A DISPERSION EQUATION OF THE CYLINDRICAL IDEAL WALL VACUUM CAVITY SINUSOIDALLY CORRUGATED IN AZIMUTHAL DIRECTION. PART I. A PHYSICALLY BASED METHOD OBTAINING OF THE DISPERSION EQUATION

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Dispersive characteristics of a cylindrical cavity with an ideally conducting outer wall has been investigated, whose radius is described by a sinusoidal-periodic dependence on the azimuth angle. From the convergence of the infinite determinant (dispersion equation), we obtain a positive definite bounded algebraic form, whose properties follow the dispersion characteristics of both a smooth and a corrugated cavity. On the basis of the obtained algebraic form, the variances of the first harmonics of a corrugated cavity with an even number of corrugations are investigated. The obtained analytical dependences correspond quantitatively to the experimental data.

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INTRODUCTION

Corrugated resonance systems are widely used in various microwave devices. For effective using of such systems, it is necessary to know exactly their eigenfields and cut-off frequencies. Traditional methods for calculating the cut-off frequencies and eigenfields fields of such complex structures require the use of simplifying assumptions about the form of the fields. The most universal method is the derivation of the dispersion equation in the form of an infinite determinant, followed by its circumcision. However, this method contains a number of obvious drawbacks. In this paper, we propose a different approach to analyzing the dispersion properties of corrugated systems. On a particular example of a sinusoidal corrugated waveguide with ideal walls, the main points of this method are shown.

1. TYPES OF OSCILLATIONS OF A CYLINDRICAL CAVITY WITH SINUSOIDAL CORRUGATED BOUNDARIES IN THE AZIMUTH DIRECTION

Consider a corrugated, ideally conducting metal cavity, in the cross section of which the radius of the lateral surface varies according to the law (see Fig. 1):

$$R(\varphi) = R_0\left(1 + \alpha \cdot \sin(M\varphi)\right), \quad (1)$$

where φ − azimuth angle in a cylindrical coordinate system, $M \gg 1$ is integer, $\alpha = \dfrac{\Delta R}{R_0} < 1$, ΔR is the depth of corrugation, R_0 is average radius of the cavity.

We consider that there is a vacuum inside the cavity. Along the axis z it is unlimited, and is located in the external, directed along the axis of the cavity, a constant magnetic field of strength $\vec{H}_0$ finite quantity.

The possible modes of oscillations of such a metal cavity can be characterized on the basis of the mode of oscillation of the anode block of the magnetron [1].

In the cross section, the cavity is a closed chain of completely identical M hollow cavities, arranged at equal distances from the axis of the cavity (under the hollow cavity we mean the recess of the corrugation).

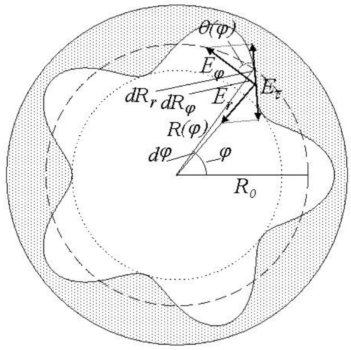

Fig. 1. Cross section and obtaining boundary conditions for the electric field strength in a cavity with an ideally conducting lateral surface. An example is taken of a cavity with a number of corrugations $M = 5$

In the frequency range under consideration, only one (lower) mode of oscillation is excited in each of these cavities. The above-described chain hollow cavity can be regarded as a ring rolled into a periodic sinusoidal retarding system, which is a kind of comb systems with a metallic base.

We assume that the resonance condition for the waves in the considered cross section of the cavity, as in any ring cavity, is the equality of an integer number of cavity wavelengths to the circumference of its mean radius [2]. If we denote the wavelength in the cavity (in the azimuth direction along the surface of the cavity of radius R_0) through λ_m, then the resonance condition in the cavity will be next:

$$2\pi R_0 = n\lambda_m, \; n = 0,1,2,3,\ldots \quad (2)$$

At the same time, condition (2) can be expressed in terms of the phase difference in any neighboring cavities:

$$\varphi_{M,n} M = 2\pi n, \; n = 0,1,2,3,\ldots \quad (3)$$

Consequently, the phase shift of oscillations between cavities can take only discrete values:

$$\varphi_{M,n} = \frac{2\pi n}{M}. \quad (4)$$

Thus, in the general case, expression (4) indicates the existence in the cavity M modes.

Analogously to the definition of the mode of oscillations in magnetron cavities adopted in [1], we consider the case, where $M = 2m$ – even number.

When $n = 0$ electromagnetic oscillations in all cavities occur synchronously (there is no phase shift). When $n = M/2 = m$ the neighboring cavities oscillate in antiphase, i.e. with a phase shift $\varphi_{M,m} = \pi$.

By analogy with oscillations in magnetrons, we will call this mode of π-type oscillations, and we will consider it as the main form of oscillations of a hollow cavity.

For even M oscillations with a phase shift for n in the range $m < n < 2m$ do not differ in physical content from those obtained for $0 < n < m$. Thus, it can be stated that all types of oscillations with $n \neq 0$ and $n \neq m$ are degenerate; pair wise have the same frequencies. It is known that degenerate modes of oscillations are not used in magnetrons. Therefore, they are of no interest for the investigation of the types of waves in the hollow cavity we are considering.

On the basis of the foregoing, we will further consider everywhere the π-type oscillations with an even number of corrugations $M = 2m$.

2. THE DISPERSION PROPERTIES OF THE CAVITY WITH SINUSOIDAL CORRUGATED BOUNDARIES IN THE AZIMUTH DIRECTION

We will assume that the dependence of the electric $\vec{E}(\vec{r},t)$ and magnetic $\vec{H}(\vec{r},t)$ fields from time and coordinates along the axis of the cavity is given by a factor $\exp(i(k_z z - \omega t))$, which will be omitted in the future.

To describe the field TE electromagnetic waves it is enough to set the component $H_z(r,\varphi)$, since the remaining components are determined by the following expressions:

$$E_r(r,\varphi) = i\omega \frac{1}{rk_\perp^2}\frac{\partial H_z}{\partial \varphi};\ E_\varphi(r,\varphi) = -i\omega \frac{1}{k_\perp^2}\frac{\partial H_z}{\partial r};\ E_z(r,\varphi) = 0;$$

$$H_r(r,\varphi) = ik_z \frac{1}{k_\perp^2}\frac{\partial H_z}{\partial r};\ H_\varphi(r,\varphi) = ik_z \frac{1}{rk_\perp^2}\frac{\partial H_z}{\partial \varphi}; \qquad (5)$$

where $k_\perp = \sqrt{k^2 - k_z^2}$ is the transverse wave number, $k = \dfrac{\omega}{c}$, c is the speed of light in vacuum.

Component $H_z(r,\varphi)$ is a solution of the homogeneous Helmholtz equation in the cross section of the cavity *S*:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \varphi^2} + k_\perp^2 H_z = 0. \qquad (6)$$

The solution of equation (6) must satisfy the boundary condition on the ideally conducting lateral surface of the cavity, which corresponds to zero tangential component of the electric field strength TE of the wave:

$$E_\tau\big|_{r=R(\varphi)} = \cos(\theta(\varphi))\cdot E_\varphi - \sin(\theta(\varphi))\cdot E_r. \qquad (7)$$

Angle value $\theta(\varphi)$ in (7) is determined according to Fig. 1 geometric constructions, where

$$dR_r = \alpha M R_0 \cos(M\varphi)d\varphi,\ dR_\varphi = R(\varphi)d\varphi,$$

$$tg(\theta(\varphi))dR_r / dR_\varphi = \alpha M R_0 (R(\varphi))^{-1}\cos(M\varphi).$$

On the cavity axis, the solution of equation (6) must be limited:

$$\left|\vec{H}(\vec{r},t)\right|_{r=0} < \infty. \qquad (8)$$

Due to the azimuth periodicity of the corrugated cavity with period π/m, we represent the solution of (6) in the form of a Fourier series with respect to the angle φ:

$$H_z(r,\varphi) = \sum_{l=-\infty}^{\infty} A_l J_{lm}(k_\perp r)e^{ilm\varphi}, \qquad (9)$$

where A_l is the amplitude of *l*-th harmonic, $J_n(x)$ is the Bessel function of the first kind *n*-th order from the argument x.

For π-type oscillations the perturbed magnetic field (9) in the neighboring corrugations oscillate in antiphase, i.e.

$$H_z\left(r,\varphi+\frac{\pi}{m}\right) = \sum_{l=-\infty}^{\infty} A_l J_{lm}(k_\perp r)e^{ilm\left(\varphi+\frac{\pi}{m}\right)} = \sum_{l=-\infty}^{\infty} e^{il\pi} A_l J_{lm}(k_\perp r)e^{ilm\varphi}.$$

Hence it follows that in order to satisfy the condition for the existence of π-type oscillations index l in (9) must take odd values: $l = 2l' - 1$, where $l' = \ldots; -3; -2; -1; 0; 1; 2; 3; \ldots$ are natural numbers. In this case, the phase opposite condition is fulfilled for the corrugation period:

$$H_z\left(r,\varphi+\frac{\pi}{m}\right) = e^{-i\pi}H_z(r,\varphi) = -H_z(r,\varphi).$$

The expression (9), taking into account the above, is transformed to the form:

$$H_z(r,\varphi) = \sum_{l'=-\infty}^{\infty} A_{2l'-1} J_{m(2l'-1)}(k_\perp r)e^{im(2l'-1)\varphi} =$$

$$= \sum_{l'=-\infty}^{\infty} A'_{l'} J_{m(2l'-1)}(k_\perp r)e^{im(2l'-1)\varphi}. \qquad (10)$$

For the projections of the electric field strength from (10) we obtain expressions:

$$E_r(r,\varphi) = i\omega \frac{1}{rk_\perp^2}\frac{\partial H_z}{\partial \varphi} = -\frac{\omega m}{rk_\perp^2}\sum_{l'=-\infty}^{\infty} lA'_{l'} J_{m(2l'-1)}(k_\perp r)e^{im(2l'-1)\varphi}$$

$$E_\varphi(r,\varphi) = -\frac{i\omega}{k_\perp^2}\frac{\partial H_z}{\partial r} = -\frac{i\omega}{k_\perp}\sum_{l'=-\infty}^{\infty} A'_{l'} e^{im(2l'-1)\varphi}\left.\frac{dJ_{m(2l'-1)}(x)}{dx}\right|_{x=k_\perp r}. \qquad (11)$$

Substituting the values of the fields (11) in condition (7), we obtain the boundary condition on the lateral surface of the cavity in the form:

$$\sum_{l'=-\infty}^{\infty} A'_{l'} e^{im(2l'-1)\varphi}\left(\begin{array}{l} x_m^2 \left.\dfrac{dJ_{m(2l'-1)}(x)}{dx}\right|_{x=x_m} + \\ + i\alpha 2m^2(2l'-1)x_0 \cos(2m\varphi)J_{m(2l'-1)}(x_m) \end{array}\right) = 0, \qquad (12)$$

where after taking the derivative of the Bessel function it is necessary to substitute

$$x = x_m = k_\perp R_0(1 + \alpha\cdot\sin(2m\varphi)).$$

The left-hand side of equation (12) is a periodic function with respect to φ with period π/m. Expanding the left-hand side of equation (12) in a Fourier series on the structure period, we obtain an infinite system of homogeneous equations with respect to the amplitudes $A'_{l'}$:

$$\sum_{l'=-\infty}^{\infty} A'_{l'} C^m_{n,l'} = 0,\ -\infty < n < \infty, \qquad (13)$$

where

$$C^m_{n,l'} = \frac{m}{\pi}\int_{-\pi/2m}^{\pi/2m}\left\{\begin{array}{l} x_m^2 \left.\dfrac{dJ_{m(2l'-1)}(x)}{dx}\right|_{x=x_m} + \\ + i\alpha 2m^2(2l'-1)x_0\, cos(2m\varphi)J_{m(2l'-1)}(x_m) \end{array}\right\} \times e^{i(l'-n)2m\varphi}d\varphi.$$

The condition for the existence of a nontrivial solution of the system of homogeneous equations (13) is the requirement that its determinant be equal to zero:

$$\det C^m_{n,l'} = 0 . \quad (14)$$

Condition (14) is the dispersion equation of the cavity with sinusoidal corrugated boundaries in the azimuth direction. It was first obtained in [3, 4] for π- and 2π-type oscillations. However, in the present paper, when obtaining the dispersion equation for the identification of π-type oscillations a physical criterion is used, consisting in the requirement to change the sign of the field strength for neighboring corrugations.

3. THE DERIVATION OF THE ALGEBRAIC FORM FOR THE CAVITY, WHICH IS CORRUGATED IN THE AZIMUTH DIRECTION

An analytic calculation of the determinant (14) is not possible. However, starting from the property of its convergence, we construct an algebraic functional that, by analogy with the functional obtained in [5], contains the dispersion characteristics of the cavity. We briefly describe the method of obtaining such an algebraic functional.

Suppose, for example, there is a zero infinite determinant $\det W_{n,l} = 0$. The equality of an infinite determinant to zero indicates its convergence. From the convergence of an infinite determinant it follows that the sum of its nondiagonal elements $\sum\limits_{n,l=-\infty}^{\infty} |W_{n,l}|$ $(n \neq l)$ and the product of the elements of the main diagonal $Q_{n,n} = 1 + W_{n,n}$ [6]. The above properties of the elements of a converging infinite determinant correspond to the convergence of an infinite product $\prod\limits_{l=-\infty}^{\infty}\left(1+\sum\limits_{n=-\infty}^{\infty}|W_{n,l}|\right)$, those

$$\prod_{l=-\infty}^{\infty}\left(1+\sum_{n=-\infty}^{\infty}|W_{n,l}|\right) = D < \infty , \quad (15)$$

where *D* is the finite number.

From the inequality $P \le D$, $P = \prod\limits_{l=-\infty}^{\infty}\left(\sum\limits_{n=-\infty}^{\infty}|W_{n,l}|\right)$ it follows that the infinite product *P* also converges.

From the absolute convergence of the infinite product *P*, by virtue of inequality $\prod\limits_{l=-\infty}^{\infty}\left(\left|\sum\limits_{n=-\infty}^{\infty} W_{n,l}\right|\right) \equiv P' < P$, follows the convergence of the infinite product P'.

Thus, on the basis of the above arguments, the convergence of (14) implies the convergence of the infinite product:

$$\prod_{l'=-\infty}^{\infty}\left(\left|\sum_{n=-\infty}^{\infty} C^m_{n,l'}\right|\right) = C_m < \infty , \quad (16)$$

and the convergence of (16) implies the convergence of the infinite product:

$$\prod_{l'=-\infty}^{\infty}\left(\left|\sum_{n=-\infty}^{\infty} C^m_{n,l'}\right|\right) = C'_m = C_m < \infty . \quad (17)$$

For compute C'_m, we use the representation:

$$\sum_{n=-\infty}^{\infty} e^{inm\varphi} = \frac{2\pi}{m}\sum_{k=-\infty}^{\infty}\delta\left(\varphi - \frac{2\pi k}{m}\right). \quad (18)$$

Using (18), we first compute the sum

$$\sum_{n=-\infty}^{\infty} C^m_{n,l'} = x_0^2 \frac{dJ_{m(2l'-1)}(x_0)}{dx} + 2i\alpha m^2 (2l'-1) x_0 J_{m(2l'-1)}(x_0) , \quad (19)$$

and then the infinite product C'_m:

$$C'_m = \prod_{l'=-\infty}^{\infty}\left| x_0^2 \frac{dJ_{m(2l'-1)}(x_0)}{dx} + 2i\alpha m^2 (2l'-1) x_0 J_{m(2l'-1)}(x_0)\right| . \quad (20)$$

Convergent infinite product C'_m in the form:

$$C'_m = \prod_{l'=-\infty}^{\infty}\left| x_0^2 \frac{dJ_{m(2l'-1)}(x_0)}{dx}\right| \cdot \prod_{l'=-\infty}^{\infty}\left|1 + C^m_{l'}\right| , \quad (21)$$

where

$$C^m_{l'} = 2i\alpha m^2 (2l'-1) J_{m(2l'-1)}(x_0)\left(x_0 \frac{dJ_{m(2l'-1)}(x_0)}{dx}\right)^{-1} .$$

We assume that the infinite product on the right-hand side of (21) $C''_m = \prod\limits_{l'=-\infty}^{\infty}\left| x_0^2 \frac{dJ_{m(2l'-1)}(x_0)}{dx}\right|$ converges, those $C''_m < \infty$. Then, using arguments analogous to the convergence of the infinite product (15), we arrive at the convergence of the infinite product $\prod\limits_{l'=-\infty}^{\infty}\left|1 + C^m_{l'}\right| = P'_m / P''_m < \infty$, from which the convergence of the infinite product follows $\prod\limits_{l'=-\infty}^{\infty}\left|C^m_{l'}\right| = P'''_m < P'_m / P''_m < \infty$. Thus, from the property of convergence of the infinite product (21), we can obtain a next bounded algebraic form:

$$\prod_{l'=-\infty}^{\infty}\left|C^m_{l'}\right| = \prod_{l'=-\infty}^{\infty}\left| 2\alpha m^2 (2l'-1) J_{m(2l'-1)}(x_0)\left(x_0 \frac{dJ_{m(2l'-1)}(x_0)}{dx}\right)^{-1}\right| = P'''_m < \infty , \quad (22)$$

where the absence of an imaginary unit in the numerator (22) follows from the property of the modulus of the product of two complex numbers [7].

It should be noted that expression (22) is valid both for positive values of the corrugation depth α, and negative. In what follows we use the following property of convergent infinite products: the discarding of one or a finite number of first factors from a convergent infinite product does not affect its convergence [6]. On this basis, we can conclude that the convergence of (22) implies the convergence of at least one factor $\left|C^m_{l_0}\right| < \infty$, where $l_0 = 1, 2, 3, \ldots$ are the harmonics of the TE oscillation of the cavity. Consider the consequences of the convergence of one factor $\left|C^m_{l_0}\right| < \infty$.

4. DISPERSION PROPERTIES OF THE CAVITY CORRUGATED IN THE AZIMUTH DIRECTION

4.1. DISPERSION PROPERTIES OF A SMOOTH CAVITY

It follows from (22) that for π-type oscillations with $\alpha \to 0$ numerator of a convergent $C^m_{l_0}$ tends to zero. Therefore, for the convergence of the infinite product (22) for the harmonic l_0 it is necessary that the denominator $C^m_{l_0}$ also aspired to zero:

$$\left.\frac{dJ_{m(2l_0-1)}(x_0)}{dx}\right|_{x_0 = k_\perp R_0} = 0 , \quad (23)$$

where due to the lack of corrugation *m* can take any values $m = 0, \pm 1, \pm 2, \ldots$, and $(2l_0 - 1)$ in expression (22), only odd. In contrast to (23), for 2π-type oscillations $2l_0$ in the expression (22) takes only even values.

Therefore, combining these two cases of oscillations (2π-type and π-type), we can conclude that the dispersion relation TE of electromagnetic oscillations in a smooth cavity is determined by the expression (23), where the order of the Bessel function of the first kind can take any integer values. The same result is obtained when solving equations (5), (6) for an ideally conducting smooth cavity as a result of applying the boundary condition (the equality of the tangential component of the electric field strength $\left(E_{\varphi}(R_0,\varphi)\big|_{\alpha\to 0}=0\right)$ on an ideally conductive lateral surface).

In the absence of corrugation $(\alpha \to 0)$ from (23) it is not difficult to obtain the eigenfrequencies of the TE electromagnetic oscillations of a smooth cavity:

$$\omega_{p,i} = c\sqrt{k_z^2 + \frac{\gamma_{p,i}'^2}{R_0^2}}\,, \qquad (24)$$

where $\gamma'_{p,i}$ – i-th zero of the Bessel function derivative $dJ_p(x)/dx$ order p $(p = 0,1,2,3,\ldots)$.

In what follows, when analyzing infinite products of the form (22), one should take into account the fact that the positive zeros of the derivative of the Bessel function n-th order are interspersed with the positive zeros of the Bessel function n-th order, i.e. are arranged as follows [8 - 10]:

$$n \le \gamma'_{n,1} < \gamma_{n,1} < \gamma'_{n,2} < \gamma_{n,2} < \ldots < \gamma'_{n,i} < \gamma_{n,i} < \gamma'_{n,i+1} < \gamma_{n,i+1} < \ldots \qquad (25)$$

4.2. DISPERSION PROPERTIES OF A CORRUGATED CAVITY WITH A FINITE DEPTH OF CORRUGATION ($\alpha < 1$)

4.2.1. DISPERSION PROPERTIES OF A CORRUGATED CAVITY FOR CUT-OFF FREQUENCIES IN THE INTERVAL $0 \le x_0 \le \gamma'_{m(2l_0-1),1}$

As noted above, the singling out of one factor in the infinite product (22), for example $\left|C^m_{l_0}\right|$, does not affect on its convergence. In this case, the following chain of transformations holds:

$$\prod_{l'=-\infty}^{\infty}\left|C^m_{l'}\right| = \left|C^m_{l_0}\right| \cdot \prod_{\substack{l'=-\infty,\\ l'\ne l_0}}^{\infty}\left|C^m_{l'}\right| = \left|C^m_{l_0}\right| D_m = P'''_m < \infty\,.$$

It follows that $\left|C^m_{l_0}\right| = P'''_m \cdot D_m^{-1}$ is a limited quantity that can be represented as a convergent infinite product [10]:

$$\left|C^m_{l_0}\right| = |\alpha| \left|\prod_{n=1}^{\infty}\left(1-\frac{x_0^2}{\gamma^2_{m(2l_0-1),n}}\right)\left(1-\frac{x_0^2}{\gamma'^2_{m(2l_0-1),n}}\right)^{-1}\right| = \frac{P'''_m}{2mD_m} \cdot = A_m < \infty \cdot \qquad (26)$$

Let us analyze the conditions for the convergence of an infinite product (26). It follows from (26) that the cutoff frequency with an increase in the depth of the corrugation should decrease to zero; natural oscillations of the cavity π-type disappear due to the connection of an ideally conducting metal of the corrugations on the cavity axis at α = 1. Fig. 2 shows the approach of the vertices of corrugations with increasing depth of ripple α for $R(\varphi)/R_0 = 1 + \alpha \cdot \cos(M\varphi)$, where $M = 2m = 4$. As follows from Fig. 2 graphs protruding toward the cavity axis of the corrugation apex approach the increase in the corrugation depth, and in the limiting case α = 1 connects.

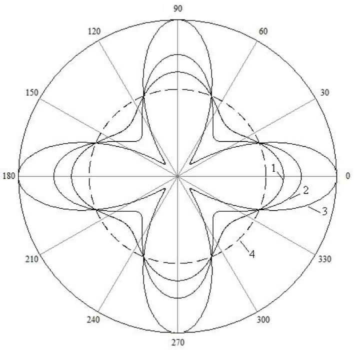

Fig. 2. Approximation of the corners of the ripple $R(\varphi)/R_0$ with an increase α for M = 4: 1 – α = 0.2; 2 – α = 0.4; 3 – α = 0.8. Dotted line 4 determines the average radius of the cavity: $R(\varphi)/R_0 = 1$

When crossing the vertices of the corrugations, the cutoff frequency for oscillations is absent, i.e. it can be set equal to zero: $\left|x_0\right|_{\alpha\to 1} \to 0$. In this limiting case the constant A_m in (26) is equal to unity: $A_m = 1$.

We use the convergence of the infinite product (26) with $A_m = 1$ for describing the dispersion properties of a corrugated cavity with a finite depth of corrugation $(\alpha < 1)$. To this end, we single out in the infinite product (26) in the interval $1 \le x_0 \le \gamma'_{m(2l_0-1),1}$ factor with a singularity, and the remaining infinite product is represented by a function $f_{m(2l_0-1)}(x_0)$, which does not have singularities in this interval:

$$\alpha\left(1-\frac{x_0}{\gamma'_{m(2l_0-1),1}}\right)^{-1} f_{m(2l_0-1)}(x_0) = \pm 1\,. \qquad (27)$$

Comparison of the analytical dependence (27) with the results of numerical calculations [3], confirmed by experimental data [4], shows that the function $f_{m(2l_0-1)}(x_0)$ monotonically increases in the interval $1 \le x_0 \le \gamma'_{m(2l_0-1),1}$, and can be represented in an asymptotic form:

$$f_{m(2l_0-1)}(x_0) = \alpha_{m(2l_0-1)} + \beta_{m(2l_0-1)}x_0 + \delta_{m(2l_0-1)}x_0^2\,, \qquad (28)$$

where $\alpha_{m(2l_0-1)}$, $\beta_{m(2l_0-1)}$ $\delta_{m(2l_0-1)}$ are the constants depending on the azimuth number m and harmonic numbers l_0. The magnitude of these constants is determined numerically or experimentally. Thus, it follows from (28) that in the interval $1 \le x_0 \le \gamma'_{m(2l_0-1),1}$ the following asymptotic dependence of the corrugation depth on the cutoff frequency of the cavity is valid:

$$\alpha = \pm\left(1-\frac{x_0}{\gamma'_{m(2l_0-1),1}}\right)\left(\alpha_{m(2l_0-1)} + \beta_{m(2l_0-1)}x_0 + \delta_{m(2l_0-1)}x_0^2\right)^{-1}. \qquad (29)$$

Representation $f_{m(2l_0-1)}(x_0)$ square trinomial is justified, because calculated at $l_0 = 1$ standard deviation $\chi^2 = \left\langle(\alpha-\alpha_{\exp})^2\right\rangle$ depth of corrugation α from the obtained by the numerical method, and confirmed experimentally $\alpha_{\exp}$ [3, 4], is less than $4.5\cdot10^{-7}$. The above is supported by the data in Table 1.

Table 1

Square deviations χ^2 of the dependence (29) on the experimental data for the different number of corrugations

l_0	m	$\alpha_{m(2l_0-1)}$	$\beta_{m(2l_0-1)}$	$\delta_{m(2l_0-1)}$	χ^2
1	2	0.89079	0.2983	- 0.05127	$1.20272\cdot10^{-7}$
1	3	0.41693	0.77469	- 0.11818	$2.57956\cdot10^{-7}$
1	4	0.3845	0.8395	- 0.10684	$4.50715\cdot10^{-7}$
1	5	0.21638	0.95621	- 0.10319	$2.05052\cdot10^{-7}$
1	6	0.17379	0.97619	- 0.09024	$1.42444\cdot10^{-7}$

In the interval $0 \le x_0 \le 1$ the approximation of the infinite product (26) with the aid of expression (29) is not applicable. Therefore, in this interval, for small values of cut-off frequencies x_0 $(x_0 << 1)$, we represent expression (26) in another asymptotic form:

$$\alpha = \pm\left(1-\lambda^2_{m(2l_0-1)}x_0^2\right), \qquad (30)$$

where the constants $\lambda^2_{m(2l_0-1)}$ (30) and their derivatives with solutions (29) and their derivatives at points with coordinates $\alpha = \hat{\alpha}_{m(2l_0-1)}$ and $x_0 = \hat{x}_{m(2l_0-1)}$.

Table 2 shows the calculated values of the coordinates of the joining points of the asymptotic solutions (29) with (30).

Table 2

Crosslinking coordinates of asymptotic expressions (29) and (30)

l_0	m	2	3	4	5	6
1	$\hat{\alpha}_{m(2l_0-1)}$	0.904	0.710973	0.71038	0.68396	0.67932
1	$\hat{x}_{m(2l_0-1)}$	0.33364	0.998	1.0399	1.17102	1.22247
1	$\lambda_{m(2l_0-1)}$	0.9287	0.53869	0.51207	0.48005	0.46323

The asymptotes (29) and (30) obtained above, each in its definition range, can be regarded as an analytical representation of the dispersion relation, since in the limiting cases of small $(\alpha = 0, m = 0)$ and large $(\alpha \to 1)$ depths of corrugation, they determine the cutoff frequencies of the TE oscillations of a smooth and corrugated cavity, respectively.

Fig. 3 shows the dispersion curves, which characterize the dependence of the corrugation depth α from cut-off frequency x_0 of azimuthally corrugated cavity for mode $l_0 = 1$ and the number of corrugations $M = 2m$, where m = 2,3,4,5,6. Thus, in the interval $0 \le x_0 \le \gamma'_{m(2l_0-1),1}$ for a given number m depth of ripple α are a monotonically decreasing function of the cut-off frequency x_0. Dispersion relations are described as not intersecting, with the exception of the point α=1, curves. When the cut-off frequency x_0 tends to zero the corrugation depth tends to unity.

4.2.2. DISPERSION PROPERTIES OF A CORRUGATED CAVITY FOR CUT-OFF FREQUENCIES IN THE INTERVAL $\gamma'_{m(2l_0-1),1} \le x_0 \le \gamma_{m(2l_0-1),1}$

In this interval of cutoff frequencies, the factor $\left|C^m_{l_0}\right|$ can be represented in the form:

$$\alpha\left(1-\frac{x_0}{\gamma_{m(2l_0-1),1}}\right)\left(1-\frac{x_0}{\gamma'_{m(2l_0-1),1}}\right)^{-1} g_{m(2l_0-1)} = \pm1, \qquad (31)$$

where $g_{m(2l_0-1)}$ are the constants.

From (31) we obtain an expression for the corrugation depth:

$$\alpha = \pm\left(1-\frac{x_0}{\gamma'_{m(2l_0-1),1}}\right)\left(1-\frac{x_0}{\gamma_{m(2l_0-1),1}}\right)^{-1} g^{-1}_{m(2l_0-1)}. \qquad (32)$$

Constants $g_{m(2l_0-1)}$ are determined from the condition that the derivatives $d\alpha/dx_0$ expressions (29) and (32) at the points $x_0 = \gamma'_{m(2l_0-1),1}$.

Constant values $g_{m(2l_0-1)}$ are given in Table 3.

Table 3

Constants $g_{m(2l_0-1)}$

l_0	m	2	3	4	5	6
1	$g_{m(2l_0-1)}$	0.30655	0.2154	0.16358	0.12811	0.101199

In the Fig. 3 numbers of curves 2,3,4,5,6 correspond to the value *m*. Curves $2', 3', 4', 5', 6'$ are mirror image of the axis x_0 curves 2,3,4,5,6.

To compare the obtained analytical dependences with the results of other authors in Fig. 3 shows the points obtained by numerical calculations (markers ◊,○,□,Δ,+) [3], and as a result of experimental studies (markers ■) [4]. In the figure in the range $0 \le \alpha \le 0.3$ a good quantitative agreement of the theoretical representations (29), (32) to the experimental and calculated data is achieved. The noted correspondence indicates the applicability of expressions (29), (32) for describing the dispersion properties of a corrugated cavity with a finite depth of corrugation in the cut-off frequency interval $0 \le x_0 \le \gamma'_{m(2l_0-1),1}$.

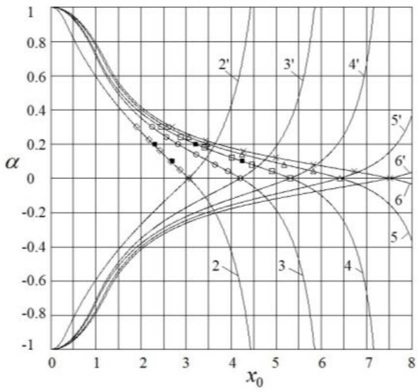

Fig. 3. Dependence of depth of ripple α from cutoff frequency x_0 for first harmonics $(l_0 = 1)$ azimuthally corrugated cavity with a number of corrugations $M = 2m$, where $m = 2, 3, 4, 5, 6$

Thus, the analytical dependencies of the corrugation depth obtained above α from cutoff frequencies x_0 determine the dispersion properties of the first harmonics of the corrugated cavity with the number of corrugations $M = 2m$ $(m = 2, 3, 4, 5, 6)$, since in the range $0 \le \alpha \le 0.3$ they coincide with a high degree of accuracy with the results of numerical calculations and experimental data of other authors.

CONCLUSIONS

The dispersion equation of an ideally conducting cylindrical vacuum cavity with sinusoidal corrugated boundaries in the azimuth direction was obtained. Cavities of this type are basic for the study of the spectra of natural oscillations in gyrotrons. Conditions are determined under which the TE electromagnetic oscillations are nondegenerate. It is shown that for non-degenerate oscillations the number of corrugations must be an even number. For an even number of corrugations, non-degenerate oscillations are π-type oscillations. For π-type oscillations the dispersion equation of the cavity with sinusoidal corrugated in the azimuth direction is obtained. The dispersion equation of such a corrugated cavity is an infinite determinant equal to zero. The equality to zero of the infinite determinant indicates its convergence. From the property of convergence of an infinite determinant, a positive definite bounded algebraic form is obtained from which it is possible to obtain the dispersion characteristics of both a smooth and a corrugated cavity. In the case of a cavity with a small depth of corrugation, analytical expressions are obtained that describe its dispersion properties. On the basis of the fact that when the depth of the corrugation tends to one, there are no natural oscillations of the cavity, an analytical description of the dispersion curves in this region of corrugation depths is offered. It is shown that the dispersion equation is symmetric with respect to the sign of the corrugation depth, i.e. the dispersion equation is valid both for the corrugation depth $\alpha = |\alpha|$, and $\alpha = -|\alpha|$. In this case, in the plane, the depth of the corrugation is the cutoff frequency, the dispersion curves of the cavity are characterized by mirror symmetry with respect to the cutoff frequency axis.

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ДИСПЕРСИОННОЕ УРАВНЕНИЕ ЦИЛИНДРИЧЕСКОГО ВАКУУМНОГО РЕЗОНАТОРА С ИДЕАЛЬНЫМИ ГОФРИРОВАННЫМИ В АЗИМУТАЛЬНОМ НАПРАВЛЕНИИ СТЕНКАМИ. ЧАСТЬ I. ФИЗИЧЕСКИ ОСНОВАННЫЙ МЕТОД ПОЛУЧЕНИЯ ДИСПЕРСИОННОГО УРАВНЕНИЯ

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Исследованы дисперсионные характеристики цилиндрического резонатора с идеально проводящими стенками, радиус которого описывается синусоидально-периодической зависимостью относительно азимутального угла. Из сходимости бесконечного определителя (дисперсионного уравнения) получена положительно определенная ограниченная алгебраическая форма, из свойств которой следуют дисперсионные характеристики как гладкого, так и гофрированного резонаторов. На основе полученной алгебраической формы исследованы дисперсии первых гармоник гофрированного резонатора с четным количеством гофров. Полученные аналитические зависимости количественно соответствуют экспериментальным данным.

ДИСПЕРСІЙНЕ РІВНЯННЯ ЦИЛІНДРИЧНОГО ВАКУУМНОГО РЕЗОНАТОРА З ІДЕАЛЬНИМИ ГОФРОВАНИМИ В АЗИМУТАЛЬНОМУ НАПРЯМКУ СТІНКАМИ. ЧАСТИНА I. ФІЗИЧНО ОСНОВАНИЙ МЕТОД ОТРИМАННЯ ДИСПЕРСІЙНОГО РІВНЯННЯ

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Досліджено дисперсійні характеристики циліндричного резонатора з ідеально провідними стінками, радіус якого описується синусоїдально-періодичною залежністю щодо азимутального кута. Зі збіжності нескінченного визначника (дисперсійного рівняння) отримана додатньо визначена обмежена алгебраїчна форма, з властивостей якої отримуються дисперсійні характеристики як гладкого, так і гофрованого резонатора. На основі отриманої алгебраїчної форми досліджені дисперсії перших гармонік гофрованого резонатора з парною кількістю гофрів. Отримані аналітичні залежності кількісно відповідають експериментальним даним.