ON THE EFFECTIVE INTERACTION OF WAVES IN INHOMOGENEOUS, NONSTATIONARY MEDIA

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The description of new conditions of effective interaction of waves in periodically inhomogeneous and periodically nonstationary media is given. New conditions, as a special case, contain the known conditions of interaction of waves (synchronism conditions). Examples of the interaction of waves which characteristics satisfy the new interaction conditions are considered. These examples allow to detect new conditions in an experiment.

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INTRODUCTION

It is known that the effective interaction of waves in weakly inhomogeneous, nonstationary and nonlinear media occurs when the following conditions are fulfilled: $\Delta \vec{k} = \sum_i \vec{k}_i = 0$, $\Delta\omega = \sum_i \omega_i = 0$. These conditions mean that the detuning of the frequencies and wave vectors of the interacting waves should be minimal (see, for example, [1 - 3]). This also means that the synchronism conditions between the interacting waves must be satisfied along each of the four axes of the four-dimensional space-time space. We note that often these four conditions are called the laws of conservation of energy and momentum in the interaction of waves. Indeed, if each of these conditions is multiplied by the Planck constant, then these are the laws of conservation of energy and momenta in the interaction of individual photons with each other. In our previous works [4, 5] it was shown that in the general case, in some distributed systems, some other relationships for the frequencies and wave vectors of the interacting waves can be performed for effective wave interaction. This possibility is due to the fact that detuning along one of the directions of the four-dimensional space can be compensated by detunings along other directions. As a result, certain lines (characteristic lines) can be identified in space along which an effective exchange of energy is possible. Effective exchange occurs, in spite of the fact that the known conditions of interaction between waves (see above) are not fulfilled. In this paper we consider the simplest examples of the realization of such a wave interaction. It is shown that in the interaction of two waves in an inhomogeneous nonstationary medium, can arise the waves, whose frequencies do not satisfy the known conditions given above.

1. PROBLEM STATEMENT. BASIC EQUATIONS

Let's consider a medium whose permittivity can be represented as two terms. The first term is a constant. The second term is assumed small, but is a periodic function of space and time. As an example, we can consider the following expression for such a permittivity:

$$\varepsilon = \varepsilon_0 + \tilde{\varepsilon},\ \ \tilde{\varepsilon} = q\cos(\vec{\kappa}\vec{r} - \Omega t),\ q << 1. \qquad (1)$$

Let two electromagnetic waves propagate in such a medium, the wave frequencies of which are different. We will be interested in the conditions for the effective interaction of these waves in such a medium. The equations for each of these waves are the Maxwell equation. From the Maxwell equations it is easy to find the equations for the electric field vectors of each of these electromagnetic waves:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2} = -\vec{\nabla}\left(\frac{1}{\varepsilon}\vec{E}\cdot\vec{\nabla}\varepsilon\right). \qquad (2)$$

By assumption, we have two waves, so we will seek the solution of (2) as the sum of two terms:

$$\vec{E} = \vec{A}_0(\vec{r},t)\exp(-i\omega_0 t + i\vec{k}_0\vec{r}) + \vec{A}_1(\vec{r},t)\exp(-i\omega_1 t + i\vec{k}_1\vec{r}), \qquad (3)$$

here $k_0^2 = \omega_0^2\varepsilon_0 / c^2$, $k_1^2 = \omega_1^2\varepsilon_0 / c^2$.

Let us consider the simplest case, which shows the most important characteristics of the new interaction conditions and which, apparently, is most easily realized in the experiment. We will assume that the interaction occurs between transverse waves, that the medium is periodically non-uniform in only one direction (in the z direction). In this case, the time detuning can be compensated only by a detuning along the z axis. Moreover, we will assume that the waves are located on one dispersion linear branch. In this case, the phase velocities of the waves and their group velocities coincide. As will be seen below, all of these restrictions are non-essential (they are imposed only to simplify the formulas) and, if necessary, can be easily removed. We will also assume that the waves propagate only in one direction – in the direction of the axis. In this case, substituting (3) in (2), we can obtain the following equations for finding the amplitudes $\vec{A}_i$:

$$\begin{cases} \left[\dfrac{\partial^2\vec{A}_0}{\partial z^2} - \dfrac{\varepsilon_0}{c^2}\dfrac{\partial^2\vec{A}_0}{\partial t^2}\right] + 2i\left[k_0\dfrac{\partial\vec{A}_0}{\partial z} + \dfrac{\varepsilon_0\cdot\omega_0}{c^2}\dfrac{\partial\vec{A}_0}{\partial t}\right] = \\ = -\dfrac{q}{2}\dfrac{\omega_1^2}{c^2}\vec{A}_1\cdot\exp\left[i\cdot\delta(z,t)\right], \\ \left[\dfrac{\partial^2\vec{A}_1}{\partial z^2} - \dfrac{\varepsilon_0}{c^2}\dfrac{\partial^2\vec{A}_1}{\partial t^2}\right] + 2i\left[k_1\dfrac{\partial\vec{A}_1}{\partial z} + \dfrac{\varepsilon_0\cdot\omega_1}{c^2}\dfrac{\partial\vec{A}_1}{\partial t}\right] = \\ = -\dfrac{q}{2}\dfrac{\omega_0^2}{c^2}\vec{A}_0\cdot\exp\left[-i\cdot\delta(z,t)\right], \end{cases} \qquad (4)$$

where $\delta(\vec{r},t) \equiv \Delta\vec{k}\cdot\vec{r} - \Delta\omega\cdot t$, $\Delta\vec{k} \equiv \left(\vec{k}_1 - \vec{k}_0 \pm \vec{\kappa}\right)$, $\Delta\omega \equiv \omega_1 - \omega_0 \pm \Omega$.

This system is regorous. It is supposed that the detuning δ though is arbitrary however it is chosen in such a way that only these two waves can interact. Characteristics of the equations (4) without the second derivatives (i.e. subcharacteristics) are parallel to straight lines:

$$\delta(\vec{r},t) \equiv \Delta k \cdot z - \Delta\omega \cdot t = const .$$

It means that derivatives along these subcharacteristics are equal to zero: $\frac{\partial \delta}{\partial \eta} = 0$, here $\eta = z + \tau$, $\tau = t \cdot c / \sqrt{\varepsilon_0}$.

The interaction of waves is due to the small inhomogeneity ($q << 1$) of the dielectric constant. It is natural to expect that the wave amplitudes will change slowly. Therefore, in the system of equations (4), we can omit the second derivatives.

It should be noted that this assumption always requires additional analysis. In particular, as a minimum, the obtaining solutions should be tested to satisfy this assumption. We note that taking into account the second derivatives in the system of equations (4), of course, opens the possibility of the appearance of new solutions, which may be interesting in their own way.

However, the questions arises: "How the presence of second derivatives can change solutions that are obtained without taking into account these derivatives." Will the solutions obtained (within the framework of accounting only the first derivatives) be stable with respect to accounting for second derivatives? "This question can be quite easily studied. Indeed, following [6], we consider one equation from system (4), in which we omit the right-hand side:

$$\left[\frac{\partial^2 A}{\partial z^2} - \frac{\partial^2 A}{\partial \tau^2} \right] - \alpha \frac{\partial A}{\partial z} - \beta \frac{\partial A}{\partial \tau} = 0 . \qquad (5)$$

In equation (5) α and β – are arbitrary constants. Add the following new variable: $\xi = \tau - z$. Equation (5) in the new variables has the form:

$$-4 \frac{\partial^2 A}{\partial \xi \partial \eta} = (\beta - \alpha) \frac{\partial A}{\partial \xi} + (\beta + \alpha) \frac{\partial A}{\partial \eta} . \qquad (6)$$

We will consider the dynamics of the jumps along the sub-characteristics: $\xi = \tau - z = \xi_0 = const$ and $\eta = \tau + z = \eta_0 = const$. For example, the amplitude jump at propagation along the subcharacteristic $\xi = \xi_0 = const$ has form:

$$s = \frac{\partial A}{\partial \xi}(\xi_0^+, \eta) + \frac{\partial A}{\partial \eta}(\xi_0^-, \eta) . \qquad (7)$$

Substituting into Eq. (6) one by one $\xi = \xi_0^+$ and $\xi = \xi_0^-$, also combining obtained the equations, we find the following equation, which describes the dynamics of jump:

$$4 \frac{\partial s}{\partial \eta} = (\alpha - \beta) s . \qquad (8)$$

From this equation it follows that the dynamics will be stable if the following conditions are fullfield: $\mathrm{Re}(\beta - \alpha) \geq 0$. Similarly, we can find the stability condition for the solution as the jump propagates along the second subcharacteristic. Finally, the stability conditions for the solutions obtained by neglecting the second derivatives will look:

$$\mathrm{Re}(\beta + \alpha) \geq 0 , \quad \mathrm{Re}(\beta - \alpha) \geq 0 . \qquad (9)$$

These conditions are quite general. They are suitable for the stability analysis in many applied problems. For example, in the case of the propagation of wave beams in inhomogeneous, nonstationary and nonlinear media. In our case, it is easy to see that the coefficients α and β purely imaginary. This means that in our case the second derivatives are unable to radically change the dynamics of the solutions obtained considering only the first derivatives.

We drop the second derivatives on the left-hand side of system (4). Then the left-hand side of these equations can be regarded as a derivative along the characteristic lines: $\eta = z + \tau = C = const$. Moreover, these directions for interacting waves coincide. We also pay attention to the fact that the right-hand sides of the system of equations (4) contain complex conjugate factors $\exp[i \cdot \delta(z,t)]$. In this case, taking into account that

$$\frac{\partial \delta}{\partial \eta} = 0 \quad \text{or} \quad \Delta\omega = \Delta k \cdot c / \sqrt{\varepsilon_0} \qquad (10)$$

in the system of equations (4) differentiating the left and right sides of the equations with respect to the new variable, we obtain the following equations for determining the amplitudes A_0 and A_1 :

$$\frac{\partial^2 A_i}{\partial \eta^2} + K^2 \cdot A_i = 0 \quad i = \{0,1\} , \qquad (11)$$

where $K^2 = q^2 (k_0 \cdot k_1) / 64$.

Solutions of equations (11) can be, for example, functions:

$$A_0 = a \cos(\mathrm{K}\eta) , \; A_1 = b \sin(\mathrm{K}\eta) .$$

This choice of solutions, in particular, can mean that there is a periodic transfer of energy from one wave to the second wave in the process of their interaction.

The interaction of waves considered above occurs when they propagate in the same direction, in the direction of the positive z axis ($k_0 > 0, \; k_1 > 0$). It is easy to show that if they propagate in the opposite direction ($k_0 < 0, \; k_1 < 0$), then the interaction will occur under the same laws of interaction. However, this interaction will occur along another subcharacteristic. Namely, along $\xi = z - \tau$. In this case, the system of equations (11) will be transformed into a system of equations:

$$\frac{\partial^2 A_i}{\partial \xi^2} + K^2 \cdot A_i = 0, \qquad (12)$$

where $\xi = z - \tau$, $i = \{0,1\}$.

2. INTERACTION IN THE LAYER

In this paper we study new conditions for the effective interaction of waves in inhomogeneous media. In addition to general theoretical considerations, it is of interest to consider some simple case in which the basic elements of the new interaction conditions under consideration were contained and that would be as simple as possible so that they could be realized in the experiment. In this section such case will be considered. It is a layer of thickness L of an inhomogeneous medium. The boundaries of the layer are ideally reflective. Inside the layer, two waves propagate perpendicular to its boundaries. Each of these waves represents the sum of two waves, one of which propagates along the z axis, and the other is a wave reflected from the boundary (Fig. 1).

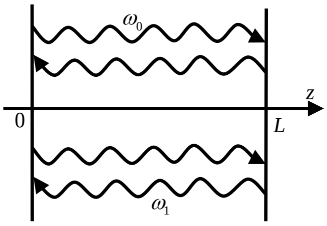

Fig. 1. Interacting waves in layer

Dynamics of interaction of each of these waves has been described above. Expression for the field components in such a system can be represented in such form:

$$\begin{aligned} E_x &= \left[\mathrm{A}\exp(ik_0 z) + B\exp(-ik_0 z)\right]\cdot\exp(-i\omega_0 t) \\ &+\left[\mathrm{C}\exp(ik_1 z) + D\exp(-ik_1 z)\right]\cdot\exp(-i\omega_1 t), \\ H_y &= \left[\mathrm{A}\exp(ik_0 z) - B\exp(-ik_0 z)\right]\cdot\sqrt{\varepsilon}\cdot\exp(-i\omega_0 t) \\ &+\left[\mathrm{C}\exp(ik_1 z) - D\exp(-ik_1 z)\right]\cdot\sqrt{\varepsilon}\cdot\exp(-i\omega_1 t). \end{aligned} \tag{13}$$

In formula (13) ε is homogeneous part (constant) dielectric permeability of the layer $0 < z < L$, $\varepsilon \neq 1$, but A, B, C, D are slow functions of time and coordinate. The analytical type of these functions is determined by the solutions of equations (11) and (12). To determine the constants of these functions, it is necessary to use the boundary conditions. In this case, they are simple:

$$z = 0: \quad A + B = 0; \; C + D = 0.$$

$$z = L: \quad \begin{aligned} &A(L)\cdot\exp(ik_0 L) + B(L)\exp(-ik_0 L) = 0; \\ &C\exp(ik_1 L) + D\exp(-ik_1 L) = 0. \end{aligned} \tag{14}$$

In the system equations (14) functions A, B, C, D can be presented in the form:

$$A = a\cdot\exp(\mathrm{i}\,\mathrm{K}\eta); \quad B = b\cdot\exp(-\mathrm{i}\,\mathrm{K}\xi);$$

$$C = c\cdot\exp(\mathrm{i}\,\mathrm{K}\eta); \quad D = d\cdot\exp(-\mathrm{i}\,\mathrm{K}\xi). \tag{15}$$

In the expressions (15) $\mathrm{a}, \mathrm{b}, c, \mathrm{d}$ constants. Taking these expressions into account, the algebraic system of equations (14) can be rewritten as follows:

$$z = 0: \quad a = -b; \quad c = -d.$$

$$z = L:$$

$$a\cdot\exp\left[i(k_0 + \mathrm{K})L + i\mathrm{K}\tau\right] = -b\cdot\exp\left[-i(k_0 + \mathrm{K})L + i\mathrm{K}\tau\right]. \tag{16}$$

We note that for the effective interaction of waves in the considered dielectric layer, it is necessary that the wave numbers satisfy the following relation:

$$\left(k_0 + \mathrm{K}\right)L = \pi n. \tag{17}$$

Taking into account the relations (16), the expression for the electric component of the total field in the layer can be expressed by the following formula:

$$E = 2ia\begin{Bmatrix} \sin\left[\left(k_0 + \mathrm{K}\right)z\right]\cdot\exp\left(-i\omega_0\tau\right) - \\ -\dfrac{k_0\sqrt{k_0 k_1}}{4k_1^2}\sin\left[\left(k_1 + \mathrm{K}\right)z\right]\cdot\exp\left(-i\omega_1\tau\right) \end{Bmatrix}\cdot \mathrm{e}^{i\mathrm{K}\tau} \tag{18}$$

or

$$\begin{aligned} \operatorname{Re} E &= a\cdot\sin\left[\left(k_0 + \mathrm{K}\right)z\right]\cdot\sin\left[\left(\omega_0 - \mathrm{K}\right)\tau\right] \\ &- a\cdot\frac{k_0\sqrt{k_0 k_1}}{4k_1^2}\sin\left[\left(k_1 + \mathrm{K}\right)z\right]\cdot\sin\left[\left(\omega_1 - \mathrm{K}\right)\tau\right]. \end{aligned} \tag{19}$$

3. INVESTIGATION OF THE DYNAMICS OF INTERACTION OF WAVES BY NUMERICAL METHODS

Conditions of the effective interaction of waves in inhomogeneous media have been verified by numerical methods. For this purpose introducing new dimensionless variables $\tau = \sqrt{k_0 k_1}\cdot t\cdot c/\sqrt{\varepsilon_0}$; $\zeta = \sqrt{k_0 k_1}\cdot z$, and also introducing new dimensionless amplitudes

$$a_0(\zeta,\tau) = \frac{1}{\sqrt[4]{k_0^{-3}k_1^3}}A_0(\zeta,\tau), \; a_1(\zeta,\tau) = \frac{1}{\sqrt[4]{k_0^3 k_1^{-3}}}A_1(\zeta,\tau)$$

from (4) (without taking into account the second derivatives) we will obtain the system of equations for the first derivatives in the form:

$$\begin{cases} \dfrac{\partial a_0}{\partial\zeta} + \dfrac{\partial a_0}{\partial\tau} = -\dfrac{q}{4i}a_1\cdot\exp\left[i\cdot\delta(\zeta,\tau)\right], \\ \dfrac{\partial a_1}{\partial\zeta} + \dfrac{\partial a_1}{\partial\tau} = -\dfrac{q}{4i}a_0\cdot\exp\left[-i\cdot\delta(\zeta,\tau)\right]. \end{cases} \tag{20}$$

Here $\delta(\zeta,\tau) = \Delta k\cdot\zeta - \Delta\omega\cdot\tau$ and introduce dimensionless detunes $\Delta k = \left(\sqrt{\dfrac{k_1}{k_0}} - \sqrt{\dfrac{k_0}{k_1}} \pm \dfrac{\kappa}{\sqrt{k_0 k_1}}\right)$,

$$\Delta\omega = \left(\sqrt{\frac{k_1}{k_0}} - \sqrt{\frac{k_0}{k_1}} \pm \frac{\sqrt{\varepsilon_0}}{c\sqrt{k_0 k_1}}\Omega\right).$$

The initial and boundary conditions are chosen in accordance with the analytical solutions. Amplitude value of the field $a_0 = 1$. The parameter $q = 0.8$, 0.1.

The results of the analytical and numerical analysis of the system of first-order equations obtained from (4) are presented in Figs. 2-4. In Fig. 2 at $q = 0.8$ presented dynamics of the interaction of waves for the case of fulfillment of known synchronism conditions ($\Delta k = \Delta\omega = 0$), also for the case of identical detunings ($\Delta k = \Delta\omega \neq 0$). As can be seen from this plot, the dynamics of the interaction of waves is practically the same. If there is a detuning along one of the directions (for example $\Delta k = 1.095$), there is no effective interaction between the waves (Fig. 3). Analytic and numerical solutions coincide with a good degree of accuracy.

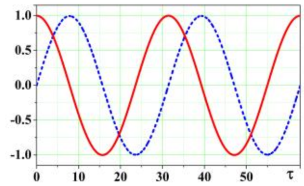

Fig. 2. Analytic and numerical values of the field amplitudes along the line $\zeta = \tau$ for detuning $\Delta k = \Delta\omega = 0$, $\Delta k = \Delta\omega = 1.095$, $\Delta k = \Delta\omega = 0.707$, $q = 0.8$, $\operatorname{Re}(a_0)$ – red line, $\operatorname{Im}(a_1)$ – blue line (dotted line)

Fig. 3 shows that the interaction of waves is practically absent. Indeed, the amplitude of the wave oscillations, which is caused by interaction with other waves in absolute magnitude, is negligible. Besides, it is seen that the frequency of these oscillations of the amplitude corresponds to the value of detuning.

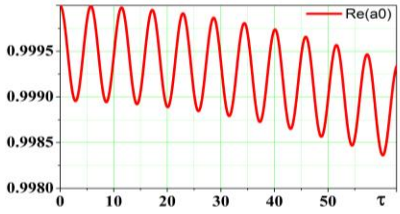

Fig. 3. Value of amplitude $\mathrm{Re}(a_0)$ *along the line* $\zeta=\tau$ *for detuning* $\Delta k=1.095$, $\Delta\omega=0$; $q=0.1$

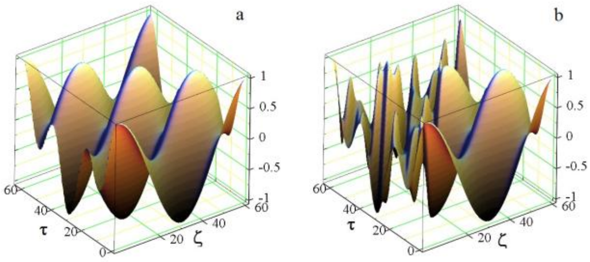

Fig. 4. Plots of amplitude $\mathrm{Re}(a_0)$

a – $\Delta k=\Delta\omega=0$, ***b*** – $\Delta k=\Delta\omega=0.707$; $q=0.8$

Fig. 4 shows that both in the case of the known synchronism conditions and in the case of equal detunings, the amplitude of the initial wave varies periodically along the characteristic line $\zeta=\tau$. Notice that under the known conditions of synchronism the real part of the amplitude of the initial wave is strictly transferred into the imaginary part of the second wave and back. When the new conditions are fulfilled, the energy exchange process is more complicated, which is represent in Fig. 4,b. It should also be noted that, unlike the known synchronism conditions, in which the energy exchange between waves can be observed in the ordinary space, the energy exchange between the waves in the presence of detunings in the general case is effectively observed along the selected subcharacteristic. This feature of the interaction of waves in the presence of detuning is clearly visible from Fig. 2 at $\Delta k=\Delta\omega=1.095$.

CONCLUSIONS

Thus, the results obtained above show that, in addition to the known conditions for the effective interaction of waves ($\Delta k=\Delta\omega=0$), there are more general conditions for energy exchange between the waves.

In the case considered above, these conditions have a simple form $\Delta k=\dfrac{\Delta\omega\sqrt{\varepsilon}}{c}$. Under these conditions, detunings Δk *and* $\Delta\omega$ are almost arbitrary values. As can be seen, these new conditions contain, as a particular case, known synchronism conditions $\Delta k=\Delta\omega=0$.

To visually see the difference in old and new conditions, in the Fig. 5 shows an example of waves which can effectively interact is presented. If to use the known conditions of interaction, then for most of these waves they can't be fulfilled. We use the new conditions of synchronicity. It is easy to see that any triplets of waves that are represented in the figure satisfy the new synchronism conditions $\Delta k=\Delta\omega\sqrt{\varepsilon}/c$. They can efficiently exchange energy. There are infinitely many such triples.

An important example of the conditions under consideration is an example of the interaction of waves in a plane layer, which was considered above. Indeed, as can be seen from the formula (19), the expression for the real component of the electric field can be easily measured in real experiments. It can be seen that the appearance of waves with new frequencies (ω_1) can easily be observed if at the initial point of time only one wave with a frequency (ω_1), in a layer there was only one wave with a frequency (ω_0).

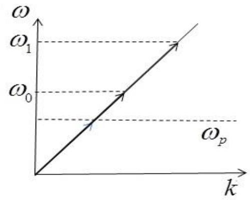

Fig. 5. Dispersion effectively interaction of waves at $\Delta k=\Delta\omega\neq 0$

If we consider an unbounded region of interaction, then the new synchronism conditions are not very convenient to observe. They can effectively manifest themselves, for example, in systems with electron beams. In other cases, it is rather difficult to observe these conditions. However, if the wave interaction region is bounded, for example, as was done in Section 3 above, then it follows from formulas (18), (19) that it is easy to observe these conditions. Indeed, it is enough to fix the probe at an arbitrary point in the layer. And in time, the probe will appear oscillations both at the frequency of the original wave and at frequencies that might not appear when the known synchronism conditions are fulfilled.

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ОБ ЭФФЕКТИВНОМ ВЗАИМОДЕЙСТВИИ ВОЛН В НЕОДНОРОДНЫХ, НЕСТАЦИОНАРНЫХ СРЕДАХ

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Дано описание новых условий эффективного взаимодействия волн в периодически неоднородных и нестационарных средах. Новые условия, как частный случай, содержат известные условия взаимодействия волн (условия синхронизма). Рассмотрены примеры взаимодействия волн, характеристики которых удовлетворяют новым условиям взаимодействия. Рассмотренные примеры позволяют обнаружить новые условия в эксперименте.

ПРО ЕФЕКТИВНУ ВЗАЄМОДІЮ ХВИЛЬ У НЕОДНОРІДНИХ, НЕСТАЦІОНАРНИХ СЕРЕДОВИЩАХ

В.О. Буц, О.П. Толстолужський

Даний опис нових умов ефективної взаємодії хвиль у періодично неоднорідних та нестаціонарних середовищах. Нові умови як окремий випадок містять відомі умови взаємодії хвиль (умови синхронізму). Розглянуто приклади взаємодії хвиль, характеристики яких задовольняють новим умовам взаємодії. Розглянуті приклади дозволяють виявити нові умови в експерименті.