_{\pm}$ (basing ourselves on Eq. (5) and Eq. (6)) in parallel to the Monte-Carlo computer studies of the coalescence. They have confirmed that the inequalities are really the case for all $v(r)$ under consideration and for all Q, maybe except $Q = 2$.

Together with the mutual repulsion of ±stripes of the same sign the inequalities (7) lead to the conclusion that +stripes or –stripes do constitute the GS superstructure in the vicinity of $1/Q$. The position of each ± stripe – a constituent of the superstructure – is determined by the stripe «coordinate», l, which is the total number of particle lines (with the PTV $\mathbf{d}_{\pm}$) between this stripe and some fixed one ($l = 0$). A set of these coordinates

determines uniquely the $2D$ FEP space structure. Therein lies the LOD.

The ground state superstructure of stripes

The GS arrangement of the ±stripes is governed by the pair potential of the stripe-stripe interaction,

$$V_{ss}^{\pm}(l) = \sum_{n=l+1}^{\infty} {\sum_{\mathbf{r}}}' u_{\xi_{\pm},-\xi_{\pm}}(\mathbf{r} - n\mathbf{f}_{\pm} - \xi_{\pm}) \qquad (8)$$

where inter-stripe «distance» $l = 1, 2, \ldots$; $\mathbf{f}_{\pm}$ is an S-crystal PTV other than $\mathbf{d}_{\pm}$, $\sum_{\mathbf{r}}'$ means the same as in Eq. (5) ($\mathbf{d}$, $\mathbf{f} = \mathbf{d}_{\pm}$, $\mathbf{f}_{\pm}$). For all $v(r)$ under consideration $\sum_{\mathbf{r}}'(n) > 0$, and $\sum_{\mathbf{r}}'(n) > \sum_{\mathbf{r}}'(n+1)$. Hence, $V_{ss}(l) > 0$ is a convex function of l. This enables us to describe the ±stripes superstructure at $\theta - Q \neq 0$ ($\theta = 1/\rho$) on the basis of the universal $1D$ algorithm [7–9], considering the stripes as the «particles» of an effective $1D$ FEP:

$$l_m = [m/c_{\pm}];\ c_{\pm} = |\theta - Q|/\sigma_{\pm},\ \sigma_{\pm} = |\mathbf{d}_{\pm} \times \xi_{\pm}| \qquad (9)$$

where $[\ldots]$ is the integral part of a number, m enumerates the ±stripes; integer l_m is the coordinate of m-th stripe, which is a pair of neighboring lines of electrons $\mathbf{r}_{m,1}(k) = k\mathbf{d}_{\pm} + l_m\mathbf{f}_{\pm} + m\xi_{\pm}$ and $\mathbf{r}_{m,2}(k) = \mathbf{r}_{m,1}(k) + \mathbf{f}_{\pm} + \xi_{\pm}$ ($k = 0, \pm 1, \ldots$). The superstructure described by Eq. (9) is thus a mixture of −stripes ($\theta - Q > 0$) or +stripes ($\theta - Q < 0$) and unperturbed stripes of the S-crystal which are parallel to $\mathbf{d}_{\pm}$, so that $c_{\pm} = N_s/\mathcal{N}_s$ is the concentration of the ±stripes; N_s is their number; $\mathcal{N}_s$ is the total number of the ±stripes and the S-crystal ones. The number of unperturbed stripes between m-th and $m+1$-th ±stripes equals $l_{m+1} - l_m - 1$.

An algorithm for arrangement of electrons' lines

Simple crystals with $\mathbf{f}_{\pm} = q_{\pm}\xi_{\pm}$, where $q_{\pm} = Q/\sigma_{\pm}$ is an integer, are of frequent occurrence. Particularly, this occurs of necessity for a triangular host lattice (Sec. 7), and also for $\sigma_{\pm} = \sigma_0$, as is typical of S-crystals on a host lattice of a lower symmetry. In such a case the above-mentioned electron lines of both types, $\mathbf{r}_{m,1}(k)$ and $\mathbf{r}_{m,2}(k)$, fall into the class of electron lines $k\mathbf{d}_{\pm} + l\xi_{\pm}$ ($k = 0, \pm 1, \ldots$; l is an integer), which can be considered as $1D$ «particles» with «coordinates» l. Their arrangment, as follows from Eq. (9), obeys the algorithm:

$$l_m = [sm],\ s = q_{\pm} \mp c_{\pm},$$

where l_m is the «coordinate» of the m-th line, s is the mean line separation measured in units of $|\xi_{\pm}|$.

5. Devil staircase

The dependence of $c_{\pm}$ (or ρ) on μ, much the same to the $1D$ FEP [7–9], is a well-developed fractal structure, a devil staircase whose steps occur at all *rational* $c_{\pm} = M/L \leq 1$ (M, L are coprime integers). At given M, L the GS configuration of the $2D$ FEP is thus a «FEP crystal» with L electrons per cell and with PTVs $\mathbf{d}_{\pm}$, $L\mathbf{f}_{\pm} + M\xi_{\pm}$.

In the commonly occurring case that $\mathbf{f}_{\pm}$ is a multiple of $\xi_{\pm}$ (Sec. 4) the steps' widths, $\Delta\mu = \Delta\mu(M/L)$, can be found by direct application of the $1D$ theory [8,9], considering the energy of the line-line repulsion,

$$\mathcal{V}(l) = \sum_{k=-\infty}^{\infty} v(|k\mathbf{d}_{\pm} + l\xi_{\pm}|)$$

(l is the distance between interacting lines), as the $1D$ pair potential. This produces

$$\Delta\mu = \mathcal{L}\sum_{m=1}^{\infty} m(\mathcal{V}(\mathcal{L}m - 1) - 2\mathcal{V}(\mathcal{L}m) + \mathcal{V}(\mathcal{L}m + 1)),$$

where $\mathcal{L} = q_{\pm}L \mp M$ is the period of the lines' pattern. The expression in the brackets is positive, since $\mathcal{V}(l)$ is a convex function in the case under consideration. Generally, $\Delta\mu(M/L)$ are expressed in terms of $V_{ss}^{\pm}(l)$ by a slight modification of the $1D$ theory.

6. J-branches and first-order transitions in the ground state of the $2D$ «frosen» electron phase

The algorithm (9) can be extended over the whole $c_{\pm}$ range, $0 < c_{\pm} < 1$, provided the crystal with one particle per cell («S′-crystal» with PTVs $\mathbf{d}_{\pm}$, $\mathbf{f}_{\pm} + \xi_{\pm}$) which arises at $c_{\pm} = 1$ ($\theta = Q \pm \sigma_{\pm}$) is stable (i.e. it is another S-crystal) or metastable. This follows from the fact that i/ owing to the coalscence of defectons macroscopically small variations in θ generate, at any $c_{\pm}$, $1D$ defects only; ii/ these $1D$ defects, according to our computer calculations, have the same PTV, $\mathbf{d}_{\pm}$, for all $c_{\pm}$.

Moreover, due to (meta)stability of the S′-crystal the algorithm (9) holds over a θ range adjacent to the interval $|Q - \sigma_-, Q + \sigma_+|$. In such a case Eq. (9) determines a mixture of stripes of new geometry which are characterized by a new triple of vectors, $\mathbf{d}_{\pm}'$, $\mathbf{f}_{\pm}'$, $\xi_{\pm}'$, the analogues of $\mathbf{d}_{\pm}$, $\mathbf{f}_{\pm}$, $\xi_{\pm}$, and the ±stripes concentration $c_{\pm}' = |\theta - Q \pm \sigma_{\pm}|/|\mathbf{d}_{\pm}' \times \xi_{\pm}'|$.

Transition from one geometry to another is continuous in θ since $c'_{\pm}$ goes to zero when $\theta \to Q + \sigma_{\pm}$.

Continuously extending the algorithm (9) in the manner shown above, we obtain the «j-branch» (we introduce the index j again) which comprises all (meta)stable structures Eq. (9) connected in continuity with the starting S-crystal. The corresponding energy, $E_j(\theta)$, can be easily found in terms of $V^{\pm}_{ss}(l)$, using Eq. (9). As a rule, there exist different S-crystals belonging to the same j-branch. On the other hand, as we have computed, intersections of different $E_j(\theta)$, and hence, *zero-temperature first-order transitions in variables* μ *or* P (a type of polymorphism), are universally present in the $2D$ FEP. (See example in Sec. 7). The dependence of E_g on θ is the function which comprises all stable portions of all $E_j(\theta)$.

Thus, owing to the LOD described above the GS of the $2D$ FEP is fully determined by the S-crystals PTVs, $m^j_{\kappa\lambda}$, the «directors», $\mathbf{d}^j_{\pm}$, and the displacement vectors, $\xi^j_{\pm}$, together with the set of $E_j(\theta)$ intersection points which are the only GS characteristics changing on small variations in $v(r)$. All these quantities can be computed on the basis of Eq. (5) and Eq. (9) by a self-consistent procedure, finding the S-crystals together with the j-branches. We have found the GS for triangular and square host lattices as well as for a number of those with central symmetry only. The computation has not revealed principal differences between GS properties of $2D$ FEP with different geometry of host lattices, at least for those which are not significantly anisotropic.

7. Example

Here we illustrate the above general results with a triangular host lattice (THL). All triangular lattices on the THL are necessarily S-crystals. This follows from the fact that it is the triangular lattice that realizes the *absolute* energy minimum of the system whose electrons are free to move. Such S-crystals are «p,q-crystals» with PTVs $p\mathbf{a}_1 + q\mathbf{a}_2$, $p\mathbf{a}_2 + q\mathbf{a}_3$ and $\theta = p^2 + q^2 - pq$ (p, q are arbitrary integers, $\mathbf{a}_{1,2,3}$ is a triple of THL PTVs which are equal in the modulus and form an angle of 120° with each other). Using the procedure discussed in Sec. 6, we have found that all $0,q$-crystals belong to the same j-branch (the main branch), which covers the range $4 \le \theta < \infty$. The S′-crystals of the $0,q$-ones are S-crystals too. They occur at $\theta = q(q+1)$ ($2 \le q < \infty$) and have PTVs $q\mathbf{a}_\kappa$, $(q+1)\mathbf{a}_\lambda$ (κ, $\lambda = 1, 2, 3$; $\kappa \ne \lambda$). The stripe structures (9) have the same PTV, $q\mathbf{a}_\kappa$, for all θ of the interval $[q(q-1), q(q+1)]$, their $\xi_{\pm}$ being $\pm\,\mathbf{a}_\lambda$ ($\kappa \ne \lambda$).

When p, $q \ne 0$, j-branches of different p,q-crystals are distinct. They do not have mutual intersections, but all intersect the main branch, the intersections occurring at rather small concentrations of the p,q-crystals' ±stripes. In other words, the intervals of p,q-crystals stability (p, $q \ne 0$), and correspondingly main-branch metastability, turn out to be narrow.

8. The limit of $\gamma >> 1$

So far, the more frequent case of $\gamma \lesssim 1$ (Sec. 3) has been discussed. Here we outline the limiting case $\gamma >> 1$. It is realized when the Coulomb interelectron forces are screened by conductors that are at distances $>> r_{ee}$ from the $2D$ FEP. Modelling such a situation by the potential $v(r) \propto r^{-1} \exp(-r/R)$ with $R >> r_{ee}$, we have computed that the energies $\epsilon_{\pm}$, $\varepsilon_{\pm}$ satisfy the inequalities $\varepsilon_- < \epsilon_- < \epsilon_+ < \varepsilon_+$, which are opposite to those of Eq. (7). Due to this fact it should be expected that the GS for $\rho - 1/Q << 1/Q$ is a superstructure of separated S-crystal *zero-dimensional* defects, but not one-dimensional ones (stripes). We have revealed that these zero-dimensional defects are «bidefectons», which are complexes consisting of two bound defectons. Well-separated bidefectons can be considered as new particles on the S-crystal as the host lattice, the mean bidefecton separation, r_d , being equal to $\sim |2(\rho - 1/Q)|^{-1/2}$. The effective pair potential of a mutual bidefecton repulsion is characterized by the same space parameter, R, as $v(r)$. If $r_d \gtrsim R$, the bidefectons, according to the general results of Secs. 3, 4, are bound to be ordered into stripes arranged by the algorithm (9). Extention of this reasoning to the case of $R >> r_d$ leads to new stripe-like superstructures consisting of zero-dimensional defects of «new» S-crystals, and so on. Eventually a well-developed fractal arises. Though details of its structure are still to be determined, it is safe to say now that the LOD does take place for $\gamma >> 1$, too.

9. Summary

The above consideration shows that the electron ordering into stripes and the effective lowering of dimension reside in the $2D$ FEP universally. In essence, a combination of discreteness of electrons' positions with a long-ranged interelectron repulsion is the only factor which gives rise to this phenomenon. For this reason it is also bound to arise with an external disorder present, the stripes being fractured and pinned by the disorder. Thus, stripe formation in $2D$ and layered narrow-band conduc-

tors can be considered to be the principle signature of a 2*D* FEP.

The charge ordering in cuprates as a manifestation of a 2D «frosen» electron phase

From the above standpoint the charge ordering in CuO_2 planes of *high-temperature superconductors* (cuprates) [11] (neutron scattering), [12] (channeling) is of especial interest. The fact that it takes place even with very low doping [12] suggests that a 2*D* FEP might be present in these systems primordially. One can envision that formation of ionized oxygen molecules, O_2^{2-}, in oxygen planes gives a certain energy gain even in cuprates of the *stoichiometric* composition*. In consequence, a part of electrons leaves the oxygen planes for *s*-orbits of Cu^{2+} ions in CuO_2 planes, resulting in formation of a number of ions Cu^+. Since the amplitude of electron hopping $Cu^+ \leftrightarrow Cu^{2+}$ is relatively small, the Cu^+ ensemble should be expected to be a 2*D* FEP, the concentration of the Cu^+ and, correspondingly, of the O_2^{2-} being determined by thermodynamic equilibrium between the 2*D* FEP and the ensemble of the O_2^{2-}. It is evident that stripe formation in the 2*D* FEP of Cu^+ ions inevitably brings to existence O^- superstructures in CuO_2 planes. Their PTVs are likely to be the same as that of the Cu^+ FEP.

In the connection with the aforesaid it should be noted that a simple explanation of the high-temperature superconductivity can be offered in terms of the 2*D* FEP taking into account the finiteness of the bandwidth [1]. It lies in the fact that a virtual exchange of 2*D* FEP elementary excitations between oxygen holes (which are known to be free charge carriers in the doped cuprates) leads inevitably to a mutual effective attraction of the holes and thereby to superconductivity (of purely Coulomb origin) with high T_c . Our preliminary studies have shown that the lowest-energy elementary excitations in the cuprate 2*D* FEP are kinks on the disorder-fractured stripes.

Some expectable features of the 2D «frosen» electron phase thermodynamics and conductivity as a consequence of the stripe formation

Our preliminary studies have shown that the effective lowering of dimension in the ground state of the 2*D* FEP accounts for a fairly interesting and unusual low-temperature thermodynamics. It is characterized by first-order transitions in T, μ-plane (T is the temperature) from the FEP crystals (Sec. 5) slightly perturbed by an ideal gas of separate defectons (they are zero-dimensional defects which arise due to thermal activation) to a strongly correlated liquid of thermally fractured stripes («FEP liquid») where there is no a long-range order. The melting temperature as the function of μ turns out to be reduced to zero at the endpoints of the intervals of the devil staircase. Therefore, at any $T \neq 0$ there is a set of alternating μ intervals which correspond to the FEP crystals or the FEP liquid.

Conduction in the 2*D* FEP liquid is expected to be by movement of kinks of the fractured stripes, each kink carrying a fractal charge (measured in units of e). That in the FEP crystals is of the common Drude type, the charge carriers being $\pm$defectons with the charge $\pm e$. With a change in μ (at a fixed T) these conduction mechanims alternate, resulting in pronounced 2*D* FEP resistivity oscillations which reflect the ground-state devil-staircase dependence of ρ on μ: the oscillations' peaks are bound to occur close to the *rational* filling factors of the FEP crystals which survive at a given T. This phenomenon is yet another distinctive mark of the 2*D* FEP. We have found it to be very similar to the resitivity oscillations of a conductive sheet in a system metal–*n*-type semiconductor–*p*-type semiconductor [3], which still remain to be explained. We are going to publish the results concerning this issue in the near future.

It is remarkable that an artificially created external perturbation localized within a small region can block up conduction over all FEP liquid, pinning only one stripe. The most appropriate systems to test this experimentally are perhaps granular thin films like those described in [4]. A similar phenomenon was reported in [5]. Yet granular films used in the experiments [5] were highly disordered, and it is unclear now whether the above theory works in such a situation.

Acknowledgments

We gratefully acknowledge discussions with Prof. M. Pepper, Prof. B. Shklovskii, and Prof. P. Wiegmann.

* A point in favor of this fact is a noticeable disorder ($\sim$ 20%) in arrangement of O^- along *c*-axis of $La_{2-x}Sr_xCuO_4$ which was revealed (by channeling method [13]) even for low x.

1. A. A. Slutskin and L. Yu. Gorelik, *Fiz. Nizk. Temp.* **19**, 1199 (1993); [*Low Temp. Phys.* **19**, 852 (1993)], A. A. Slutskin, *Usp. Fiz. Nauk* **165**, 707 (1995) [*Physics Uspekhi,* **38**, 669 (1995)].
2. W. Heywang, *Amorfe und Policrystalline Halbleiter*, Springer Verlag (1984); A. A. Slutskin and V. I. Makarov, in: *Proc. of Electroceramics* **5**, September 1996, Aveiro Portugal, p. 267.
3. M. Pepper. *J. Phys.* **C12**, L617 (1979).
4. R. G. Osifchin, W. J. Mahoney, J. D. Bielefeld, R. P. Andres, J. I. Henderson, and C. P. Kybiak, *Superlattices and Microstructures* **18**, 283 (1995).
5. H. Nejoh and M. Aono, *Apll. Phys. Lett.* **64**, 2803 (1994).
6. V. L. Pokrovsky and A. L. Talapov, *Theory of Incommensurate Crystals*, N. Y., Harward Acad. (1984), I. F. Ljuksutov, A. G. Naumovets, and V. L. Pokrovsky, *Two-Dimensional Crystals*, Naukova Dumka, Kiev (1988).
7. J. Hubbard, *Phys. Rev.* **B17**, 494 (1978).
8. P. Bak and R. Bruinsma, *Phys. Rev. Lett.* **49,** 249 (1982).
9. Ya. G. Sinai and S. Ye. Burkov, *Usp. Matemat. Nauk* **38**, 205 (1983).
10. M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, *Phys. Rev.* **B54**, 1853 (1996).
11. J. M. Tranquada, B. J. Sternlieb, J. D. Axe, Y. Nakamura, and S. Uchida, *Nature* (*London*) **375**, 561 (1995).
12. A. S. Borovik, E. A. Kovaleva, and V. I. Makarov, *Phys. Lett.* **A172**, 457 (1993).
13. V. I. Makarov, R. P. Slabospitsky, N. A. Skakun, A. S. Borovik, A. P. Voronov, A. Yu. Grinchenko, V. S. Malishevscy, and V. A. Oleinik, [*Fiz. Nizk. Temp.* **17**, 476 (1991) [*Sov. Low Temp. Phys.* **17**, 251 (1991)].