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Quantum computing with bits made of electrons on a helium surface

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We describe a quantum computer based on electrons supported by a helium film and localized laterally by small electrodes. Each quantum bit (qubit) is made of combinations of the ground and first excited state of an electron trapped in the image potential well at the surface. Mechanisms for preparing the initial state of the qubit, operations with the qubits, and a proposed readout are described. This system is, in principle, capable of 10^5 operations in a decoherence time.

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1. Introduction

In a quantum computer two stationary states of a quantum system are identified with classical bits 0 and 1. Each quantum bit (qubit) is made up of a superposition of these two quantum states. The state of the nth qubit

$$\psi = a\,|0\rangle + b\,|1\rangle\,;\quad |a|^2 + |b|^2 = 1\;. \qquad (1)$$

In the majority of operations $a_n = 0$, 1 or $2^{-1/2}$.

A quantum computer is a superposition of all qubit states. A simple superposition is a product of individual qubit states. The general state of a quantum computer is an entangled state, a state that cannot be made of a product of individual qubits. An example is the state

$$2^{-1/2}\,(|01\rangle + |10\rangle). \qquad (2)$$

One method of producing an entangled state is by using a controlled NOT (CNOT) gate. This gate consists of a control bit and a target bit. If a control bit is $|0\rangle$ the target bit is unchanged, while if the control bit is $|1\rangle$ the components of the target bit $|0\rangle$ and $|1\rangle$ change to $|1\rangle$ and $|0\rangle$, respectively. A CNOT gate is described below. An example of an entangling operation is with the first bit as the control bit

$$2^{-1/2}(|0\rangle + |1\rangle)|1\rangle = 2^{-1/2}(|01\rangle + |10\rangle)\;. \qquad (3)$$

The general state is written as

$$\Psi = \sum_j \alpha_j \left|x_j\right\rangle\,,\quad \sum_j |\alpha_j|^2 = 1, \qquad (4)$$

where each basis vector $\left|x_j\right\rangle$ is one of the 2^n permutations of the zeros and ones representing separate qubits. In the final state one of these basis vectors is the answer to the calculation. The entangled state collapses when one of the qubits is read. For the most general algorithms all qubits must be read out simultaneously.

We describe here a proposed quantum computer that uses laterally confined electrons on the surface of a liquid helium film as qubits and describe operations with these qubits including a simultaneous readout. A full description of quantum computing is beyond the scope of this paper [1]. This concept was first introduced by Platzman and Dykman [2].

2. Electrons on helium

Electrons are bound to the surface of liquid helium by the dielectric image potential. A repulsive Pauli potential that can be represented to a good approximation as $V = \infty$ for $z < 0$ prevents them from penetrating into liquid helium. The hydrogenic-like potential for $z > 0$ is

$$V = -\Lambda e^2/4\pi\varepsilon_0 z;\quad \Lambda = (\kappa - 1)/4(\kappa + 1)\;, \qquad (5)$$

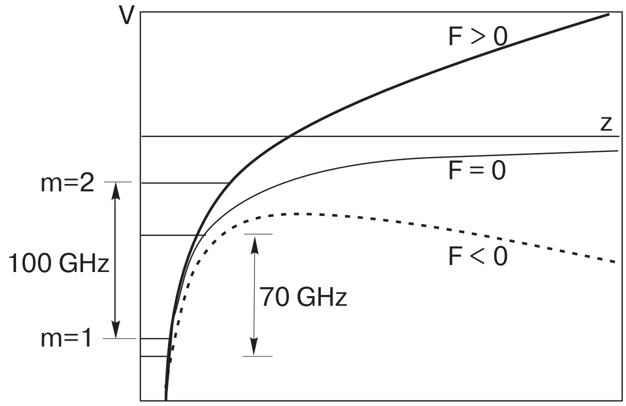

Fig. 1. The potentials and energy levels with and without an electric field F applied normal to the surface. The ground ($m = 1$) and excited ($m = 2$) energy levels are indicated schematically for each potential. The potential for an extracting field $F < 0$ is also shown as a dashed line.

where z is the coordinate normal to the surface, and κ is the dielectric constant of helium. The energy levels form a Rydberg spectrum, $E_n = -R/n^2$. The parameters for liquid ^{3}He are $\Lambda_3 = 0.00521$, $R_3 = 0.37$ meV, and the effective Bohr radius is $a_B = 10.2$ nm. The average separations of the electron from the surface are $\langle z \rangle = 15.3$ and 61 nm for the ground and first excited state, respectively. The transition frequency between the ground and first excited state is 70 GHz. These transitions can be shifted with a Stark field applied normal to the surface. The potentials are shown in Fig. 1 for applied electric fields $F > 0$, $F = 0$ and $F < 0$.

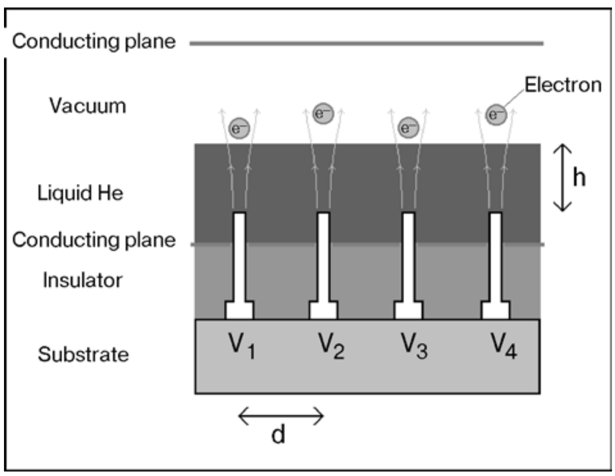

Fig. 2. The geometry of a four-qubit system with electrons above the microstructure and the helium film. The drawing is not to scale. Optimal dimensions are $d \approx h \approx 1$ μm. Control potentials V_n are applied on the microelectrodes.

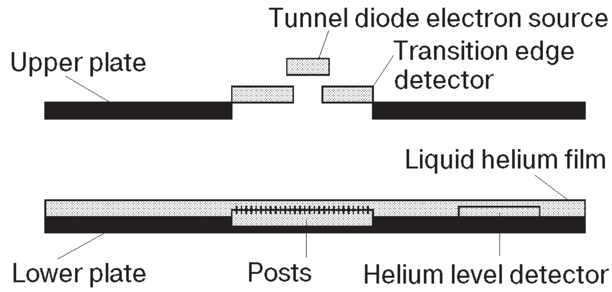

Fig. 3. Schematic of the cell. The upper plate includes detectors used in the readout. The lower plate includes the posts and is covered with the helium film. The electrons float over the posts about 10 nm above the surface of the helium film.

3. Design of the computer

We identify the ground and first excited states of these electrons with the states $|0\rangle$ and $|1\rangle$, respectively. In order to address and control the qubits each electron must be localized laterally. This will be accomplished by locating electrons above microelectrodes (posts) that are separated by about 1 μm. The electrons will be separated from the posts by an ≈ 1 μm thick helium film. Lateral confinement results from the image potentials of the posts and the potential applied on the posts. The electron will be in the ground state for lateral motion.

A schematic of posts and electrons for a four-qubit system is shown in Fig. 2. A voltage applied to a given post controls the Stark field for the corresponding electron. An array of posts with leads has been fabricated.

A schematic of our cell is shown in Fig. 3. The plates form the top and bottom of an enlarged waveguide that transmits sub-mm radiation to the electrons. Superconducting micro-bolometers will be located at the top of the guide to detect electrons that are allowed to escape from the posts. A tunnel-diode electron-emission source will be located above the electron detectors. Electrons will be loaded onto the film through a hole in the detector chip, and one electron will be trapped over each post by image and applied potentials. The helium film thickness will be measured with a capacitor made of metal strips deposited on the ground plane. The system will be operated at 10 mK to increase coherence times of the qubit states.

4. Operations

Data input: The operation would normally begin with all qubits in the ground state. Then initial data will be input by preparing each qubit in some admixture of states $|0\rangle$ and $|1\rangle$ by Stark shifting individual qubits into resonance with microwave radiation for a predetermined time. The state of the qubit n will be

$$\Psi_n = \cos(\theta_n/2)\,|0\rangle - i\sin(\theta_n/2)\,|1\rangle\,, \qquad (6)$$

where $\theta_n = \Omega\tau_n$, $\Omega = eE_{\mathrm{rf}}\langle 1|z|2\rangle/\hbar$ is the Rabi frequency, E_{rf} is the strength of the rf field, and τ_n is the time the nth qubit is in resonance with the microwave field.

Quantum gates: In general, computations will be implemented by applying pulses of radiation to interacting qubits. We illustrate a potential operating mode of the system by describing two qubits operated as a SWAP gate. The interaction is the Coulomb interaction between neighboring electrons. The dipolar component of the direct interaction potential between qubits i and j is

$$V(z_i, z_j) \sim (e^2/8\pi\varepsilon_0 d^3)(z_i - z_j)^2, \qquad (7)$$

where d is the electron separation, and z_i is the separation of the ith electron from the helium surface. Start with one qubit in the state $|0\rangle$ and the other in the state $|1\rangle$. Next apply the same Stark fields to both qubits so that the states $|01\rangle$ and $|10\rangle$ would be degenerate. In this condition the system will oscillate between the two states at a frequency given by the interaction energy, which in first order is given by $e^2a_B{}^2/4\pi\varepsilon_0 d^3$. This frequency is $\sim 10^8$ Hz for a separation of 1 μm. By leaving the electric fields in this condition for one half cycle of this oscillation, the two qubits will swap states. It will be difficult to tune neighboring qubits to precisely identical Stark shifts, and in practice we may sweep the Stark shift of one qubit through resonance with a neighboring qubit. In this case, the final state of the qubit will depend on the rate at which the electric field is swept through the resonance condition.

A two-qubit CNOT gate can be operated as follows. The energy for the target bit to make a transition depends whether the separation of the electrons is increased or decreased in the transition. The transition frequency is

$$\Delta\nu = \Delta\nu_0 \pm (e^2/8\pi\varepsilon_0 d^3 h)(z_2 - z_1)^2, \qquad (8)$$

respectively, when the control bit is in the $|1\rangle$ or $|0\rangle$ state. Here $\Delta\nu_0$ is the frequency in the absence of interactions, h is Planck's constant, and subscripts refer to the ground and excited states. By applying radiation at one of the frequencies, a transition will or will not occur depending on the state of the control bit. The control bit is Stark shifted out of resonance.

Readout. For the general case the states of all qubits must be read within the time scale is set by the

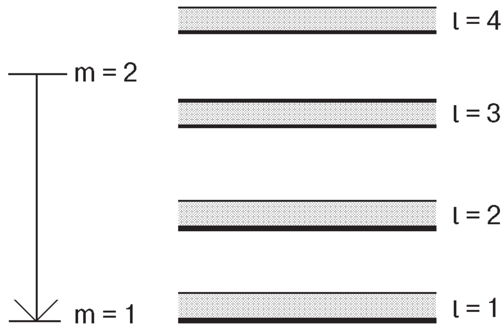

Fig. 4. Energy level schematic. The integer m labels the hydrogenic-like states, and the integer l labels lateral states, which represent harmonic oscillator states or Landau levels in a magnetic field. The hatched region indicates the band of plasma oscillations associated with each level.

plasma frequency ~ 100 GHz. We describe here our initial proposal for a destructive readout pending research into other schemes. We will apply a short, ~ 1 ns, ramp of an extracting electric field to all qubits. The potential for a fixed value of extracting field is shown in Fig. 1. The tunneling probability is exponential in the time-dependent barrier height and width. All electrons in the upper $|1\rangle$ state will tunnel through the barrier within a short period of time when this probability becomes sufficiently large. For this extracting field the tunneling probability will be negligibly small for electrons in the ground $|0\rangle$ state. After the ramp is removed the remaining electrons will be in the ground state. Subsequently, an extracting field sufficiently large so that electrons in the ground state will tunnel from the surface [3] will be applied sequentially to each post. A $|0\rangle$ will be registered for each electron detected by the transition-edge bolometer and a $|1\rangle$ for those states that are empty.

5. Decoherence

T_1: Logic operations must be accomplished in less time than it takes for the interactions of the qubit with the environment to destroy the phase coherence of the state functions. For electrons on ^{4}He the lifetime of the excited state $|1\rangle$ is limited by interactions with ripplons and coupling to phonons in the bulk liquid. These processes are discussed in detail by Dykman et al. [4]. The electron-ripplon coupling Hamiltonian is

$$\mathcal{H}_{\mathrm{er}} = eE_\perp\delta, \qquad (9)$$

where $E_\perp$ is the normal component of the electric field that includes both the applied field and variations in the helium dielectric image field due to surface distortions, and δ is the amplitude of the surface height variation. The average rms thermal fluctuation of the surface is

$$\delta_T = (k_B T/\sigma)^{1/2} \cong 2\cdot 10^{-9}\ \text{cm}. \qquad (10)$$

The transition from the excited to ground state requires a ripplon with a wave vector $\cong a_B^{-1}$. For a single electron on bulk helium a radiationless transition occurs with the energy absorbed by electron plane-wave states for motion parallel to the surface and momentum absorbed by ripplons. For this case a calculation of T_1 yields

$$T_1^{-1} \cong \Delta\nu(\delta_T/a_B)^2, \qquad (11)$$

where $\Delta\nu$ is the transition frequency. At T = 10 mK, $\delta_T/a_B \cong 10^{-3}$, and $T_1 \cong 10$ μs.

For electrons confined by posts, the lateral states are harmonic oscillator states of the image potential well of the posts. These are separated in energy by $\hbar\omega_1 \approx \hbar(eE_\perp/4\pi\varepsilon_0 mh)^{1/2} \sim 1$ K for $E_\perp = 500$ V/cm. For interacting electrons there is a band of plasma oscillations associated with each harmonic oscillator level, which for a crystal array with a separation of 1 μm has a bandwidth that is 300 mK. The frequency ω_1 can be tuned so the transition $|1\rangle \to |0\rangle$ is incommensurate in energy with the excitation to the plasmon band of any harmonic level and suppresses this channel for decay. This is illustrated in Fig. 4. The separation of energy levels for lateral motion can also be accomplished with the application of a magnetic field.

Decay can occur with the emission of two ripplons with opposite wavevectors and the excitation of a harmonic level nearest in energy to the $|1\rangle$ state plus plasmons. This yields a value of $T_1 \sim 1$ msec.

The dominant T_1 decay mechanism is the emission of a phonon into the bulk liquid with the excitation of harmonic energy levels. The coupling is through phonon-induced modulation of the image potential of an electron and leads to a decay time of ~ 30 μsec.

T_2: The phase of the wave-function varies a $[U(t) + e\phi(t)]t/\hbar$, where U and ϕ are the energy of a state and the electrostatic potential, respectively. Dephasing occurs due to a decay of the phase difference between the two qubit states $|1\rangle$ and $|0\rangle$. The dominant dephasing mechanism is estimated to be Johnson noise in the micro-electrodes. For a 25 Ω resistor at T = 1 K attached to the posts, the dephasing time is estimated to be $T_2 \sim 100$ μsec. A two-ripplon scattering process modulates the energy of the states and leads to a value $T_2 \sim 10$ msec.

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