## Influence of damping parameter on reflection of bulk spin waves from the uniaxial multilayer ferromagnetic structure

Yu.I.Gorobets, S.A.Reshetnyak\*, T.A.Homenko\*

Institute of Magnetism, National Academy of Sciences of Ukraine and Ministry of Education and Sciences of Ukraine,
36-b Vernadskogo av., Kyiv, 03142, Ukraine
\*National Technical University of Ukraine "Kyiv Polytechnical Institute",
37 Prospect Peremohy, Kyiv, 03056, Ukraine

## Received October 18, 2007

The influence of a damping parameter on the reflection coefficient of spin wave from a multilayered uniaxial ferromagnetic medium with inhomogeneous distribution of magnetic parameters has been studied. Variations of the reflection coefficient with the period of the structure, external magnetic field and damping parameter have been investigated.

Исследовано влияние слабого затухания на коэффициент отражения объёмных спиновых волн от одноосной мультислойной ферромагнитной структуры с неоднородным распределением магнитных параметров. Исследованы зависимости коэффициента отражения от периода структуры, параметра затухания и внешнего магнитного поля.

Consider a system consisting of three parts, planes of whose contacts are parallel to yz plane. The first and third (along the direction of x-axis) parts each are a uniform uniaxial semi-infinite ferromagnets; they are interposed by a N-layered ferromagnet with modulated parameters of exchange interaction  $\alpha$ , uniaxial magnetic anisotropy  $\beta$ , Gilbert damping parameter  $\eta$  and saturation magnetization  $M_o$ . Layers are arranged parallel to the yz plane and have thicknesses a and b. Quantities  $\alpha$ ,  $\beta$ ,  $M_o$ ,  $\eta$  assume values  $\alpha_I$ ,  $\beta_I$ ,  $\eta_I$ ,  $M_{oI}$  and  $\alpha_2$ ,  $\beta_2$ ,  $\eta_2$ ,  $M_{o2}$  in corresponding layers. The axis of easy magnetization is parallel to the direction of the external permanent magnetic field  $H_o$  and to the z-axis. We will now calculate the coefficient of reflection of spin waves from such a structure.

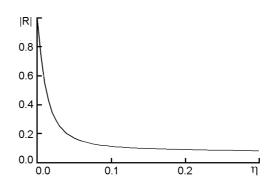
We will use the spin density formalism [5], according to which the magnetization can be written in the form:

$$\mathbf{M}_{j}(\mathbf{r},t)\!=\!M_{0j}\Psi_{j}^{+}(\mathbf{r},t)\sigma\Psi_{j}(\mathbf{r},t)\,,\;j\!=\!1,2, \tag{1}$$

where  $\Psi_j$  are quasi-classical wave functions playing the role of the order parameter of the spin density; r is the radius-vector of the Cartesian coordinate system; t is the time, and  $\sigma$  are Pauli matrices.

Note that in the exchange approximation, at the condition L >> l = a + b, where L is the characteristic length of the material, the energy density in every one of homogeneous layers takes the form:

Fig. 1. Variation of amplitude of reflection with Gilbert damping parameter at  $\alpha_1 = \alpha_2 = 2 \cdot 10^{-11} \text{m}^2$ ,  $\beta_1 = 20$ ,  $\beta_2 = 100$ ,  $M_{01} = 90$  G,  $M_{02} = 95$  G,  $\omega = 5.2 \cdot 10^{11} \text{ s}^{-1}$ ,  $\alpha = 3.8 \text{ } \mu\text{m}$ ,  $b = 0.2 \text{ } \mu\text{m}$ ,  $H_0 = 2.3 \text{ kOe}$ .



$$w_{j} = \frac{\alpha(x)}{2} \left[ \frac{\partial m_{j}}{\partial x_{k}} \right]^{2} + \frac{\beta(x)}{2} \left[ m_{jx}^{2} + m_{jy}^{2} \right] - H_{0} M_{jz}$$
(2)

Here it is taken into account that in the ground state the material is magnetized parallel to  $\mathbf{e}_z$ ;  $M_j^2(\mathbf{r},t) = const$  and  $\mathbf{M}_j(\mathbf{r},t) = M_{0j}\mathbf{e}_z + \mathbf{m}_j(\mathbf{r},t)$ , where  $\mathbf{m}_j(\mathbf{r},t)$  is a small deviation from the ground state.

The Lagrange equations for  $\Psi_i$  have the form:

$$i\hbar \frac{\partial \Psi_{j}(\mathbf{r},t)}{\partial t} = -\mu_{0} \mathbf{H}_{ej}(\mathbf{r},t) \sigma \Psi_{j}(\mathbf{r},t) + \frac{\eta_{j}\hbar}{2} \frac{\partial \left(\Psi_{j}(\mathbf{r},t)\sigma \Psi_{j}(\mathbf{r},t)\right)}{\partial t} \sigma \Psi_{j}(\mathbf{r},t) , \qquad (3)$$

where  $\mu_0$  is the Bohr magneton,  $\hbar$  is Plank constant,  $\eta_j$  is Gilbert damping parameter,  $\mathbf{H}_{ej} = -\frac{\partial w_j}{\partial \mathbf{M}_j} + \frac{\partial}{\partial x_k} \frac{\partial w_j}{\partial \left(\partial \mathbf{M}_j/\partial x_k\right)}.$ 

Then, using the linear perturbation theory [1], the solution of Eq.(3) can be written as following:

$$\Psi_{j}(\mathbf{r},t) = \exp(i\mu_{0}H_{0}t/\hbar) \cdot \begin{pmatrix} 1 \\ \chi_{j}(\mathbf{r},t) \end{pmatrix}, \tag{4}$$

where  $\chi_j(\mathbf{r},t)$  is a small correction that characterizes the deviation of magnetization from the ground state. Linearizing the Eq.(3) and carrying out the Fourier transformation in terms of time and coordinates y, z, we obtain:

$$\left[\alpha(x)\frac{d^2}{dx^2} + \Omega_j - \alpha(x)k_{\perp}^2 - \beta(x) - \tilde{H}_{0j} + i\eta_j\Omega_j\right]\chi_{j\omega\mathbf{k}_{\perp}}(x) = 0.$$
(5)

Here  $\tilde{H}_{0j} = H_0/M_{0j}$ ,  $\Omega_j = \omega \hbar/2\mu_0 M_{0j}$ ,  $\omega$  is the frequency,  $\mathbf{k}_\perp = (0, k_y, k_z)$ , j = 1, 2,  $\chi_{j\omega \mathbf{k}_\perp}(x)$  is Fourier transform of function  $\chi_j(\mathbf{r}, t)$ .

Following to [6], the amplitude of reflection of a spin wave from a multilayered structure consisting of N layers can be written in the form:

$$R_N = R \frac{1 - \exp(2iqlN)}{1 - R^2 \exp(2iqlN)},\tag{6}$$

where R is the amplitude of reflection from the semi-infinite multilayered structure:

$$R = \frac{\sqrt{(\rho+1)^2 - \tau^2} - \sqrt{(\rho-1)^2 - \tau^2}}{\sqrt{(\rho+1)^2 - \tau^2} + \sqrt{(\rho-1)^2 - \tau^2}},$$
(7)

q is the Bloch quasi-wave vector, which is determined by the equation:

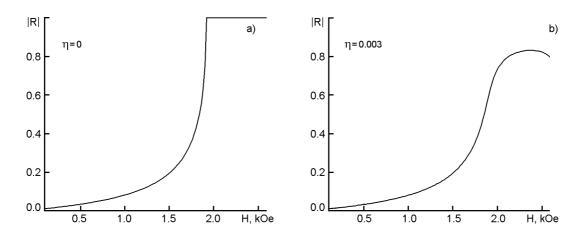


Fig. 2. Variation of amplitude of reflection with external magnetic field at  $\alpha_1$  =  $\alpha_2$  =  $2 \cdot 10^{-11} \text{m}^2$ ,  $\beta_1$  = 20,  $\beta_2$  = 100,  $M_{01}$  = 90 G,  $M_{02}$  = 95 G,  $\omega$  =  $5.2 \cdot 10^{11}$  s<sup>-1</sup>,  $\alpha$  = 3.8  $\mu$ m, b = 0.2  $\mu$ m.

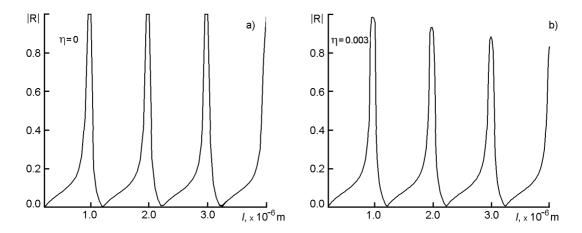


Fig.3. Variation of amplitude of reflection with period of the structure at  $\alpha_1 = \alpha_2 = 2 \cdot 10^{-11} \text{m}^2$ ,  $\beta_1 = 20$ ,  $\beta_2 = 100$ ,  $M_{01} = 90 \text{ G}$ ,  $M_{02} = 95 \text{ G}$ ,  $\omega = 5.2 \cdot 10^{11} \text{ s}^{-1}$ ,  $\alpha = 3.8 \text{ } \mu\text{m}$ ,  $b = 0.2 \text{ } \mu\text{m}$ ,  $H_0 = 2.3 \text{ kOe}$ .

$$\exp(iqlN) = rac{\sqrt{\left( au + 1
ight)^2 - 
ho^2} + \sqrt{\left( au - 1
ight)^2 - 
ho^2}}{\sqrt{\left( au + 1
ight)^2 - 
ho^2} - \sqrt{\left( au - 1
ight)^2 - 
ho^2}} \,,$$

l=a+b is the period of the structure;  $\rho$  and  $\tau$  are respectively complex amplitudes of reflection and transmission for a single symmetric (with respect to its center) period.

The Eq.(5) describes the dynamics of magnetization in the exchange approximation. Its solution is to be continuous and to have a continuous derivative  $\frac{\partial \chi_{\omega, \mathbf{k}_{\perp}}(x)}{\partial x}$ . Boundary conditions will have the following form (indices  $\omega$  and  $\mathbf{k}_{\perp}$  are skipped):

$$\chi_{1}(x_{0}-0) = \chi_{2}(x_{0}+0),$$

$$\alpha(x_{0}-0)\chi_{1}'(x_{0}-0) = \alpha(x_{0}+0)\chi_{2}'(x_{0}+0).$$
(8)

We associate the function  $\chi_I = \exp\left(\left|ik_1^+ - k_1^-\right|x\right)$  to the falling wave, the function  $\chi_\rho = \rho \exp\left(-\left[ik_1^+ - k_1^-\right]x\right)$  to the reflected wave, and the function  $\chi_\tau = \tau \exp\left(\left[ik_1^+ - k_1^-\right]x\right)$  to the wave that transmitted through a separate period. Substituting in (8) these expressions as well as the expression  $\chi_{layer} = C_1 \exp\left(\left|ik_2^+ - k_2^-\right|x\right) + C_2 \exp\left(-\left[ik_2^+ - k_2^-\right]x\right)$ , which describes the wave in an intermediate layer, we come to the expressions for amplitudes of reflection and transmission of a spin wave:

$$\rho = \exp(ik_1^+b) \cdot \exp(-k_1^-b) \frac{E_-F_-}{GE_+ - F_+E_-},$$

$$\tau = \exp(-ik_1^+a) \cdot \exp(k_1^-a) \cdot \frac{2G}{GE_+ - F_+E_-}.$$
(9)

Here

$$\begin{split} G &= 2\alpha_1\alpha_2 \big(ik_1^+ - k_1^-\big) (ik_2^+ - k_2^-)^{,} \\ F_\pm &= a_1^2 \Big(ik_1^+ - k_1^-\Big)^2 \pm a_2^2 \Big(ik_2^+ - k_2^-\Big)^2 \,, \\ E_\pm &= e^{\Big[ik_2^+ - k_2^-\big]\alpha} \pm e^{-\Big[ik_2^+ - k_2^-\big]\alpha} \,, \\ k_j^\pm &= \sqrt{\Big(\pm s_j + \sqrt{s_j^2 + n_j^2}\Big)/2} \,, \\ s_j &= \Big(\Omega_j - \beta_j - \alpha_j k_\perp^2 - \tilde{H}_{0j}\Big)/\alpha_j \,, \; n_j = \eta_j \Omega_j / \alpha_j \,. \end{split}$$

As for amplitudes of reflection for the semi-infinite multilayered structure and for structure consisting of N layers, they are described by the Eqs. (6.7).

Taking into account the damping affects substantially the reflection of spin waves from a multilayered structure and changes qualitatively the characteristic sections of curves illustrating the variation of the amplitude of reflection with the period of the structure and external magnetic field. In particular, the evening-out of the curves is observed near forbidden zones and decrease of gradients in vicinities of such regions. Just regions with sharp dependencies as known are the perspective in the sense of the develop of quick-response elements and sensors. Therefore, these processes have to be taken into account at the development of devices of spin-wave microelectronics.

## Referenses

- 1. Yu.I. Gorobets, A.E. Zyubanov, A.N. Kuchko, K.D. Shejuri, Fiz. Tverd. Tela, 34, 1486 (1992).
- 2. V.V. Kruglyak, A.N. Kuchko, *Phys. B. Condens. Matter*, **339**, 130 (2003).
- 3. V.V. Kruglyak, R.J. Hichken, A.N. Kuchko, V.Yu. Gorobets, J. Appl. Phys., 98, 014304 (2005).
- 4. A.M. Zyuzin, A.G.Baganov, S.N.Sabaev, S.S. Kidyaev, Fiz. Tverd. Tela, 42, 1279 (2000).
- 5. V.G. Bar'yakhtar, Yu.I. Gorobets, Bubble Domains and Their Lattices. Nauka, Kiev (1988), p. 168 [in Russian].
- V. K. Ignatovich, Uspekhi Fisz. Nauk, 150, 145(1986).

## Вплив параметра згасання на відбиття об'ємних спінових хвиль від одновісної багатошарової феромагнітної структури

Ю.І.Горобець, С.О.Решетняк, Т.А.Хоменко

Досліджено вплив слабкого згасання на коефіцієнт відбиття об'ємних спінових хвиль від одновісної багатошарової феромагнітної структури з неоднорідним розподілом магнітних параметрів. Досліджено залежності коефіцієнта відбиття від періоду структури, параметра згасання та зовнішнього магнітного поля.