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Dynamical regimes reconstruction in three-component bistable system with dispersive relaxation time

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A three-component dynamic system describing a quantum cavity electrodynamic device with a pumping and nonlinear dissipation is studied. Various dynamical regimes are investigated in terms of divergent trajectories approaches and fractal statistics. It has been shown that stable and unstable dissipative structures type of limit cycles can be formed in such system, with variation of pumping and nonlinear dissipation rates. Transitions to chaotic regime and the corresponding chaotic attractor are studied in detail.

Изучается трех-компонентная динамическая система, описывающая лазерное излучение под влиянием внешней накачки и нелинейной диссипации. Проводится исследование различных динамических режимов поведения, анализируя разбегание траекторий и фрактальные свойства. Показано, что при изменении параметров внешней накачки и нелинейной диссипации в системе возможно формирование устойчивых и неустойчивых диссипативных структур типа предельного цикла. Переходы в хаотический режим и свойства соответствующего аттрактора изучаются подробно.

The most interesting phenomenon in the theory of nonlinear dynamical systems is a transition from a regular dynamics to an irregular one. The most popular dynamical model manifesting a chaotic regime is the Lorenz system [1], initially derived for the description of atmospheric flows. Later on, it was widely used in various kinds of physical systems (in laser physics [2], in hydrodynamics (turbulence study) [3], in solid state physics [4], to name just a few); its generalization was used to describe effects of self-organized criticality [5], etc.). As is well known, the chaotic regime is related to the interplay between the instability and nonlinearity. The instability is responsible for the exponential divergence of two neighboring trajectories, while the nonlinearity bounds trajectories within a finite volume of the system phase space. The combination of those two mechanisms results in high system sensitivity to the initial conditions.

Optoelectronic devices are known to be the simplest systems that manifest chaotic mode. In practice, the control of dynamical modes in optoelectronic devices is a very important problem because different types of radiation can be used for different areas of human activity [6, 7]. Recently, the problem of chaos control in laser dynamics has attracted a lot of attention. Currently, new mechanisms resulting in chaotic mode generation were found out [8-10]. According to laser physics, the self-organization processes with formation of dissipative structures or generation of chaotic signal can be stimulated by introducing an additional nonlinear medium into the Fabry-Perot cavity [2, 6, 11]. Experimental studies of the chaotic mode in such systems allow to generalize several theoretical models, and to identify additional nonlinear sources, in order to provide a more detailed theoretical analysis and to predict various types of laser dynamics [12-15].

From a theoretical viewpoint, the problem of recognizing the chaotic mode in dynamical systems and the chaos control is of a special interest in nonlinear dynamical system theory. Moreover, due to the chaotic dynamics, the chaotic systems are characterized by various attractor types. Hence, another very interesting problem is to investigate properties of such attractors. In this paper, we study a chaotic mode in a generalized model of a dynamical system (the initial model doing not exhibit it) for a three-component dynamical system realized in optoelectronic devices. Our aim is to study the possibility of the well known two-level laser system type of Lorenz-Haken model [2, 7], which can manifest chaotic mode only if an additional nonlinear medium (type of washout filter or electro-optical modulator) is inserted into the Fabry-Perot cavity. In our investigation, we use the assumptions of an absorptive optical bistability and consider a three-component system in order to show that a transition towards chaotic mode can be controlled by pumping and bistability parameter. The latter is related to nonlinear dependence of the relaxation time for one of dynamical variables on its value. It will be shown that by varying the system parameters, it is possible to obtain stable and unstable dissipative structures, the period doubling bifurcation and the chaotic regime realization. The corresponding chaotic attractor is studied using various approaches.

First, a model for our system will be presented incorporating a nonlinear dissipation term. The next step will consist in the stability analysis of the system. Further, various dynamical modes and the phase diagram of the system will be considered. Then the main properties of the chaotic behavior of the system will be discussed. The main results and prospects for the future are presented in the conclusions.

Thus, let us consider a three-component dynamical model used widely to describe the self-organization processes:

$$\begin{cases} \dot{\eta} = -\eta + h, \\ \sigma\dot{h} = -h + \eta S, \\ \varepsilon\dot{S} = (r - S) - \eta h, \end{cases} \tag{1}$$

where dot stands for time derivative. This model can be simply derived from the well-known chaotic Lorenz system [1] using the following relations: $t' = \sigma t$, $\eta = X/\sqrt{b}$, $h = Y/\sqrt{b}$, $S = r - Z$, $\varepsilon = \sigma/b$. Here X, Y, Z are dynamical variables of the Lorenz system; σ, b, and r are related constants; we drop the prime in the time t for convenience. The dynamical system (1) can be obtained in the laser theory using the Maxwell equations for electro-magnetic field and the density matrix evolution equation. This results in a system of the Maxwell-Bloch type that is reduced to the Lorenz-Haken model (1) for two-level laser system (see, e.g., [2, 6, 16]). In such a case, η, h, and S are reduced to the electric field strength, polarization, and the inverse population difference of energy levels, respectively. The models of Eq.(1) type are widely used to describe synergetic transitions in complex systems [4], where the above variables η, h, S acquire a physical meaning of order parameter, conjugated field and control parameter, respectively. The quantity r relates to the pumping intensity and measures the environmental influence.

A naive consideration of dynamic modes in the model (1) shows that at $r > 0$ and $\sigma \approx \varepsilon \approx 1$, the behavior is well defined, despite the fact that the model contains nonlinearities in both second and third equations. In this work, we consider a more general case of an absorptive optical bistability model [2, 11]. From the physical viewpoint, the absorptive optical bistability is related to the possibility of the additional medium in the Fabry-Perot cavity (phthalocyanine fluid, gases SF_6, $BaCl_3$ and CO_2 [2]) to transit a radiation at high intensities only, absorbing weak signals. Models of such type were considered for the single component case, where only a stationary behavior of the system was analyzed (see [17] and references therein). We will focus on describing possible dynamic modes in the three-component model. Mathematically, our model takes the form

$$\begin{cases} \dot{\eta} = -\eta + h + f_\kappa, \quad f_\kappa = -\dfrac{\kappa\eta}{1+\eta^2} \\ \sigma\dot{h} = -h + \eta S, \\ \varepsilon\dot{S} = (r - S) - \eta h \end{cases} \tag{2}$$

Here, the additional force f_κ is introduced to take into account the additional medium influence; κ is the so-called bistability parameter proportional to the atomic density. Formally, this means a dispersion of the relaxation time for the variable η. In fact, if we rewrite the first equation as $\dot{\eta} = -\eta/\tau(\eta) + h$, then the dispersive relaxation time is $\tau(\eta) = 1 - \kappa/\left(1+\kappa+\eta^2\right)$.

Our main goal is to study possible scenarios of dynamic mode reconstruction in such a three-component system under the influence of pumping r and the nonlinear dissipation, controlled by κ. We will study the conditions of a chaotic mode appearance. The corresponding irregular dynamics will be studied in detail. It will be shown that chaotic mode can be controlled by pumping and the additional medium properties.

Now let us consider conditions when relaxation time values for all variables η, h, and S are commensurable, i.e., $\sigma \sim \varepsilon \sim 1$. Let the system behavior be presented in the phase space (η, h, S). At first, let us consider the steady states. Setting $\dot{\eta} = 0$, $\dot{h} = 0$ and $\dot{S} = 0$, fixed points can be found described by the values $\eta_0^{(k)}$, $h_0^{(k)}$ and $S_0^{(k)}$, where $k = 1,2,3$. The first one has coordinates

$$\left(\eta_0^{(1)}, h_0^{(1)}, S_0^{(1)}\right) = (0,0,r) \tag{3}$$

and exists always; two symmetrical points, denoted with superscripts 2 and 3, respectively, have coordinates

$$\left(\eta_0^{(2,3)}, h_0^{(2,3)}, S_0^{(2,3)}\right) = \left(\pm\sqrt{r-\kappa-1}, \frac{\pm\sqrt{r-\kappa-1}}{r-\kappa}, \frac{r}{r-\kappa}\right). \tag{4}$$

These two points are realized if condition $r \geq \kappa + 1$ is satisfied. This means that $r_c = \kappa + 1$ is a critical point for the bifurcation of doubling.

Let us perform a local linear stability analysis using the stationary states [18]. Within the framework of the standard Lyapunov exponents approach, the time-depended solutions of the system (2) are assumed to have the form $\vec{u} \propto e^{\Lambda t}$, $\Lambda = \lambda + i\omega$, where $\vec{u} \equiv (\eta, h, S)$, λ controls the stability of the stationary points, ω defines the frequency, as usual. The values of real and imaginary Λ parts are calculated according to the Jacobi matrix elements $M_{ij} \equiv \left(\partial f^{(i)}/\partial u_j\right)_{u_j = u_{j0}}$, $i,j = \eta, h, S$; $f^{(i)}$ represents the right-hand side of the corresponding dynamic equations in (2); the subscript 0 relates to the stationary value. As a result, the Jacobi matrix takes the form

$$M=-\begin{pmatrix} 1+\kappa\dfrac{1-\eta_0^2}{\left(1+\eta_0^2\right)^2}+\Lambda & -1 & 0 \\ -S_0 & 1+\Lambda & -\eta_0 \\ h_0 & \eta_0 & 1+\Lambda \end{pmatrix}. \tag{5}$$

First, let the stability of the fixed point (3) be determined. The solution of the eigenvalue problem yields that the three eigenvalues are real and negative:

$$\begin{aligned} \Lambda_1 &= \lambda_1 = -1 < 0 \\ \Lambda_2 &= \lambda_2 = -\frac{1}{2}\kappa - 1 - \frac{1}{2}\sqrt{\kappa^2+4r} < 0, \\ \Lambda_3 &= \lambda_3 = -\frac{1}{2}\kappa - 1 + \frac{1}{2}\sqrt{\kappa^2+4r} < 0. \end{aligned} \tag{6}$$

It follows that at $r<r_c$, there is only node (3) in the phase space (η,h,S). On the other hand, at $r>r_c$, all three fixed points are realized in the phase space. It follows from our analysis that the fixed point (3) changes its stability and becomes a saddle ($\Lambda_3>0$).

The stability of other two points can be determined from solutions of the cubic equation

$$\begin{aligned} &\Lambda^3+\left[3+\kappa\frac{2+\kappa-4}{(r-\kappa)^2}\right]\Lambda^2+\left[2+\kappa-r-\frac{r}{r-\kappa}+2\kappa\frac{2+\kappa-r}{(r-\kappa)^2}\right]\Lambda \\ &+\left[2+\kappa+\kappa\left(\frac{2+\kappa-r}{r-\kappa}\right)^2\right]=0. \end{aligned} \tag{7}$$

Analytically, we can define only a border of the domain of system parameters where periodic solutions are realized. To consider a possibility of limit cycles formation in the phase space, let us assume $\Lambda=\pm i\omega$ in Eq.(7). This yields the equation for the corresponding border in the form

$$\begin{aligned} &2\kappa\frac{2+\kappa-r}{(r-\kappa)^2}\left[\kappa-r-6+\frac{r}{2}\frac{1}{r-\kappa}\right]-2\kappa^2\left[\frac{2+\kappa-r}{(r-\kappa)^2}\right]^2 \\ &+4(\kappa-1)+3r\left(\frac{1}{r-\kappa}-1\right)=0. \end{aligned} \tag{8}$$

We will show below that stable periodic solutions are realized inside the domain limited by solutions of Eq.(8) in the plane (r,κ).

To describe the dynamic modes of the system under consideration, let the phase diagram shown in the plane (r,κ) be considered, see Fig.1. Here, the dashed line $r=\kappa+1$ divides the plane (r,κ) into two domains denoted I and II and relates to the critical values of r and κ. Inside the domain I, only a node is realized. Above the dashed line, a set of dynamic modes can be observed. Inside the domain II, the node point is transformed into a saddle point and two additional stable appear. In the domain bounded by the solid line, the stable limiting cycles are formed. This line is obtained as a solution of Eq.(8) under assumption that dynamic solutions of the system (2) are periodical, i.e. $\Lambda=\pm i\omega$. If we move from the domain II to the domain III, then the above focuses lose their stability and, as a result, stable limiting cycles are realized. All phase trajectories are attracted by these

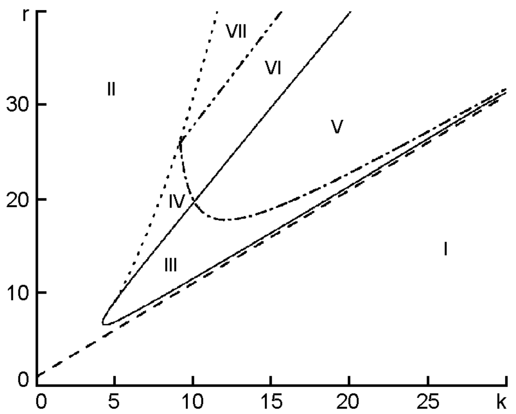

Fig.1. Phase diagram of the system (2) at $\sigma = \varepsilon = 1$

manifolds. The corresponding limiting cycles with unstable focuses are shown in Fig.2a. The domain IV bounded by solid, dotted, and dash-and-dot lines corresponds to the case when two stable and the corresponding two unstable limiting cycles appear (see Fig.2b). Such a transformation of the system dynamic behavior (transition from domain III into domain IV) is a result of double Hopf bifurcation. Here, unstable focuses shown in Fig.2a lose their instability, which results in formation of an unstable limiting cycles inside the stable ones (see Fig.2b). If we cross the dotted line (transition from the domain II into domain IV), then the stability of focuses remains unchanged. Here, two stable limiting cycles and two unstable ones are formed (Fig.2b). The scenario of such a bifurcation is as follows. With an increase in the one of two system parameters r or κ in a point of the dotted line, a new manifold of semi-stable limiting cycle type is formed [3]. If we move to the domain IV, then this semi-stable limiting cycle decomposes into an outer stable one and inner unstable one. Moving in opposite direction (IV $\rightarrow$ II), one of the well known type of Hopf bifurcation occurs: an annihilation of both stable and unstable limiting cycles. To obtain dotted and dash-dotted lines, we use the following approach. We compute numerically a square of the minimum distance $\Delta(t) = \min_{t>0}\left(\eta(t+\tau) - \eta(\tau)\right)^2$. It is obviously that if $\lim_{t\to\infty}\left(\eta(t) - \eta(0)\right)^2 = const$, then the corresponding phase trajectory tends to a stable focus. This criterion is used to obtain the dotted line in Fig.1. The dash-and-dot line corresponding to the period doubling bifurcation can be found by comparing such minimal distances where one of two parameters, r or κ, vary. The period doubling bifurcation point can be found if condition $\Delta(t) \approx \Delta(2t)$ is satisfied.

Let us consider a change of the system behavior when we move from the domain III to the domain V limited by solid and dash-and-dot lines. When the dash-and-dot line is crossed, the period doubling bifurcation occurs. Here, two stable limiting cycles with unstable focuses shown in Fig.2a are transformed into the one limiting cycle (Fig.2c), its period equals two periods of the one cycle shown in Fig.2a. The system behavior inside the domain V is the same as shown in Fig.2c. When the line dividing both domains V and VI is crossed, the stability of focuses shown in Fig.2c changes. This results in formation of two unstable limiting cycles and the additional period doubling bifurcation. In the domain VII, the phase trajectories become irregular (see Fig.2d) due to the period doubling bifurcations. In such a case, a chaotic attractor is formed in the phase space. Its properties will be studied below. Moving from the domain IV to the domain VI, a standard doubling period bifurcation occurs. The transition from the domain II into the domain VII is described by the following scenario. In the line separating the domains II and VII, the following situation is realized: a transversal intersection of both stable and unstable manifolds of the hyperbolic stationary point (saddle) occurs. This indicates a homoclinic structure and a stochastic layer appearance. In that case, the phase space is characterized by two unstable limiting cycles and the irregular behavior of trajectories outside of them (see Fig.2d). It is to note that the dash-dot-dot line was obtained using the maximal (global) Lyapunov exponent, which takes positive values inside the domain VII. The maximal (global) Lyapunov exponent was calculated as

$$\Lambda_m \equiv \Lambda\left(\vec{\delta}(\tau_0)\right) = \overline{\lim_{T\to\infty}} \frac{1}{T} \ln \left\| \frac{\delta \vec{u}(\tau)}{\delta \vec{u}(0)} \right\|$$

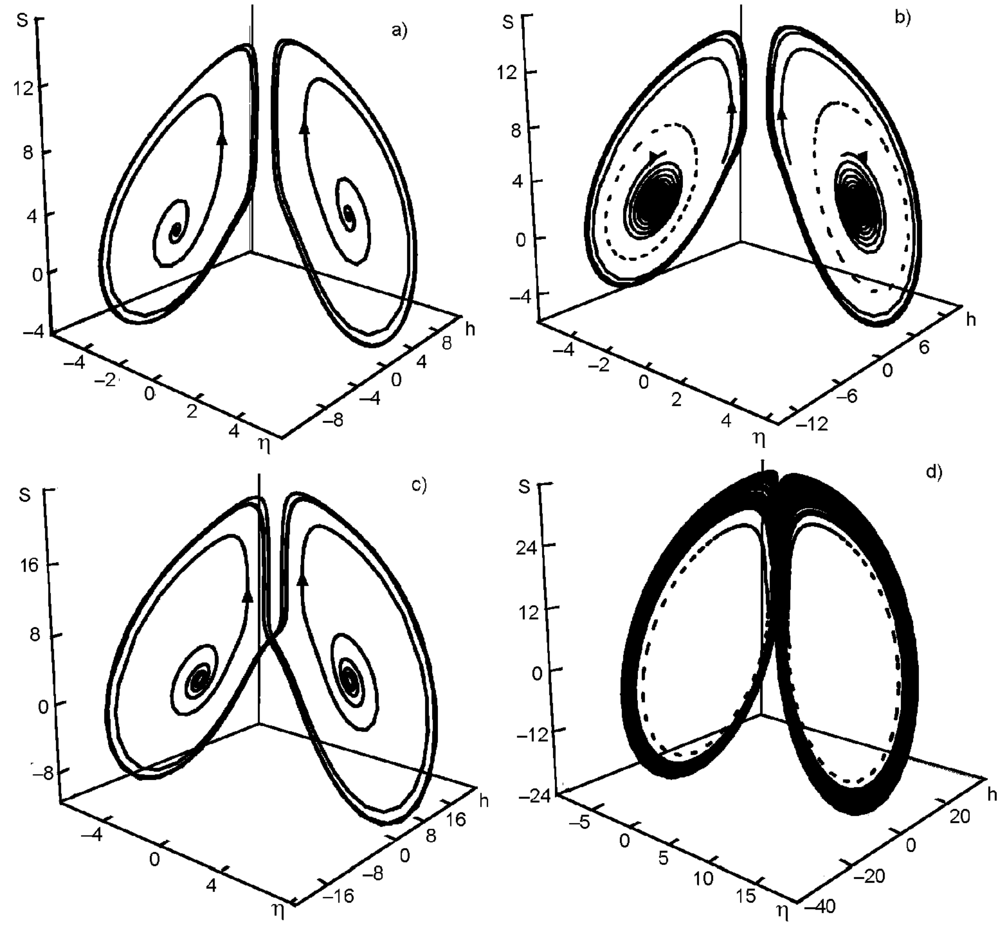

Fig.2. Phase portraits of the system (2) at : (a) $r=15.0$, $\kappa=10.0$; (b) $r=20.0$, $\kappa=9.0$; (c) $r=25.0$, $\kappa=15.0$; (d) $r=39.0$, $\kappa=11.5$

where $\overline{(\ldots)}$ means an upper limit; $\left\|\vec{u}\right\|$ is a norm. The quantity Λ_m allows to consider a divergence of any two initially nearby trajectories in the phase space, starting from points $\vec{u}(\tau)$ and $\vec{u}(\tau')$. It follows that the divergence of such trajectories is given by the dependence $\delta\vec{u}(\tau)=\delta\vec{u}(\tau_0)e^{\Lambda_m t}$. It is seen that at $\Lambda_m<0$, trajectories of dynamical system converge, and, accordingly, the regular dynamics is observed. Here, the trajectories move towards fixed points in the phase space. In the special case of $\Lambda_m=0$, we get the limiting cycle in the phase space. If $\Lambda_m>0$, the corresponding trajectories diverge and the system shows a chaotic regime. To determine the maximal Lyapunov exponent, we have used the Benettin algorithm [19].

Now let us consider in detail the statistical properties of the system irregular behavior (the related phase portrait is shown in Fig.2d). The chaotic mode is known to be characterized by fast decay of an auto-correlation function and continuous character of the frequency spectrum. To demonstrate that the attractor obtained acquires such properties, let the auto-correlation function $C(\tau)=(2/T)\int_0^{T/2}\eta(t)\eta(t+\tau)dt$ be calculated for the corresponding process $\eta(t)$. Another criterion to show the chaotic mode is the continuous frequency spectrum. To obtain it, let us use the Fourier transformation of the correlation function that gives the spectral density function $S(\omega)=\sqrt{2/\pi}\int_0^{T}C(\tau)e^{i\omega\tau}d\tau$. The auto-correlation function and the corresponding frequency spectrum are shown in Fig.3. It is seen that the auto-correlation function $C(\tau)$ has the fast dropping

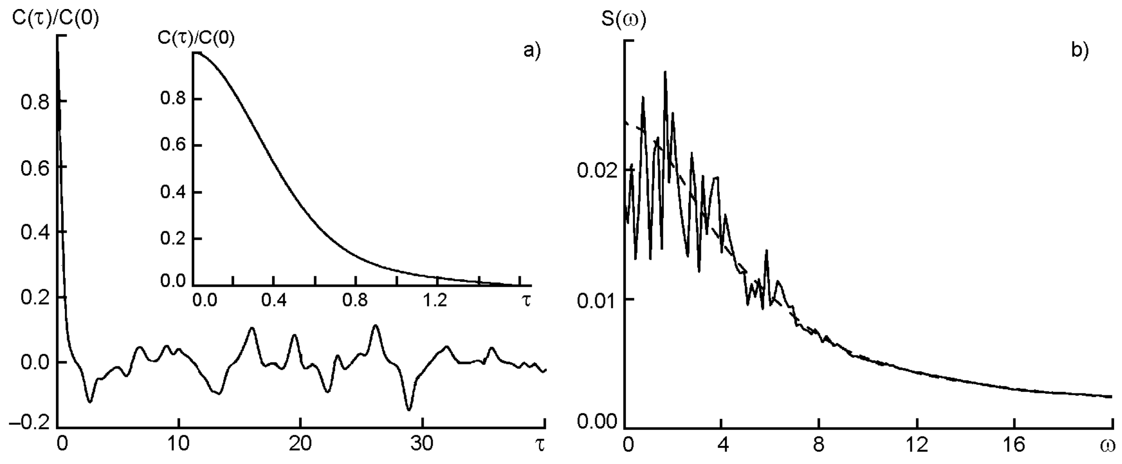

Fig.3. Auto-correlation function $C(\tau)$ (a) and spectral density $S(\omega)$ (b) for process $\eta(t)$ at $r = 39.0$, $\kappa = 11.5$. The corresponding dependences at short time are presented using an inset and dashed curve.

character. Thus, the system loses its memory very quickly. The spectral density function has a continuous character. Such a picture is typical of chaotic modes [20]. In our calculation of $S(\omega)$, we have used full protocol of the correlation function dependence (solid line in Fig.3b). The existence of main peaks in Fig.3b is related to the fact that there are fixed regions in the phase space where the phase trajectory spends more time than in other regions [3]. Considering the protocol $C(\tau)$ at small times (see inset in Fig.3a), the spectral density takes a form of the Lorenzian (see dashed curve in Fig.3a).

Considering chaotic systems, it is of interest to find a one-dimensional law that allows to represent the genuine chaotic behavior. To that end, we calculate the first return map $S_{n+1} = f(S_n)$ by fixing the consecutive maxima values of the function $S(t)$. Mathematically, these maximal values are S-coordinates of points in the Poincarй cross-section of the surface $(r - S) - \eta h = 0$. The corresponding one-dimensional law $S_{n+1} = f(S_n)$ is shown in Fig.4. It is seen that an increase in r yields in an acute peak appearance in dependence $S_{n+1}(S_n)$. This fact indicates that a system manifests chaotic mode.

It is well known from theory of dynamic systems that irregular behavior of the system is characterized by positive value of the maximal (global) Lyapunov exponent. As it was pointed out above, such irregular behavior is realized in the domain VII of the system parameters in Fig.1.

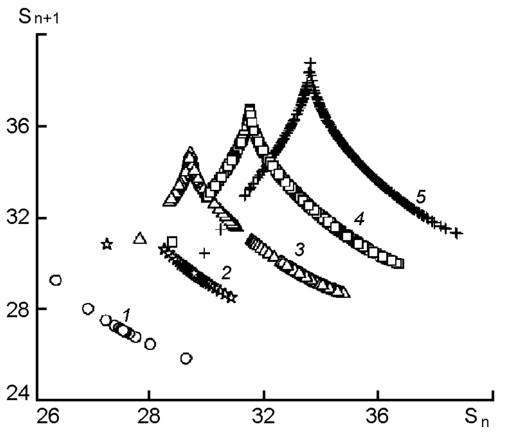

Fig.4. The one-dimensional law $S_{n+1} = f(S_n)$ for the system (2) at $\kappa = 11.5$

Let us describe the behavior of all three Lyapunov exponents vs pumping intensity r at fixed κ in detail. The corresponding graph as a cross-section of plane Fig.1 at $\kappa = 12.0$ is shown in Fig.5a. It is seen that if the pumping r varies at fixed κ (starting from domain III, corresponding to the limit cycle formation) then $\Lambda_1 = 0$, $\Lambda_2 < 0$, $\Lambda_3 < 0$. When the dash-dot-dot line is crossed, the system is characterized by the set $\Lambda_1 > 0$, $\Lambda_2 = 0$, $\Lambda_3 < 0$, that corresponds to the chaotic behavior of the phase trajectory. The chaotic mode is observed inside the hatched domain, bounded by $r_a < r < r^a$ at $\kappa = 12.0$. Knowing the full set of Lyapunov exponents containing one positive exponent, one can calculate the fractal Lyapunov dimensionality for the corresponding chaotic attractor. Using the ap-

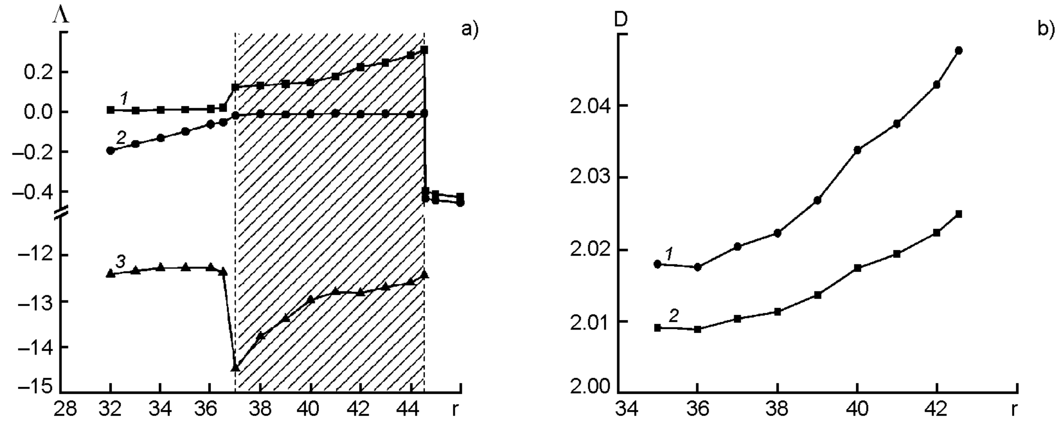

Fig.5. Dependence of Lyapunovs exponents (a) and Lyapunovs fractal dimensions (b) on pumping intensity at $\kappa = 12.0$

proach proposed in [21] for the set $\vec{\Lambda} = (\Lambda_{max}, 0, \Lambda_{min})$, the fractal Lyapunov dimensionality can be calculated using the formula $D_{KY} = 2 - \Lambda_{max}/\Lambda_{min}$. In our case, for the attractor at $r = 39.0$ and $\kappa = 11.5$ (see. Fig.2d), we get $\Lambda_{min} = -12.278$, $\Lambda_{max} = 0.178$, $D_{KY} = 2.0145$. An alternative formula proposed by Sprott [22], $D_{\Sigma} = 1.5 + 0.5\sqrt{1 - 8\Lambda_{max}/\Lambda_{min}}$, gives $D_{\Sigma} = 2.028$. The variation character of fractal dimensionalities D_{KY} and D_{Σ} is shown in Fig.5b. It is seen that as the pumping intensity r increases from the critical value $r = r_a$ toward $r = r^a$ (see Fig.5a), the corresponding fractal dimensionalities increase.

To conclude, we have studied various types of dynamical modes for the three-component system with external pumping and nonlinear dissipation. It has been shown that the system under consideration is related to absorptive optical bistability systems. Using Lyapunov exponents approach, we have demonstrated that various kinds of stable dynamic behavior could be realized by varying the system parameters. It has been found that controlling the pumping and/or properties of additional nonlinear force associated with the absorptive medium in optical systems, both unstable and stable dissipative structures of various types can be formed. A set of bifurcations from the limit cycle is studied in detail, with consideration for dynamics in the three-dimensional phase space. It has been revealed that in the system under consideration, a strange chaotic attractor is realized if additional nonlinear medium is introduced into the optical cell. Properties of such a chaotic attractor have been investigated using Lyapunov exponents, auto-correlation and spectral density functions, one-dimensional first return map, and fractal analysis. The corresponding frequency spectrum is continuous, the temporal auto-correlation function decays fast. We have shown that the Lyapunov fractal dimension of the strange chaotic attractor depends on the system parameters. It follows from our analysis that the chaotic regime can be controlled by nonlinear dissipation rate and pumping intensity. By varying such parameters, we can obtain a stable focus and stable limiting cycle from the chaotic mode.

Our results correspond qualitatively to the results of experimental and theoretical studies of semiconductor lasers [8, 23, 24] where the period doubling bifurcations and the chaos in laser radiation were observed. It has been shown previously that chaos in such systems can be controlled by variations in the angle between two crystals introduced into the cell [23], overall feedback strength [8], pumping [24], and by other ways (see [25] and references therein). Considering the system ability to manifest different dynamical modes, we investigated a model suitable to study the laser dynamics. We have shown that the chaotic mode can be controlled in our case by the bistability parameter describing nonlinear properties of additional medium. Our results may be used to predict the dynamic reconstruction in optoelectronic devices. Comparing our results with existing theoretical investigations (see, e.g., [17]), we claim that parameters $r \sim \kappa \sim 10$ (in dimensionless units) have the same ranges as in other studies (see [6, 17, 26]).

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Перебудова динамічних режимів у трьохкомпонентній бістабільній системі з дисперсійним часом релаксації

Є.Д. Білоколос, В.О. Харченко

Вивчається трьохкомпонентна динамічна система, що описує лазерне випромінювання під впливом зовнішньої накачки та нелінійної дисипації. Проводиться дослідження різних динамічних режимів поведінки, аналізуючи розбігання траєкторій та фрактальні властивості. Показано, що при зміні параметрів зовнішньої накачки та нелінійної дисипації у системі є можливим формування стійких та нестійких дисипативних структур типу граничного циклу. Переходи до хаотичного режиму та властивості відповідного атрактора вивчаються детально.