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Backward electromagnetic waves in a magnetodisordered dielectric

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A nonlinear-evolution set of equations of the hydrodynamic type describing a magnet with a noncollinear arrangement of spins is investigated. An explicit expression invariant to right and left spin rotations is used for the energy density. The model under consideration can be interpreted as a continuum limit of a system of distributed symmetric tops. In the three-dimensional case exact solutions for the spin density are obtained in the form of helical waves for the quadratic-biquadratic energy density (in terms of Cartan's invariant functions). Solutions are also obtained for the magnon fields inducing these waves. The existence of backward helical waves is predicted. Energy transport may occur at an angle greater than $\pi/2$ relative to the direction of the helical waves. The analytical dependences of the wave vector and of the frequency on the helical wave amplitude, magnetic susceptibility, rigidity, and other constants of the model are found. The predicted property would allow for the construction of backward wave generators based on the use of disordered magnetic materials. The backward electromagnetic waves in a layered disordered magnetodielectric are considered. The relationship between the parameters of electromagnetic waves of the (e) layer and of the (i) layer is obtained.

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Introduction

The spin excitations in magnetic media with a noncollinear arrangement of spins are investigated using the hypothesis of spontaneous symmetry breaking of the statistical equilibrium state [1,2]. Using this hypothesis, Halperin and Hohenberg [3] proposed a hydrodynamic approach which was used to derive dynamic equations for magnetic media with a spontaneously broken symmetry with respect to spin rotations. Linear dynamical equations were obtained by Halperin and Saslov [4,5], while nonlinear dynamics was considered in the Lagrangian approach by Volkov and Zheltukhin [6] and by Andreev and Marchenko [7]. Dzyaloshinskii and Volovik used the Hamiltonian formalism for this purpose [8]. Peletminskii and co-workers developed this formalism for different magnetic structures [9,10].

The dynamical variables describing the nonequilibrium state of magnetic media with a spontaneously broken symmetry include the spin density $s_\alpha(\mathbf{x}, t)$ $(\alpha = x, y, z)$ and the order parameter, i.e., the orthogonal rotation matrix $a_{\alpha\beta}(\mathbf{x}, t)$. In the long-wavelength limit, where spatial nonuniformities of the dynamical variables are small, we investigate the dynamics and take into consideration the possible nonlinear interactions of spin waves, using the concept of spontaneous breaking of the $SO(3)$-symmetry of spin rotations that leave the exchange interactions invariant. We shall assume that the energy density is a function of s, a, and ∇a or, which is the same, of the variables $\underline{s}_\alpha \equiv a_{\alpha\beta}\, s_\beta$ and $\underline{\omega}_{\alpha k} \equiv \frac{1}{2}\, e_{\alpha\beta\gamma} a_{\beta\lambda} \nabla_k a_{\gamma\lambda}$, which is Cartan's right form. The evolution equations in terms of $\underline{s}_\alpha$ and $\underline{\omega}_{\alpha k}$ assume the form of equations with constraints [10,11]:

$$
\begin{aligned}
&\partial_t\, \underline{s}_\alpha = -\nabla_k \partial_{\underline{\omega}_{\alpha k}} \varepsilon + e_{\alpha\beta\gamma}\, (\underline{s}_\beta \partial_{\underline{s}_\gamma} \varepsilon + \underline{\omega}_{\beta k} \partial_{\underline{\omega}_{\gamma k}} \varepsilon)\ , \\
&\partial_t\, \underline{\omega}_{\alpha k} = -\nabla_k \partial_{\underline{s}_\alpha} \varepsilon + e_{\alpha\beta\gamma}\, \underline{\omega}_{\beta k} \partial_{\underline{s}_\gamma} \varepsilon\ , \\
&\nabla_k\, \underline{\omega}_{\alpha i} - \nabla_i\, \underline{\omega}_{\alpha k} = e_{\alpha\beta\gamma}\, \underline{\omega}_{\beta k}\, \underline{\omega}_{\gamma i}\ , \\
&s_\alpha = a_{\beta\alpha}\, \underline{s}_\beta\ .
\end{aligned}
\qquad (1)
$$

In these equations, $\partial_{\underline{s}_\alpha} \varepsilon = -\underline{\omega}_\alpha$, where $\underline{\omega}_\alpha = \frac{1}{2}\, e_{\alpha\beta\gamma}\, (\partial_t a a^T)_{\gamma\beta}$ is the right form associated with the time derivative. The set (1) determines the dynamical properties of the system without taking dissipation into account and describes the low frequency dynamics with an exchange interaction, when, for long enough times, rigid spin complexes are formed because of the strong exchange. These complexes remain practically undeformed, and their orientation is determined by the orthogonal rotation matrix $a_{\alpha\beta}(\mathbf{x}, t)$. It follows from set of equations (1) that the energy density ε and the momentum components $\pi_i = \underline{s}_\alpha \, \underline{\omega}_{\alpha i}$ are conserved locally:

$$\partial_t \varepsilon = -\nabla_k \partial_{\underline{s}_\alpha} \varepsilon \, \partial_{\underline{\omega}_{\alpha k}} \varepsilon \, , \quad \partial_t \pi_\alpha = -\nabla_k t_{ik} \, ,$$
$$t_{ik} = -\delta_{ik}(\varepsilon - \underline{s}_\alpha \partial_{\underline{s}_\alpha} \varepsilon) + \underline{\omega}_{\alpha i} \partial_{\underline{\omega}_{\alpha k}} \varepsilon \, , \qquad (2)$$

where t_{ik} is the momentum flux density tensor. In practice we used the following expression for the energy density:

$$\varepsilon = \varepsilon_i + \varepsilon_a \, ,$$

where

$$\varepsilon_i = \frac{1}{2\chi} \underline{s}_\alpha^2 + \frac{\rho}{2} \underline{\omega}_{\alpha k}^2 + \frac{1}{4\chi_1} \underline{s}_\alpha^4 + \frac{\rho_1}{4} \underline{\omega}_{\alpha k}^4 + \frac{q}{2} \pi_i^2 \qquad (3)$$

is the isotropic component and

$$\varepsilon_a = \frac{\rho_2}{4} (\underline{\omega}_{\alpha x}^2 \, \underline{\omega}_{\alpha y}^2 + \underline{\omega}_{\alpha x}^2 \, \underline{\omega}_{\alpha z}^2 + \underline{\omega}_{\alpha y}^2 \, \underline{\omega}_{\alpha z}^2) \qquad (4)$$

is the «anisotropic» component (without taking into account the differential equations of coupling between $\underline{\omega}_{\alpha x}$, $\underline{\omega}_{\alpha y}$, $\underline{\omega}_{\alpha z}$) of the energy; χ is the magnetic susceptibility; ρ is the «stiffness» constant, and χ_1 , ρ_1 , ρ_2 , and q are phenomenological coupling constants. This energy density is invariant to left and right spin rotations. The general set of equations (1) has been studied by us previously (see Ref. [11]). Ivanov [12] used the Lagrangian approach for a quadratic dependence of the energy density (amorphous magnet) to obtain topological solitons in spin glasses.

Backward spin density waves

Let us determine the exact nonlinear solutions of stationary profile of the system (1). These solutions are helical waves with helical vector $\mathbf{k}$ and frequency ω [13]:

$$s_x = c_3 \sin \varphi_0 \sin \theta + c_1 \cos \varphi_0 \, ,$$
$$s_y = -c_3 \cos \varphi_0 \sin \theta + c_1 \sin \varphi_0 \, , \qquad (5)$$
$$s_z = c_3 \cos \theta$$

where

$$\mathbf{k} = \frac{c_1}{\rho} \left[\frac{|C_3|}{2c_3} \pm \left(\frac{C_3^2}{4c_3^2} + \frac{\mathbf{q}^2 \rho}{\chi} \right)^{1/2} \right] \mathbf{q} \, ,$$
$$\omega = \frac{c_1}{\chi} \, , \ \theta(\mathbf{x}, t) = \omega t - \mathbf{k}\mathbf{x} + \theta_0 \, , \qquad (6)$$
$$\mathbf{s}^2 = c_1^2 + c_3^2 \, ,$$

c_1 , c_3 , C_3 , θ_0 , $\mathbf{q}$ are constants.

We have used the parametrization of the orthogonal rotation matrix $a_{\alpha\beta}(\mathbf{x}, t)$ with Eulerian angles φ, θ, ψ [13,14]. For the sake of simplicity we have not written out the contribution of the biquadratic terms (see Ref. [13]). The self-consistent magnetic field $\mathbf{h}$ forming a helical spin density wave $s(\mathbf{x}, t)$ is given by the relation $h_\alpha \equiv \partial_{s_\alpha} \varepsilon$ and is defined as

$$\mathbf{h} = \frac{1}{\chi} \mathbf{s} \, . \qquad (7)$$

It follows from formulas (5) and (7) that the self-consistent static magnetic field has the form $\mathbf{h}_0 = (c_1/\chi \cos \varphi_0 \, , c_1/\chi \sin \varphi_0 \, , 0)$ and determines the eigenfrequency of the magnetic moments. Obviously, the magnitude of this frequency equals $|c_1|/\chi$, $\chi > 0$.

According to Eqs. (2), the energy flux density is defined as

$$j_k = \partial_{\underline{s}_\alpha} \varepsilon \, \partial_{\underline{\omega}_{\alpha k}} \varepsilon \, . \qquad (8)$$

Let us now determine the cosine of the angle w between the direction of wave propagation $\mathbf{k}$ and the direction of the energy flux density $\mathbf{j}$:

$$\cos w = \frac{\mathbf{k} \cdot \mathbf{j}}{|\mathbf{k}||\mathbf{j}|} \, . \qquad (9)$$

Formula (9) assumes a simple form for all positive phenomenological coupling constants:

$$\cos w = \frac{|q_x|}{(q_x^2 + q_y^2 + q_z^2)^{1/2}} \operatorname{sgn} c_1 \, . \qquad (10)$$

Since the constant c_1 can be positive or negative in the model under consideration, we arrive at the conclusion that helical waves in a disordered magnet can propagate in the direction opposite (at an angle greater than $\pi/2$) to the energy transport direction.

Electromagnetic waves in a layered magnetodisordered dielectric

In a dielectric medium without free charges and currents, the electromagnetic vectors **e**, **h** obey the Maxwell equations

$$\text{rot}\,\mathbf{e} = -\frac{1}{c}\,\partial_t \mathbf{b}\ ,\quad \text{div}\,\mathbf{b} = 0\ ,$$
$$\text{rot}\,\mathbf{h} = \frac{1}{c}\,\partial_t \mathbf{d}\ ,\quad \text{div}\,\mathbf{d} = 0\ , \tag{11}$$

where c is the speed of light; $\mathbf{b} = \mu\mathbf{h}$ is the magnetic induction, μ is the magnetic permeability; $\mathbf{d} = \varepsilon\mathbf{e}$ is the electric displacement, and ε is the dielectric permittivity.

Since $\omega^2 = (c^2/\varepsilon\mu)\mathbf{k}^2$, as follows from Eqs. (11), taking into account formula (6), we find

$$\frac{|C_3|}{c_3} = \pm\left(1 - \frac{\varepsilon\mu\rho}{c^2\chi}\right)\left(\frac{\varepsilon\mu\rho}{c^2\chi}\right)^{-1/2} . \tag{12}$$

It is evident that this ratio tends to zero if $\varepsilon \to 1$, $\mu \to 1$, and $\rho/\chi \to c^2$. From the equation div $\mathbf{b} = 0$, we obtain two conditions:

$$k_x \sin\varphi_0 = k_y \cos\varphi_0\ ,\quad k_z = 0\ ,\quad |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}\ . \tag{13}$$

The fact that the solution is independent of the space variable z in the three-dimensional space indicates that the waves are «cylindrical». According to Eqs. (5), (7), and (11), the self-consistent electric field in the magnetodielectric has the form

$$e_x = -\frac{c c_3 k_y}{\omega\chi\varepsilon}\cos\theta + e_{0x}\ ,$$
$$e_y = \frac{c c_3 k_x}{\omega\chi\varepsilon}\cos\theta + e_{0y}\ , \tag{14}$$
$$e_z = \frac{c c_3 |\mathbf{k}|}{\omega\chi\varepsilon}\sin\theta + e_{0z}\ ,$$

where $\mathbf{e}_0 = (e_{0x}\,,\, e_{0y}\,,\, e_{0z})$ is a self-consistent static electric field in the dielectric.

In this phenomenological approach, we cannot determine the parameters c_1, $\mathbf{q}$, $\mathbf{e}_0$, but the boundary conditions

$$\mathbf{n}\times(\mathbf{h}^e - \mathbf{h}^i) = 0,\quad \mathbf{n}\,(\varepsilon^e\mathbf{e}^e - \varepsilon^i\mathbf{e}^i) = 0\ ,$$
$$\mathbf{n}\times(\mathbf{e}^e - \mathbf{e}^i) = 0,\quad \mathbf{n}\,(\mu^e\mathbf{h}^e - \mu^i\mathbf{h}^i) = 0 \tag{15}$$

define the relationship between the parameters of the dielectric medium (i) and the medium (e). Here **n** is the unit vector normal to the boundary surface of the medium (i) and medium (e).

From relations (15) for the boundary surface $z = 0$, $\mathbf{n} = (0,0,1)$ we obtain, after eliminating the coordinates of the boundary surface, the following:

$$c|\mathbf{k}^e|\left(\frac{\omega^i}{\omega^e} - \cos(\varphi_0^i - \varphi_0^e)\right) + c|\mathbf{k}^i|\left(\frac{\omega^e}{\omega^i} - \cos(\varphi_0^i - \varphi_0^e)\right) + (\varepsilon^i e_{0z}^i - \varepsilon^e e_{0z}^e)\sin(\varphi_0^i - \varphi_0^e) = 0,\quad \sin(\varphi_0^i - \varphi_0^e) \neq 0\ . \tag{16}$$

$$\left(\frac{1}{\mu^i\varepsilon^i}\frac{\mathbf{k}^i}{\omega^i} - \frac{1}{\mu^e\varepsilon^e}\frac{\mathbf{k}^e}{\omega^e}\right)(\mathbf{e}_0^i - \mathbf{e}_0^e) = 0,\quad k_x^e k_y^i - k_y^e k_x^i \neq 0\ . \tag{17}$$

Now we consider the case when the static electric field in the dielectric medium is $\mathbf{e}_0 = (0,\,0,\,e_{0z})$. Since the energy flux density in the dielectric is $\mathbf{j} = (c/4\pi)\mathbf{e}\times\mathbf{h}$, as follows from Eqs. (11), we find

$$\cos w^i = \frac{\mathbf{k}^i\cdot\mathbf{j}^i}{|\mathbf{k}^i||\mathbf{j}^i|} = \frac{\sqrt{\mu^i/\varepsilon^i}\,(c_3^{i^2}/\chi^{i^2})\,\text{sgn}\,\omega^i + e_{0z}^i(c_3^i/\chi^i)\sin\theta^i}{\left[\left(\frac{\mu^i}{\varepsilon^i}\frac{c_3^{i^2}}{\chi^{i^2}} + 2e_{0z}^i\sqrt{\mu^i/\varepsilon^i}\,\frac{c_3^i}{\chi^i}\,\text{sgn}\,\omega^i\sin\theta^i\right)\frac{(c_1^{i^2} + c_3^{i^2})}{\chi^{i^2}} + e_{0z}^{i^2}\frac{c_1^{i^2}}{\chi^{i^2}} + e_{0z}^{i^2}\frac{c_3^{i^2}}{\chi^{i^2}}\sin^2\theta^i\right]^{1/2}}\ . \tag{18}$$

$\theta^i \equiv \omega^i t - \mathbf{k}^i\mathbf{x} + \theta_0^i$, and $\mathbf{x}$ belongs to the medium (i).

Equation (16) has the real solution

$$\frac{\omega^i}{\omega^e} = \frac{-b \pm \sqrt{b^2 - 4|\mathbf{k}^i||\mathbf{k}^e|}}{2|\mathbf{k}^e|}\ ,$$
$$b \equiv (\varepsilon^i e_{0z}^i - \varepsilon^e e_{0z}^e)\,\frac{k_y^i k_x^e - k_x^i k_y^e}{|\mathbf{k}^i||\mathbf{k}^e|} - (|\mathbf{k}|^i + |\mathbf{k}^e|)\,\frac{k_x^i k_x^e + k_y^i k_y^e}{|\mathbf{k}^i||\mathbf{k}^e|} \gtrless \pm 2\sqrt{|\mathbf{k}^i||\mathbf{k}^e|} \tag{19}$$

with due regard for (13). Solution (19) points to the existence of the backward electromagnetic waves according to (18), provided that, for example,

$$\sqrt{\mu^i/\varepsilon^i}\; c_3^i \operatorname{sgn} \omega^i + e_{0z}^i \sin\theta^i < 0, \quad c_3^i > 0 .$$

If we choose the boundary surface $y = 0$, $\mathbf{n} = (0,1,0)$ then we obtain the following conditions of coupling between the parameters in the model under consideration:

$$\frac{c}{\varepsilon^e}|\mathbf{k}^e|\left(\mu^i \frac{\omega^i}{\omega^e} - \mu^i \cos\varphi_0^i \cos\varphi_0^e - \mu^e \sin\varphi_0^i \sin\varphi_0^e\right) +$$
$$+ \frac{c}{\varepsilon^i}|\mathbf{k}^i|\left(\mu^e \frac{\omega^e}{\omega^i} - \mu^e \cos\varphi_0^i \cos\varphi_0^e - \mu^i \sin\varphi_0^i \sin\varphi_0^e\right) +$$
$$+ (e_{0z}^i - e_{0z}^e)(\mu^e \sin\varphi_0^i \cos\varphi_0^e - \mu^i \cos\varphi_0^i \sin\varphi_0^e) = 0 . \tag{20}$$

$$\mu^e \sin\varphi_0^i \cos\varphi_0^e - \mu^i \cos\varphi_0^i \sin\varphi_0^e \neq 0 .$$

$$\left(\frac{k_x^i}{\omega^i} - \frac{k_x^e}{\omega^e}\right)(e_{0x}^i - e_{0x}^e) + \left(\frac{k_y^i}{\omega^i} - \frac{\varepsilon^i}{\varepsilon^e}\frac{k_y^e}{\omega^e}\right)e_{0y}^i -$$
$$-\left(\frac{\varepsilon^e}{\varepsilon^i}\frac{k_y^i}{\omega^i} - \frac{k_y^e}{\omega^e}\right)e_{0y}^e = 0 , \quad \varepsilon^e k_x^e k_y^i - \varepsilon^i k_x^i k_y^e \neq 0 . \tag{21}$$

Formulas (7), (14), (16), and (17) can be applied to a layered magnetodisordered dielectric medium.

Conclusion

According to Eq. (5), the exact nonlinear solutions presented here are helical waves. The contribution of biquadratic terms to the energy density (3), (4) increases with the spin density in the system [13]. Energy transport can occur at an angle greater than $\pi/2$ with respect to the direction of propagation of the helical spin wave. Formulas (7) and (14), together with the boundary conditions (15), can be verified in an experiment. The relation between the parameters of the electromagnetic waves and the properties of the layer (e) and the layer (i) for a flat boundary (formulas (16), (17)) is established.

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