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Proximity effect at the interface He II–He I in microgravity environment

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The proximity effect causes the existence of some transition area with the gradual variation of the density of superfluid component instead of the sharp boundary at the level where the hydrostatic pressure realizes the phase transition He II–He I. In the microgravity environment the characteristic length of this effect increases, and more convenient conditions arise for measurements in the transition area. The problem of the expansion of thermodynamical potential in power series in the vicinity of He II–He I interface is considered. The critical values of the size of the superfluid area are determined.

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Introduction

According to the phase diagram of ^{4}He the hydrostatic pressure causes the phase transition He II→He I but, because of the proximity effect, instead of abrupt boundary between the superfluid and normal liquid at the level, where the λ-pressure is reached, there is formed the transition zone where the density of superfluid component varies gradually. The superfluidity penetrates into the normal area [1–3]. The characteristic length of this effect is $\xi_g = 6.5 \cdot 10^{-3}$ cm [1].

Microgravity

The width of the transition zone (its height) may be estimated as several times ξ_g, i.e., it is of the order of 10^{-2} cm. This value is quite macroscopic but nevertheless so small that nobody could carry out the experimental study of this area. The sole attempt [4] was unsuccessful.

The more convenient conditions for measurements in transition area at the boundary He II–He I are achieved in the microgravity environment since $\xi_g = \xi_0^{3/5}\alpha^{-2/5}$ where $\xi_0 = 2.73 \cdot 10^{-8}$ cm $\cdot$ K$^{2/3}$, $\alpha = \rho g / |dP_\lambda / dT|$. I.e., the 10^5 times decrease of g entails the increase of the width of transition area to centimeters, and $3 \cdot 10^7$ times decrease of g is necessary to reach the height of order of 10 cm.

The unit of wave function and «Ψ = 0 problem»

The Ginzburg–Pitaevskii (GP) equation for ^{4}He being under hydrostatic pressure may be written down in the form (we do not consider some flow and Ψ is a positive function):

$$\frac{\hbar^2}{2m}\frac{d^2\Psi}{dz^2} + A\Psi - B\Psi^3 - C\Psi^5 = 0, \qquad (1)$$

where A and B are dependent on the distance z from the boundary ($z > 0$ is the superfluid area, $z < 0$ is the normal one): $A = \pm A_0(\alpha|z|)^{4/3}$ (the sign minus is for the normal area), $B = B_0(\alpha|z|)^{2/3}$. The asymptotic solution far in the superfluid area ($z >> \xi_g$) is

$$\Psi_a = (\alpha z)^{1/3}\Psi_0, \quad \Psi_0^2 = -\frac{B_0}{2C} + \sqrt{\left(\frac{B_0}{2C}\right)^2 + \frac{A_0}{C}}. \qquad (2)$$

It is convenient to introduce the parameter M: $M = C\Psi_0^4 / A_0 = 1 - B_0\Psi_0^2 / A_0$ and to express coefficients A_0, B_0 as

$$A_0 = \frac{1.11 \cdot 10^{-16}}{1 + M/3}\ \mathrm{erg} \cdot \mathrm{K}^{-4/3},$$

$$B_0 = 3.52 \cdot 10^{-39}\frac{1 - M}{1 + M/3}\ \mathrm{erg} \cdot \mathrm{cm} \cdot \mathrm{K}^{-2/3}.$$

Then Eq. (1) may be transformed into the dimensionless form

$$\frac{d^2\psi}{d\zeta^2} \pm |\zeta|^{4/3}\psi - (1-M)|\zeta|^{2/3}\psi^3 - M\psi^5 = 0, \quad (3)$$

where $\zeta = z/\xi_{gM}$, $\psi = \Psi/\Psi_{gM}$:

$$\xi_{gM} = \xi_g (1 + M/3)^{3/10},$$

$$\Psi_{gM} = \sqrt{\xi_0/\xi_g}(1 + M/3)^{1/10}\Psi_0 \ [3].$$

The GP equation is founded on the expansion of the thermodynamical potential in power series. If this expansion is performed in respect to the ratio of wave function to its equilibrium value then it becomes invalid at the He II–He I boundary where this ratio becomes infinite [3]. This problem does not exist for Eq. (3) and for corresponding expansion in power series in respect to Ψ/Ψ_{gM} because Ψ_{gM} is temperature and coordinate independent (if we compare Ψ_{gM} with Ψ_a, the quantity αz in Eq. (2) is substituted by $(\xi_0/\xi_g)^{3/2}(1 + M/3)^{3/10}$). These quantities have the dimension of temperature, but the latter is temperature and coordinate independent.

In these units the asymptotic solution (Eq. (2)) has the form $\psi_a = \zeta^{1/3}$.

Critical sizes of superfluid area

The superfluid area must content the superfluid component enough to entail the superfluidity in normal area. That is why the size (the height) of the superfluid area H_s has the critical value H_{sc} such that if $H_s < H_{sc}$ then the density of the superfluid component is zero in the whole vessel (more exactly for the case $M > 1$ see below). It is determined by the equation:

$$J_{0.3}\left(\frac{3}{5}h_{sc}^{5/3}\right) I_{-0.3}\left(\frac{3}{5}h_n^{5/3}\right) + J_{-0.3}\left(\frac{3}{5}h_{sc}^{5/3}\right) I_{0.3}\left(\frac{3}{5}h_n^{5/3}\right) = 0, \quad (4)$$

the first variant of which is obtained in [1]; $h_{sc} = 2.29$ when $h_n = \infty$ [2,5] and $h_{sc} = 2.55$ when $h_n = 0$ [5] ($-H_n < z < H_s$, H_n is the size of normal area, $h = H/\xi_{gM}$). Let us consider the case $h_s - h_{sc} << h_{sc}$ and $h_n = 0$. Employing the method suggested in [6] we obtain the approximate solution of Eq. (3) of the form $\psi = c\sqrt{\zeta}J_{0.3}(\zeta^{5/3}j/h_s^{5/3})$ where $j = 2.8541$. The analyses of the coefficient c shows that in the case $M > 1$ the size h_{sc} is not critical and that there exist two other critical sizes: $h_{\min}$ above which the superfluidity becomes possible, and h_{tr} above which the superfluidity becomes stable, $h_{tr} > h_{\min}$:

$$h_{\min} = h_{sc}\left[1 + \frac{(M-1)^2}{4M}\frac{I_2^2}{I_1 I_3}\right]^{-3/10},$$
$$h_{tr} = h_{sc}\left[1 + \frac{3}{16}\frac{(M-1)^2}{M}\frac{I_2^2}{I_1 I_3}\right]^{-3/10}, \quad (5)$$

where
$$I_1 = \int_0^j x J_{0.3}^2(x)dx = 0.91627,$$
$$I_2 = \int_0^j x^{6/5} J_{0.3}^4(x)dx = 0.37001,$$
$$I_3 = \int_0^j x^{7/5} J_{0.3}^3(x)dx = 0.16790.$$

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