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Jo Sterten, Assistant Professor

Norwegian University of Science and Technology, Gjovik, Norway

NUMERICAL SIMULATION OF DYNAMIC OBJECT BASED ON CONVOLUTION OPERATIONS

A recursive digital filter construction method is considered for simulation of inertial element as a typical component of complex dynamic object. New computational formulas are obtained. Their high accuracy as compared to traditional ones is shown. Decomposition of initial model of simulated object by convolution operations with several typical exponential kernels is proposed instead of traditional operations of integration and differentiation.

Key words: *mathematical model, transfer function, Volterra integral operator, convolution operator, digital filter.*

Introduction. A wide range of control system units can be described with acceptable accuracy as a linear stationary dynamic object with lumped parameters. There are several mathematical descriptions of such an object. Common higher order linear differential equation is its traditional mathematical model.

Well-known software packages for simulation of continuous objects (SPSCO), such as MATHLAB, SIMULINK, CC, PSPACE, MCAP, SIGNAL, EURIKA, MATCAD, as well as domestic SPSCO, DISPAS, are based on computational solution of differential equations. They have limited accuracy and low tolerance to noise interference or rapidly changing signals, that is especially evident while solving inverse problems of systems dynamics, in particular signal recovery problem [1].

In Laplace image space, traditional differential model of continuous object is associated with rational fractional transfer function that can be decomposed into partial fractions. Then, on the basis of the convolution theorem and tables of Laplace transform basic functions one can pass to the originals. As a result, we obtain an equivalent linear integral equation based on convolution operator with a complex kernel in the form of superposition of exponential and power functions.

Integrated mathematical model has a number of advantages as compared to the differential one [1, 2]. In particular, when it is used, accuracy of numerical simulation of the object under investigation is enhanced. The algorithms obtained are characterized by deeper parallelization. The last of these properties is very important for simulation of an object in real time, when the computation should be completed during the step of discretization of input and output signals.

Digital filtration techniques. Application ofdigital filters seems to be very promising for simulation of continuous objects.The simplest and most common technique is to synthesize a transversal digital filter with finite high-order impulse response (usually one or two hundred units). This leads to emergence of technical difficulties. Advantage of this method consists in simplicity of organization of computer-aided calculation of transversal filters [3]. Very promising perspective stems from development of recursive digital filters [4] with simple technical implementation (usually they have low orders — one, two or three units) and high level of computational process parallelization.

Traditionally, an analog prototype is used for development of a recursive digital filter. It means formally that with equivalent algebraic manipulations, operator of primary continuous mathematical model can be reduced to a form suitable for realization in analog integrators, that are replaced thereafter with digital ones.

Image $F(p)$ of continuous original function $f(t)$ according to definition of integral Laplace transform can be find as follows:

$$F(p) = \int_0^\infty \exp(-pt) f(t) dt,$$

where t is argument of the original function (as a rule, it is time); p is a mapping function argument (frequency or spatial coordinate) [5].

It is convenient to use this transformation, as well as Fourier transform, for spectral analysis of harmonic signals. Infinitely long in time sine waves are transformed in images into infinitely narrow by spatial coordinate spectral lines. Dynamic operations of differentiation or integration, complicated in the originals, are reduced to simple multiplication operations in images. This method makes possible significant simplification of analytical calculations of complex dynamical system response to an arbitrary continuous input signal. Sometimes performance of reverse transformation for transition to the originals causes significant difficulties.

Image $F(z)$ of the original discrete function $f(nh)$ according to definition of Jury discrete transform (z-transform) is found as follows:

$$F(z) = \sum_{n=0}^{\infty} f(nh) z^{-n},$$

where n is argument of the original function (formally it is number h of discretization step); z is argument of the image function [5].

It is very convenient to use Jury transform when developing recurrence formulas for numerical determination of values of the original function in equidistant sampling nodes. It is enough to calculate value of the function in some start nodes. Digital filters do work on this principle. They make it possible to simulate complex dynamic objects in real time, i.e. to calculate response of such an object to an arbitrary input signal. Next output signal value is determined from several input and output values of the object in previous discretization time points. Simplicity of inverse transformation that leads to recurrence formulas is an important advantage of Jury transform.

From the formulas that determine Laplace transform and Jury transform formally follows that $z = \exp(-ph)$, or $p = (1/h)\ln(z)$. Substitution of continuous differentiation operator p with finite difference expression leads to an infinite series. As a rule, truncation of the logarithm expansion into Laurent series is used. Thus bilinear z-transform, that uses the first two terms of expansion into Laurent series, is widely used: $\ln(z) = {}$ $=2\,(z-1)\,/\,(z+1)$. Hence $p = (2\,/\,h)\,(z-1)\,/\,(z+1)$. This method of recursive digital filter synthesis is also called Tustin's method [3].

This is how traditional digital filter synthesis method for simulation of continuous dynamic object is built [3]. Its main drawback consists in low accuracy of calculation of the output signal of simulated dynamic object in discretization nods, caused by incorrect substitution of continuous operator with discrete one. This is because the source model of dynamic object has been defined as a rational fractional transfer function, i.e. as an initial decomposition of continuous operator of object on differentiation stage.

However, primary continuous operator of dynamic object can be represented as a decomposition of rational fraction expression into partial fractions. It corresponds in the originals to decomposition of continuous operator into several convolution operations with several type kernels (exponential, exponential power and exponential trigonometric) [6]. Analog model of dynamic object can be developed without integrators, solely on inertial and oscillation elements [7].

It should be noted that integration is a subcase of convolution operation, when kernel is a singular constant function [2, 5].

Convolution method. Replacement of continuous convolution operator with finite difference expression can be performed more accurately. Kernel of convolution operator is often an exponential function, that leads to very low orders of difference equation of the desired digital filter. Let's prove a simple but important theorem.

Theorem. Continuous operator $1\,/\,(Tp+1)$ in Laplace image space is associated with discrete operator $(l-g)\,z\,/\,(z-g)$, $g = \exp\,(-h\,/\,T))$ in z-image space.

Proof. Output signal Y of inertial element is related with input signal X by a simple equation in Laplace images

$$Y(p) = V(p)X(p), V(p) = 1/\left(Tp+1\right),$$

where T is inertial element time constant. Product of two functions in images is associated in originals with convolution of two functions, that is reduced to the problem of realization of Volterra integral operator:

$$y(t) = \int_0^t V\left(t-s\right)x(s)ds.$$

Convolution operation is formally defined for infinite integration limits. However, since the argument of signal is time and not a spatial coordinate, and kernel $V(t) = 0$ for $t < 0$, one can choose finite integration limits. Signal start ($t = 0$) can be taken as a reference-starting point, upper limit of integration will be variable here.

Let's find kernel function $V(f)$ in originals according to Laplace Transform Table. Let's reduce the $V(p)$ function in images to tabular style:

$$V(p) = (1/T)/(p+(1/T)).$$

Its parallel in the originals

$$V(t) = (1/T)\exp(-t/T).$$

Consequently, passage of signal through inertial element in the originals is associated with operation of the signal function convolution with kernel in the form of exponential function.

Let's discretize time t at increments of h, and move on to discrete convolution:

$$y(nh) = \sum_{k=0}^{n} v((n-k)h)x(kh)h.$$

Keep in mind that the value ds in continuous operator is associated with increment of h in discrete operator. Thus, $\nu(nh) = (h / T) \exp(-(nh/T))$. In z-images we obtain

$$Y(z) = V(z)x(z), V(z) = (h/T)z/(z-g), g = \exp(-h/T).$$

Let's move on to discrete originals through the shift theorem:

$$Y(z)(z-g) = (h/T)zx(z),$$

$$Y(z)(1-gz^{-1}) = (h/T)x(z),$$

$$Y(z) = gz^{-1}Y(z) + (h/T)x(z),$$

$$y_n = gy_{n1} + (h/T)x_n, y_n = y(nh), x_n = x(nh).$$

Let's perform correction of digital filter static balance mode for the case $y_n = y_{n-1} = x_n$ with $T \to 0$. As a result, we obtain a working formula for numerical simulation of inertial element:

$$y_n = gy_{n1} + (1+g)x_n.$$

Next value of discrete output signal is calculated from the current value of input signal and one of the previous values of the output signal.

Coming back to z-images, we get $V(z) = (1 - g)\, z / (z - g)$. Thus, from Laplace images we can pass to z-images through the following substitution:

$$1/(Tp+1) \Rightarrow (1-g)z/(z-g), g = \exp(-h/T).$$

The theorem is proven.

Accuracy of simulation of the primary continuous operator depends solely on length of the filter order grid and according to [6] does not depend on discretization increment (unlike Tustin's method). The validity of this statement can be verified by analyzing disparity between calculations of inertial element transient response obtained with digital filtering formula and analytical formula $y(t) = 1 - \exp(-t / T)$.

If Tustin's method is used, significant dynamic error is observed in numerical simulation of continuous object. Magnitude of this error decreases with decrease of discretization increment. Computational formula structure is more complex [2].

Concepts of direct current static transmission factor for continuous and discrete transfer functions are substantially different. In the first case, this value is equal to one (constant), and in the second case it is close to zero and has not physical, but mathematical meaning. In order to ensure equal levels of input and output signals of dynamic object in equilibrium state, this value can be chosen in such a way that its discrete transfer function shall be equal to one for $z = 1$. This condition ensures physical equality to one of transmission factor of continuous object discrete model.

Conclusions and generalization. If input signals used are smoother than a jump, minor error in numerical simulation of inertial element output signal can emerge as compared to analytical calculation. However, this error can be eliminated with additional weak transversal filtering. In this case, signal averaging in several adjacent points is actually performed in accordance with high-order quadrature formulas.

Thus, convolution method can be considered the most efficient method for synthesis of recursive digital filter to simulate linear stationary object with lumped parameters. Structure of its recurrence computational formulas is simpler than traditional one, and accuracy of numerical simulation of object is higher, even with error down to zero with respect to analytical calculation.

Depending on the kind of roots (simple, multiple, complex conjugate) of denominator polynomial of the primary rational fractional (in Laplace images) transfer function of the channel we get several kinds of partial fractions. We restricted ourselves to the case of one simple root, since all cases of complex roots generally can be reduced to it. Thus, the case of multiple root corresponds to a serial connection of several identical inertial elements [1], and the case of complex conjugate roots corresponds to loop joint of two inertial elements [7]. Parameters of each elementary unit of dynamic objects can have functional dependences on time or input signal amplitude, that allows for simulation of nonstationary or nonlinear objects respectively.

Thus, it may be concluded that instead of common integration or differentiation operations for development of dynamic models of continuous objects it makes sense to use more complex operations in the form of convolutions with exponential power kernels. It improves accuracy of numerical simulation of continuous objects.

A recursive digital filter construction method is considered for simulation of inertial element as a typical component of complex dynamic object. New computational formulas are obtained. Their high accuracy as compared to traditional ones is shown. Decomposition of initial model of simulated object by convolution operations with several typical exponential kernels is proposed instead of traditional operations of integration and differentiation.

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Розглянуто метод побудови рекурсивного цифрового фільтра для імітації ланки — типового елементу складного динамічного об’єкта. Отримано нові розрахункові формули. Показано їх високу точність порівняно з традиційними та доцільність декомпозиції вихідної моделі об’єкта, що імітується, за операціями згортки з декількома типовими експоненціальними ядрами замість традиційних операцій інтегрування та диференціювання.

Ключові слова: *математична модель, передатна функція, інтегральний оператор Вольтерри, оператор згортки, цифровий фільтр.*

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