Influence of the director anchoring energy on a light induced Freedericksz transition threshold in the bounded light beams

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Director transition threshold values are numerically calculated for the homeotropic nematic cell under the field of spatially bounded light beams in depending on the director anchoring energy with the cell surface. Analytical expressions for the threshold in the wide and narrow light beams are obtained for the cases of strong and weak director anchoring. It is shown that influence of the anchoring energy finite values increase with decreasing of the beam cross-section. It is appear to be the most strong at the weak director anchoring.

Численно рассчитаны значения порога переориентации директора в гомеотропной нематической ячейке в поле пространственно ограниченных световых пучков в зависимости от энергии сцепления директора с поверхностью ячейки. В пределе сильного и слабого сцепления директора найдены аналитические выражения для порога в широких и узких пучках света. Показано, что влияние конечных значений энергии сцепления возрастает с уменьшением поперечного размера пучка. Оно оказывается наиболее сильным при слабом сцеплении директора.

Threshold reorientation of the director of the nematic liquid crystals (NLC) in the light fields – a light induced Freedericksz transition (LIFT) [1–3] attracts an increased attention due to a potential of the effect for applications in electrooptical devices. Magnitude of the threshold is the most important parameter of LIFT and therefore its dependence on different parameters of the liquid crystal cell and the light beam was studied in many papers. In particular, there were considered influence of the limited cross size of the light beam on the LIFT threshold [1,2], possibility of the considerable decreasing of a threshold in the nematic which contains small concentration of the impurity molecules [4–7], the threshold reorientation of the director in the light field with spatially modulated intensity [8,9]. Influence of the director finite anchoring energy with the cell surface on LIFT was also studied in papers [7–9]. In particular, the dependence of the threshold on the anchoring energy value was found but only for the case of the infinitely wide incident light beams. The conventional light beams possess a finite width and in the qualitative sense it is clear that in this case the dependence of the director reorientation threshold on its anchoring energy value must be stronger. In current paper the quantitative results are obtained which allow estimating of the degree of dependence of the LIFT threshold on the director anchoring energy value for the beams with finite cross section.

Let we have the planeparallel cell of NLC bounded by the planes $z = 0$ and $z = L$ with the initial homeotropic director orientation along the Oz axis. Plane-polarized along the Ox axis monochromatic light wave is normally incident on the cell. We suppose that the cross size of the light beam is limited along the Oy axis. As long as the threshold of the orientational instability is reached the director reorientation takes place in the xOz plane. Then taking into account that system is homogeneous in the Ox direction we can seek the director in the cell bulk in the form

$$\mathbf{n} = \mathbf{e}_x \cdot \sin\varphi(y,z) + \mathbf{e}_z \cdot \cos\varphi(y,z), \tag{1}$$

where $\mathbf{e}_x$, $\mathbf{e}_z$ are the Cartesian unit vectors, φ is the director deviation angle from its initial direction.

Minimizing the free energy of the NLC cell one can obtain the next linearized in the angle φ equation to determine the director reorientation threshold [2]:

$$K_2\,\frac{\partial^2\varphi}{\partial y^2} + K_3\,\frac{\partial^2\varphi}{\partial z^2} + \frac{\varepsilon_a\,\varepsilon_\perp}{8\,\pi\,\varepsilon_{||}}\,I(y)\varphi = \eta\,\frac{\partial\varphi}{\partial t}\,, \qquad I(y) = |E_0(y)|^2, \tag{2}$$

where K_2, K_3 are the Frank's elastic constants, $\varepsilon_a = \varepsilon_{||} - \varepsilon_\perp > 0$ is the anisotropy of nematic dielectric permittivity at the frequency of the incident light, $E_0(y)$ is the electric field amplitude of the incident monochromatic light wave, η is the relaxation constant, $\eta \sim (10^{-2} \div 1)$ P [2].

Describing interaction of the director with the cell surface in the approximation of Rapini potential [10] one can also obtain the linearized bound conditions to equation (2)

$$\begin{aligned} &\left[K_3\,\frac{\partial\varphi}{\partial z} + W\,\varphi\right]_{z=L} = 0, \\ &\left[K_3\,\frac{\partial\varphi}{\partial z} - W\,\varphi\right]_{z=0} = 0, \end{aligned} \tag{3}$$

where $W > 0$ is the director anchoring energy with the cell surface.

Let the distribution of intensity in the light beam cross section is described by the function

$$I(y) = \begin{cases} I_0 = \text{const}, & \text{if} \quad |y| \leqslant a, \\ 0, & \text{if} \quad |y| > a. \end{cases} \tag{4}$$

Then solving equation (2) with the variable separation method and demanding the bound conditions (3) to be satisfied we obtain

$$\varphi(y,z,t) = Y(y)\sum_{n=1}^{\infty} C_n\left(\cos(\lambda_n z) + \frac{W}{K_3\lambda_n}\sin(\lambda_n z)\right)e^{\Gamma_n t}\,, \tag{5}$$

where C_n is a constant of integration, $\Gamma_n = \dfrac{1}{\eta}(K_2 q^2 - K_3\lambda_n^2)$, and the function $Y(y)$ takes a form

$$Y(y) = \begin{cases} \cos\left(\sqrt{\xi^2 - q^2}\,y\right), & \text{if} \quad |y| \leqslant a, \\ \cos\left(\sqrt{\xi^2 - q^2}\,a\right)\exp\left[-q(|y| - a)\right], & \text{if} \quad |y| > a. \end{cases} \tag{6}$$

Here $\xi^2 = \dfrac{\varepsilon_a\varepsilon_\perp I_0}{8\pi\varepsilon_{||}K_2}$, q is determined by equation $\sqrt{\xi^2 - q^2}\,\tan\left(a\sqrt{\xi^2 - q^2}\right) = q$, and λ_n denote the nontrivial solutions to equation

$$\tan(\lambda L) = \frac{2WK_3\lambda}{(\lambda K_3)^2 - W^2}\,. \tag{7}$$

The director orientational instability threshold can be obtained from condition $\Gamma_1 = 0$, where λ_1 is the least module root to equation (7). In general case the threshold value can be obtained only numerically. Therefore initially we will consider the limiting cases when it is possible to obtain also the analytical expressions for the threshold.

Let us suppose that the director anchoring with the cell surface is strong so that the dimensionless anchoring energy $w = \dfrac{WL}{K_3} \gg 1$. In this case it appears from equation (7) that $\lambda_1 = \dfrac{\pi}{L}\left(1 - \dfrac{2}{w}\right)$. Then for the wide light beams such that $a \gg L$ we can obtain from equation $\Gamma_1 = 0$ the next expression for the threshold

$$I_{0p} = I_\infty \left(1 + \frac{L^2 K_2}{4a^2 K_3} - \frac{4}{w}\right), \tag{8}$$

where $I_\infty = \dfrac{8\pi^3 \varepsilon_{||} K_3}{\varepsilon_a \varepsilon_\perp L^2}$ is the threshold value in the field of the infinitely wide light beam at the infinitely strong director anchoring with the cell surface [1].

For the narrow beams when the condition $a \ll L$ is fulfilled, equation $\Gamma_1 = 0$ at the strong anchoring ($w \gg 1$) gives the threshold value

$$I_{0p} = I_\infty \left[1 + \frac{L}{\pi a}\sqrt{\frac{K_2}{K_3}} - \frac{4}{w}\left(1 + \frac{L}{2\pi a}\sqrt{\frac{K_2}{K_3}}\right)\right]. \tag{9}$$

The results (8), (9) coincide with the results of paper [2] in the case of the infinitely strong director anchoring with the cell surface($w = \infty$) and with the results of paper [7] in the case of the infinitely wide light beams ($a = \infty$). Comparing formulas (8) and (9) one can see that in the case of the narrow beams the contributions to the threshold value from the director anchoring energy and light beam cross size are not additive unlike the case of the wide beams.

If the director anchoring with the cell surface is weak so that parameter $w \ll 1$, one can obtain from (7) that $\lambda_1 = \dfrac{\sqrt{2w}}{L}$. In this case it follows from equation $\Gamma_1 = 0$ that the threshold value equals

$$I_{0p} = I_\infty \frac{2w}{\pi^2}\left[1 + \frac{K_2}{2wK_3}\frac{\pi^2}{\left(2a/L + \sqrt{2K_2/wK_3}\right)^2}\right] \tag{10}$$

for the case of the wide beams ($a \gg L$) and

$$I_{0p} = I_\infty \frac{2w}{\pi^2}\left(1 + \frac{L}{a}\sqrt{\frac{K_2}{2wK_3}}\right) \tag{11}$$

for the case of the narrow beams ($a \ll L$).

If elastic constant K_2 is not anomalous small then at the weak anchoring the condition $\sqrt{\dfrac{K_2}{wK_3}} \gtrsim 1$ fulfills. In this case for the wide beams at the finite values of parameter w the condition $\dfrac{a}{L} \gg \sqrt{\dfrac{K_2}{wK_3}}$ can fulfill and, as follows from (10), the threshold approximately equals $I_{0p} \approx I_\infty \dfrac{2w}{\pi^2}$. If the opposite inequality fulfills $1 \ll \dfrac{a}{L} \ll \sqrt{\dfrac{K_2}{wK_3}}$, then expression for the threshold contains an additional multiplier $1 + \dfrac{\pi^2}{4}$ and is described by formula $I_{0p} \approx I_\infty \dfrac{2w}{\pi^2}\left(1 + \dfrac{\pi^2}{4}\right)$. For the narrow beams at $\sqrt{\dfrac{K_2}{wK_3}} \gtrsim 1$ one can neglect the first item in formula (11) and expression for the threshold takes form $I_{0p} \approx I_\infty \dfrac{L}{\pi^2 a}\sqrt{\dfrac{2wK_2}{K_3}}$. In this case depending on the ratio of values of parameters $\dfrac{L}{a} \gg 1$ and $w \ll 1$ the threshold value can be both smaller and greater than I_∞.

Note, that for the narrow beams in the limit $a \to 0$ the linear intensity of the incident light beam $S_p = \int_{-a}^{a} I(y)\,dy$ has the threshold value which equals at the strong director anchoring ($w \gg 1$)

$$S_p = I_\infty \frac{2L}{\pi}\sqrt{\frac{K_2}{K_3}}\left(1 - \frac{2}{w}\right) \tag{12}$$

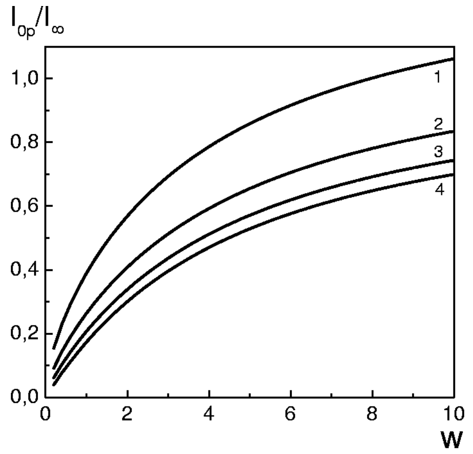

Fig.1. The dependence of the threshold intensity I_{0p}/I_∞ of the one-dimensionally bounded light beam (4) via the anchoring energy value w; $a/L = 0.5(1), 1(2), 2(3), \infty(4)$.

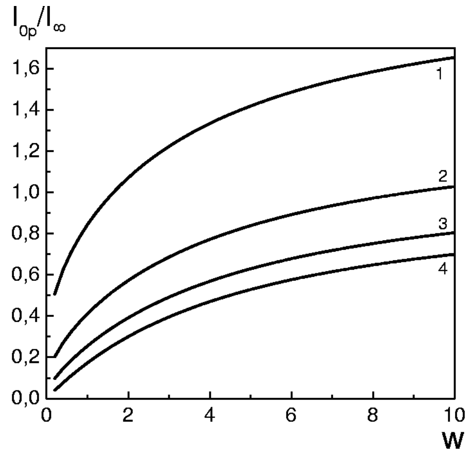

Fig.2. The dependence of the threshold intensity I_{0p}/I_∞ of the two-dimensionally bounded light beam (14) via the anchoring energy value w. $\rho_0/L = 0.5(1), 1(2), 2(3), \infty(4)$.

and at the weak anchoring ($w \ll 1$)

$$S_p = I_\infty \frac{2L}{\pi}\sqrt{\frac{K_2}{K_3}}\frac{\sqrt{2w}}{\pi}. \qquad (13)$$

At Fig.1 the dependence of values of the dimensionless threshold intensity of the incident light I_{0p}/I_∞ via the director anchoring energy value with the cell surface w is presented for several values of ratio a/L. We obtained it numerically in the one constant approximation ($K_2 = K_3 = K$). Value of the threshold intensity I_{0p} decreases monotonically with increasing of ratio a/L and in accordance with formulas (8), (10) approaches in the limiting case $a \to \infty$ to value $I_{0p} = \dfrac{8\pi\varepsilon_{||}K_3\lambda_1^2}{\varepsilon_a\varepsilon_\perp}$ (curve 4).

If the incident light wave is polarized along the Oy axis, it is necessary to replace the elastic constant K_2 by K_1 in the obtained above expressions for the threshold.

At Fig.2 the dependence of values of the director reorientation threshold via its anchoring energy with the cell surface is presented for the case when the light intensity distribution in the beam cross section is bounded along the both coordinates x and y and has a form

$$I(\rho) = \begin{cases} I_0 = \text{const}, & \text{if} \quad \rho \leqslant \rho_0, \\ 0, & \text{if} \quad \rho > \rho_0, \end{cases} \qquad (14)$$

where ρ is a distance to the beam axis, ρ_0 is the beam radius. For calculations there was used equation analogous to equation (2) but taking into account that the director reorientation angle φ is a function of ρ and z. As in the previous calculations the one constant approximation for the Frank's elastic constants was used.

For the two-dimensionally bounded beam (14) the dependence $I_{0p}(w)$ has qualitatively the same form as in the case of the one-dimensionally bounded beam (4), but at the same anchoring energy the dependence of the threshold on the beam width, as one could wait, is stronger in the two-dimensional case. Asymptotic analytic expressions for the director reorientation threshold in the light beam field (14) have the next form:

a) at the strong director anchoring ($w \gg 1$)

$$I_{0p} = I_\infty\left(1 + \frac{J_{01}^2 L^2}{\pi^2\rho_0^2} - \frac{4}{w}\right), \qquad (15)$$

if $\rho_0 \gg L$, and

$$I_{0p} = I_\infty \left\{ 1 - \frac{2L^2}{\pi^2 \rho_0^2 \ln(\pi\rho_0/L)} - \frac{4}{w}\left[1 + \left(\frac{L}{\pi\rho_0 \ln(\pi\rho_0/L)}\right)^2\right]\right\}, \quad (16)$$

if $\rho_0 \ll L$;

b) at the weak director anchoring ($w \ll 1$)

$$I_{0p} = I_\infty \frac{2w}{\pi^2}\left(1 + \frac{L^2 J_{01}^2}{2w\rho_0^2}\right), \quad (17)$$

if $\rho_0\sqrt{2w} \gg L$ and

$$I_{0p} = I_\infty \frac{2w}{\pi^2}\left(1 - \frac{L^2}{w\rho_0^2 \ln(\sqrt{2w}\rho_0/L)}\right), \quad (18)$$

if $\rho_0 \ll L$. Here J_{01} is a first root of the Bessel function $J_0(x)$.

At $\rho_0 \to 0$ the threshold linear intensity of the two-dimensionally bounded beam $S_p = 2\pi \int_0^{\rho_0} I(\rho)\rho\, d\rho$ increases as $\frac{1}{|\ln \rho_0|}$.

Thus, influence of the director anchoring energy with the cell surface on the director reorientation threshold increases with decreasing of the cross size of the light beam incident on the cell. At that for the narrow beams the contribution to the threshold value from the anchoring energy is not additive with the contribution from the beam cross size. It turns out to be the most strong at the relatively weak director anchoring ($w \lesssim 1$).

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Вплив енергії зчеплення директора на поріг світлоіндукованого переходу Фредерікса в обмежених світлових пучках

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Чисельно розраховані значення порогу переорієнтації директора у гомеотропній нематичній комірці у полі просторово обмежених світлових пучків в залежності від енергії зчеплення директора з поверхнею комірки. У граничних випадках сильного і слабого зчеплення директора знайдено аналітичні вирази для порогу у широких та вузьких пучках світла. Показано, що вплив скінченних значень енергії зчеплення зростає із зменшенням поперечного розміру пучка. Він виявляється найбільш сильним при слабому зчепленні директора.