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MULTIAGENT HEURISTIC ALGORITHMS FOR SOLVING PROBLEMS OF SLOPE REINFORCEMENT PARAMETERS OPTIMIZATION

Problem of finding optimized placement of bored concrete piles to reinforce soil slope has been considered. Processes in soil were modelled by two-dimensional strain-stress model which takes filtration processes into account. Genetic and particle swarm optimization algorithms were used to find optimized values of soil reinforcement parameters. Multiagent parallel algorithm is proposed to scale search process onto large parallel computational environments.

Key words: *slope reinforcement, optimal placement, genetic algorithm, particle swarm optimization, parallel algorithms, multiagent algorithms.*

Introduction. Reinforcement of slopes is an important engineering problem with stabilization by means of bored piles [1, 2] as one of its practical solutions. Efficiency and optimal configuration of stabilizing constructions are estimated using both theoretical approaches [3–5] and approaches that use numerical methods of mathematical modelling: boundary elements method [6, 7], finite elements [8, 9], finite differences [10] and Monte-Carlo method [11]. Search of optimized placement parameters of stabilizing constructions is mostly studied comparing several variants, potentially optimal from practical point of view. Issues of automated optimization for engineering decision making support are poorly studied.

Algorithms of constructive parameters optimization while strengthening slopes using concrete piles are considered in the paper in the case of filtration processes presence and additional loading applied on the soil. According to [12], water affects slopes by creating pore pressure that influence strain-stressed state; by changing density and chemistry of soils; by developing erosion. Regarding sophisticated models of erosion and chemical processes, only first of the influencing factors has been studied.

Taking into account complexity of mathematical models of soil consolidation and deformation, use of heuristic methods, such as genetic algorithms [13, 14] and particle swarm optimization (PSO) [15, 16] is proposed for solving optimization problem. Such heuristic procedures include solving large number of direct problems with different values of parameters, thus use of two-dimensional mathematical models disregarding plastic deformation processes has been considered to decrease computational complexity of algorithms.

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Even with simplified mathematical models, finding optimal values of form parameters requires excessive amount of time, so use of parallel algorithms run on distributed systems can be efficient here. To run on such systems, algorithms need modification in the way to independently solve several direct problems with different parameter values. Another approach, that allows doing heuristic search on massively parallel system such as voluntary computing, consists in considering particle in a swarm or element in a population of genetic algorithm as an autonomous agent in a multiagent environment.

Mathematical statement of direct problem. Concrete piles reinforcements' influence on slope stability was considered on the base of the model which describes soils' strain-stressed state in the presence of filtration processes, similar to [17]:

$$\begin{cases} \tilde{\mu}\dfrac{\partial h}{\partial t} - \dfrac{\partial}{\partial x}\left(K(x,y,t)\dfrac{\partial h}{\partial x}\right) - \dfrac{\partial}{\partial y}\left(K(x,y,t)\dfrac{\partial h}{\partial y}\right) = 0, \\ \rho\dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial}{\partial x}\left[\lambda(x,y)\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) + 2\mu(x,y)\dfrac{\partial u}{\partial x}\right] - \\ \qquad - \dfrac{\partial}{\partial y}\left[\mu(x,y)\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)\right] - \dfrac{\partial P}{\partial x} = 0, \\ \rho\dfrac{\partial^2 v}{\partial t^2} - \dfrac{\partial}{\partial y}\left[\lambda(x,y)\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) + 2\mu(x,y)\dfrac{\partial v}{\partial y}\right] - \\ \qquad - \dfrac{\partial}{\partial x}\left[\mu(x,y)\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)\right] - \dfrac{\partial P}{\partial y} = -\rho g, \\ P = \rho_w g(h - y), \\ K(x,y,t) = \begin{cases} K_0(x,y,t), \ P \le 0, \\ K_0(x,y,t)e^{-kP}, \ P > 0, \end{cases} \\ \qquad (x,y,t) \in \Omega_T, \Omega_T = \Omega \times (0,T], \end{cases} \tag{1}$$

where h is a water head, P — pressure, u, v — components of soil displacement vector, $\tilde{\mu}$ — moisture capacity, $K_0(x,y,t) = \bar{K}_0(x,y)e^{\varepsilon(\theta)/(0.18\varepsilon_t - 0.048)}$, $\bar{K}_0(x,y)$ — piecewise linear filtration coefficient, $\varepsilon(\theta) = \varepsilon_0 + \theta(1-\varepsilon_0)$, $\varepsilon_0 = 0.64$, $\varepsilon_t = 1.06$, $\theta = \partial u/\partial x + \partial v/\partial y$ — volume deformation, $\lambda(x,y), \mu(x,y)$ — piecewise linear Lame coefficients, ρ — soil density, ρ_w — water density, g — gravitational acceleration, k — given constant.

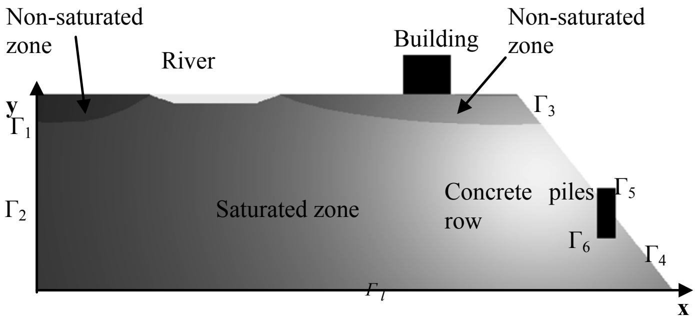

***Fig. 1.** Domain of solution*

Boundary conditions for model (1) on the domain depicted on fig.1 are stated as follows:

- on river boundary Γ_w : $h = y + h_0$, $\sigma_n = 0$, $\tau_s = 0$, where σ_n, τ_s are normal and tangent components of strain vector, h_0 — depth of the river;
- on contact boundary between building and soil Γ_b : $\frac{dh}{dn} = 0$, $\sigma_n = -\gamma_0$, $\tau_s = 0$, where γ_0 is a pressure imposed by the building on the soil;
- on upper boundary of the domain (except Γ_w and Γ_b) and on external boundary of concrete pile Γ_5 : $\frac{dh}{dn} = 0$, $\sigma_n = 0$, $\tau_s = 0$;
- on left boundary of the domain: $u = 0$, $\tau_s = 0$, $\frac{dh}{dn} = 0$ on Γ_1, $h = y$ on Γ_2;
- on lower boundary Γ_l of the domain: $\frac{dh}{dn} = 0$, $u = 0$, $v = 0$;
- on right boundary of the domain (the slope): $\sigma_n = 0$, $\tau_s = 0$, $\frac{dh}{dn} = 0$ on Γ_3, $h = y$ on Γ_4;
- coupling conditions on Γ_6 boundary of contact between concrete pile and a soil: $[\sigma_n] = 0$, $[u_n] = 0$, $[\tau_s] = 0$, $\{\tau_s\}^{\pm} = r[u_s]$, $[q_n] = 0$, $\{q_n\}^{\pm} = \bar{k}[h]$, $q_n = K(x, y, t)\left(\frac{\partial h}{\partial x} n_x + \frac{\partial h}{\partial y} n_y\right)$, where u_n, u_s are normal and tangent components of soil displacement vector and n_x, n_y are components of contact boundary normal.

Physical parameters and parameters of domain for solving testing problems where taken as follows:

- soil filtration coefficient: $\overline{K}_0 = 0.3\, m/day$;
- concrete filtration coefficient: $\overline{K}_0 = 4.32 \cdot 10^{-6}\, m/day$;
- exponential relationship coefficient $k = 0.005$;
- soil density $\rho = 1940\, kg/m^3$;
- concrete density $\rho_c = 2300\, kg/m^3$;
- soils elasticity modulus $E = 50 MPa$, Poisson coefficient $\nu = 0.3$;
- concrete elasticity modulus $E = 3 \cdot 10^4 MPa$, Poisson coefficient $\nu = 0.2$;
- river depth $h_0 = 2m$;
- pressure imposed by building on the soil $\gamma_0 = 10 MPa$;
- coefficients of coupling conditions set on the contact boundary between soil and concrete: $r = 0.1 \dfrac{MPa}{m}$, $\overline{k} = 10^{-5} m / day$;
- length of solution domain — $140m$, height — $50m$, slope — $55°$;
- river width — $15m$, building width — $10m$, concrete reinforcement width — $1m$;
- Time period used for modelling was $0 - 30$ *days* .

Local soil assurance factor η (soil durability is considered insufficient when $\eta < 1$) was calculated using Coulomb-More model [18] as

$$\eta = \frac{(\sigma_1 + \sigma_2 + 2c \cdot ctg\varphi)}{(\sigma_1 - \sigma_2)\cos^2 \varphi} - \frac{\sin^2 \varphi}{\cos^2 \varphi},$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm 1/2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2},$$

$$\sigma_x = \lambda(\partial u / \partial x + \partial v / \partial y) + 2\mu \partial u / \partial x,$$

$$\sigma_y = \lambda(\partial u / \partial x + \partial v / \partial y) + 2\mu \partial v / \partial y,$$

$$\tau_{xy} = \mu(\partial u / \partial y + \partial v / \partial x),$$

where φ is a friction angle and c is soil cohesion coefficient.

Optimization problem statement and solution algorithms. Optimization problem regarding the model (1) can be stated as a problem of finding such geometric configuration of concrete reinforcement (its position on the slope and depth) that minimizes soil instability risk.

Optimization parameters ζ are:

- coordinates of concrete reinforcement placement on the slope x_c, y_c ;

- depth of concrete pile installation h_c, $h_c < h_m$.

Goal function is stated as follows:

$$f(\zeta) = \frac{1}{T}\int_0^T \left(h_m S(\Omega_1) + h_c\right) dt, \quad \Omega_1 = \{(x,y) : \eta(x,y) < 1\}, \qquad (2)$$

where Ω_1 is soil instability region with $\eta < 1$, $S(\Omega_1)$ — its area, $[0,T]$ — period of time during which soil stability estimation increase must be assured. Minimization of goal function in the form (2) results in concrete reinforcement of minimal volume that ensures minimal instability risk during given period of time.

As optimization parameters are parameters of solution domain form, solving such problem with classical methods is complex enough to make urgent usage of heuristic algorithms. We propose to solve the problem with such heuristic search algorithms as genetic algorithms [13, 14] and PSO [15, 16].

Used genetic algorithm with candidate solutions represented as floating point values has the following peculiarities:

- each candidate solution γ is a set of optimization parameters γ_i. In the case of considered problem $i = 1,...,3$;
- crossover operation is defined as $\overline{\gamma_i} = R\gamma_i^{(1)} + (1-R)\gamma_i^{(2)}$, where $R \in [0,1]$ is a random number and $\gamma^{(1)}$, $\gamma^{(2)}$ are weighted randomly (with inverse goal function values as weights) chosen solution;
- candidate solution resulting from crossover substitutes a solution with the biggest goal function value in the case when its value of goal function is lower. This makes population size remain constant during iteration process;
- mutation consists in changing random optimization parameter of random solution per value not bigger than a given maximal one;
- generation of initial population can be done randomly or in a specified way to cover biggest possible part of the domain in which optimization parameter values are permissible to change;
- iteration process finishes when difference between maximal and minimal goal function value in the population becomes less than a given number;
- number of iterations is restricted with a given number.

Considered PSO algorithm with local choice of movement direction can be interpreted as generalization of the genetic algorithm and has the following peculiarities:

- coordinates p_i of swarm particle p are the values of optimization parameters. Particle $p^{(i)}$ moves with velocity $v^{(i)}$;

- on each iteration, velocities of all particles except the one with the lowest value of goal function are recomputed the following way: $\overline{v}_j^{(i)} = \alpha v_j^{(i)} + \beta R(\overline{p}_j - p_j^{(i)})$, where $\overline{v}_j^{(i)}$ is a new value of particle i -s' velocity, $R \in [0,1]$ — random number, $\overline{p}$ — coordinates of another particle weighted randomly chosen with inverse goal function values as weights, α, β — algorithms' parameters;
- after computing new values for velocities, coordinates of particles are also recomputed: $\overline{p}_j^{(i)} = p_j^{(i)} + \overline{v}_j^{(i)}$, where $\overline{p}_j^{(i)}$ are a new coordinates of particle i ;
- generation of initial population and stopping criterion remains the same as in the case of genetic algorithm.

Such PSO algorithm become equal to above descript genetic when $\alpha = 0, \beta = 1$.

Parallel algorithms. To run on distributed memory systems, such as clusters, algorithms where modified in the way to solve several direct problems with different parameter values on each iteration.

We propose to single out a managing process of distributed program while other processes doing remote solving of direct problems with parameter values obtained from the managing one.

On algorithms' iterations, new sets of optimization parameter values are generated using crossover and mutation (for genetic algorithm) or by changing particles' velocities and position (for PSO). After that, managing process sends them to other processes which independently solve direct problems sending results (values of goal function) back. For genetic algorithms, after receiving goal function values, managing process changes population substituting existing solutions with higher goal function values by newly generated ones.

In this scheme, scalability is limited by global nature of communications (from managing process to all other) thus making it efficient only for small networks or clusters but not for massively parallel systems.

To overcome this limitation, the algorithms can be modified to have an ability to be executed on the system of autonomous intellectual agents [19, 20]. One of the approaches to such modification uses the assumption about local nature of evolutionary processes. Convergence of the «local» algorithms depends on completeness of agents' communication graph.

In the case of genetic algorithm, we propose to use the approach [21] which depicts locality assumption in the following way:

- each agent represents a single changing candidate solution;
- agent are organized in the form of arbitrary network;
- crossover is done only between connected agents changing current candidate solutions of one of the agents and retaining population size;

- each agent does independent iterative search exchanging information about best found solution with neighbouring agents;
- the worst solution remains in the population to preserve its size.

In the case of PSO algorithm, similar scheme is used where each agent operates a single particle. When changing particles' velocity and position, weighted random choice of the «locally» best particle is done only between particles from the neighbourhood of the agent.

Agents are organized in a network and communicate with each other using the following requests:

- request for connecting to the network;
- request for starting computations which includes algorithms' parameters and direct problem statement. Request is retranslated by agents to the neighbouring ones. Each agent remembers the sender of the request as a «parent» thus building cover tree over the network. At first, request must be sent by some «root» agent which can resend it over the network to rebuild cover tree if some agents have stopped and to append new connected agents to the tree;
- request for sending back the best goal function and optimization parameters values in the subtree. After completing crossover and mutation (in case of genetic algorithm) or velocity and position changing (in case of PSO), each agent computes the best goal function value from solution obtained by itself and by its «children» and sends it to the «parent» agent. As the result, «root» agent knows the best value of goal function in the network and can take decision about stopping computation process;
- request for stopping computations sent by «root» agent and retranslated by others;
- request for sending back current goal function and optimization parameters values sent by an agent to its neighbours while executing crossover operation (in the case of genetic algorithm) or doing particles' position change (in the case of PSO).

Direct problem solving. Direct problems were solved using finite elements method on the quadrangle mesh that yields in better accuracy similarly to described in [17].

As optimization parameters are changing in the process of search thus changing domain of direct problem solution, new mesh generation should be carried out while solving direct problem on each heuristic algorithms' iteration. On this stage, mesh of quadrangles was built from triangle mesh obtained by Delauney triangulation with further application of smoothing procedure [22].

Sparse linear systems obtained after finite element discretization were solved by BiCGSTab [23]. As speeds of processes in soil and concrete substantially differ, the linear systems are ill-conditioned resulting in the need of preconditioners usage in order to obtain accurate enough solutions. While solving, preconditioners built on the base of Schur decomposition and basis matrix method [24] were used.

Computational experiments results. Problem of finding optimized concrete reinforcement parameters on the base of model (1) was solved be the following algorithms:

1. Genetic algorithm with fixed initial population;
2. Genetic algorithm with random initial population;
3. Multiagent genetic algorithm;
4. PSO with $\alpha = 0.5,\ \beta = 0.5$ and fixed initial population;
5. PSO with $\alpha = 0.5,\ \beta = 0.5$ and random initial population;
6. PSO with $\alpha = 0.75,\ \beta = 0.25$ and random initial population;
7. PSO with $\alpha = 0.25,\ \beta = 0.75$ and random initial population;
8. Multiagent PSO with $\alpha = 0.5,\ \beta = 0.5$.

Coordinated of upper left corner of concrete reinforcement lied on $(25,50)-(49.5,15)$ segment and its depth was restricted by $h_c \in [7.5m, 15m]$. Goal function values on the iterations of algorithms are depicted on fig.2.

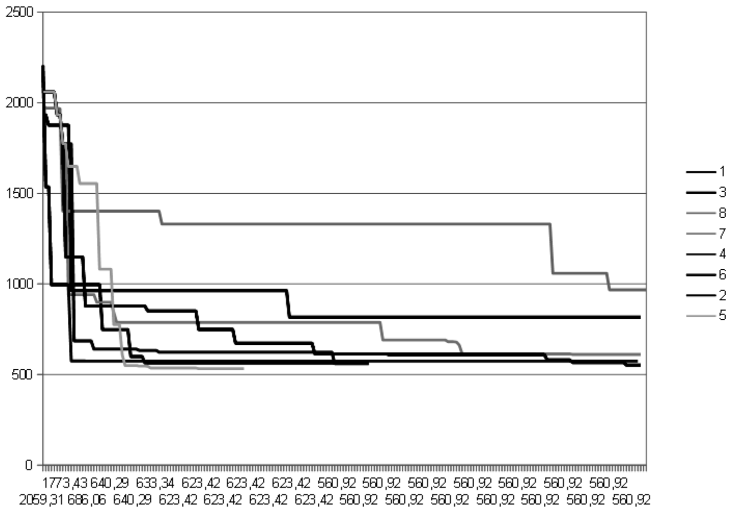

Fig. 2. *Best goal function values on algorithms' iterations (1–8 — numbers of algorithms)*

Obtained results show that algorithms with fixed initial population converge faster than algorithms with random one due to proximity of local minima to the boundary of the domain of permissible optimization parameters values. Convergence speed of genetic algorithms in the conducted experiments was lower than for the PSO. Multiagent modifications of algorithms converge slower than basic because of their “locality”. Considered heuristic algorithms have converged to different but close local minima.

In the averaged optimized parameter set obtained for the testing problem, depth of reinforcement was 8.5m with coordinates of upper left corner equal to (44.6m, 22m). Found optimized solution points on necessity to block filtration in the lower part of the slope and to strengthen a zone located above a zone of biggest slope deformations.

Water head field obtained in the situation of with concrete reinforcement presence for $T = 30\ days$ is depicted on fig. 3, soil displacement (multiplied by 20) — on fig.4 and soil assurance factor values — on fig. 5.

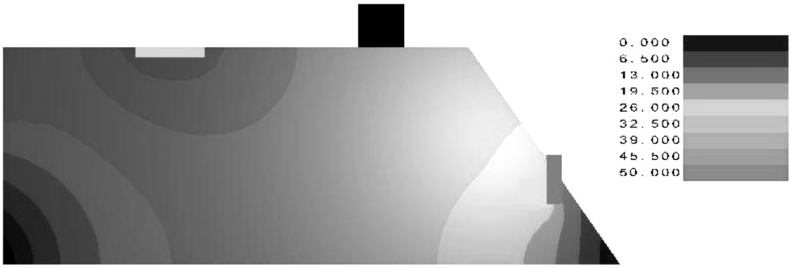

Fig. 3. *Water head field*

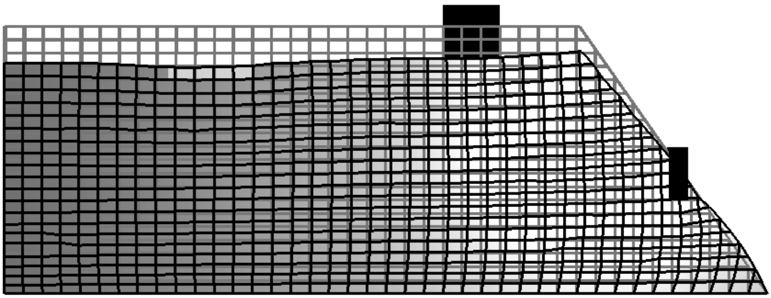

Fig. 4. *Soil displacements*

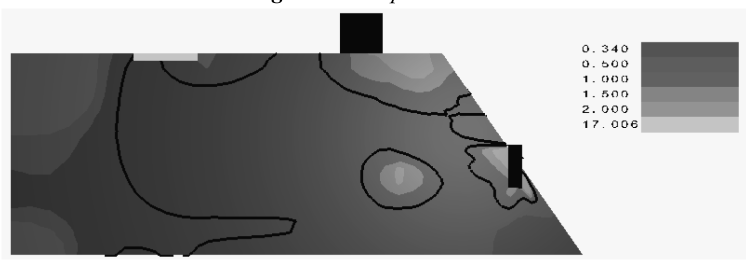

Fig. 5. *Soil assurance factor distribution*

Conclusions. Appliance of heuristic algorithms for finding concrete reinforcement optimized placement on the slope in order to minimize soil instability risks has been studied comparing genetic and PSO algorithms.

Computational experiments results have shown that PSO algorithm variations converges to better solutions than genetic using less amount of time. All obtained solutions in the situation of substantive filtration and additional loading give a recommendation to block filtration flow in the lower part of the slope and apply strengthening to the zone located above highest deformations area.

Regarding high computational complexity of considered problems that urges using high performance, especially distributed memory, systems for their numerical solution, variations of genetic and PSO algorithm has been developed for executing them in multiagent environment which significantly increase their scalability and allows conducting optimization on volunteer computing systems. On the other hand, experiments have shown that convergence speed of multiagent algorithms is lower than of basic ones.

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У роботі розглядається задача знаходження оптимізованого розміщення бетонних паль у ґрунтовому схилі для його зміцнення. Процеси, що протікають у ґрунтах, моделювались за двовимірною моделлю напружено-деформованого стану з урахуванням фільтрації. Для пошуку оптимізованих значень параметрів конструкцій, що укріплюють ґрунтовий масив, були використані генетичні алгоритми та алгоритми рою частинок. Пропонується мультиагентний паралельний алгоритм для масштабування процесу пошуку у паралельних обчислювальних середовищах великого розміру.

Ключові слова: *укріплення схилів, оптимальне розміщення, генетичний алгоритм, оптимізація роєм частинок, паралельні алгоритми, мультиагентні алгоритми.*

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