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DISCRETE-TIME STEADY-STATE CONTROL OF INTERCONNECTED SYSTEMS BASED ON PSEUDOINVERSION CONCEPT

Introduction. *The problem of controlling interconnected systems subjected to arbitrary unmeasurable disturbances remains actual up to now. It is important problem from both theoretical and practical points of view. During the last decades, the internal model control principle becomes popular among other methods dealing with an improvement of the control system. A perspective modification of the internal model control principle is the so-called model inverse approach. Unfortunately, the inverse model approach is quite unacceptable if the systems to be controlled are square but singular or if they are nonsquare. It turned out that the so-called pseudoinverse (generalized inverse) model approach can be exploited to cope with the noninevitability of singular square and also nonsquare system.*

The purpose *of the paper is to generalize the results obtained by the authors in their last works which are related to the asymptotic properties of the pseudoinverse model-based method for designing an efficient steady-state control of interconnected systems with uncertainties and arbitrary bounded disturbances and also to present some new results.*

Results. *In this paper, the main effort is focused on analyzing the asymptotic properties of the closed-loop systems containing the pseudoinverse model-based controllers. In the framework of the pseudoinversion concept, new theoretical results related to the asymptotic behavior of these systems are obtained. Namely, in the case of nonsingular gain matrices with known elements, the upper bounds on the ultimate norms of output and control input vectors are found. Next, in the case of nonsquare gain matrices whose elements are also known, the asymptotic behavior of the feedback control systems designed on the basis of pseudoinverse approach are studied. Further, the sufficient conditions guaranteeing the boundedness of the output and control input signals for the linear and certain class of nonlinear interconnected systems in the presence of uncertainties are derived.*

Conclusion. *It has been established that the pseudoinverse model-based concept can be used as a unified concept to deal with the steady-state regulation of the linear interconnected discrete-time systems and of some classes of nonlinear interconnected systems with possible uncertainties in the presence of arbitrary unmeasured but bounded disturbances.*

Keywords: *discrete time, feedback, pseudoinversion, interconnected systems, optimality, stability, uncertainty.*

INTRODUCTION

The problem of controlling interconnected systems subjected to arbitrary unmeasurable disturbances stated several decades ago in the work [1] remains actual up to now [2, 3]. It is important problem from both theoretical and practical points of view [4, 5]. During the last decades, the internal model control principle becomes popular among other methods dealing with an improvement of the control system. Based on this method, interconnected control problem was first approached in [6]. A perspective modification of the internal model control principle is the so-called model inverse approach. The perfect output control performance is an important interconnected control problem closely related to inverse systems. Since the pioneering work [7], the problem of inversion of linear time-invariant interconnected systems has attracted an attention of several researches. See [8–11]. Recently, a significant progress in this research area has been achieved in [2, 3, 12]. Most of these works except [3, 12] dealt with continuous-time interconnected systems.

An inverse model approach to ensuring perfect steady-state regulation in linear discrete-time interconnected systems was first advanced in [13] and independently in [14]. Similar discrete-time counterpart of interconnected process control systems containing the table inverse model was proposed in [15]. The steady-state control of linear interconnected system discussed in [11] in the framework of the problem of minimal inversion, has also been studied in the paper [16] dealing with nonlinear discrete-time interconnected control systems. Unfortunately, the inverse model approach is quite unacceptable if the systems to be controlled are square but singular or if they are nonsquare. Several researches whose works are cited in [17] observed that the inverse model-based controller may be also not admissible for designing some process control systems which contain ill-conditioned plants since they may become (almost) noninvertible in the presence of an uncertainty.

It turned out that the so-called pseudoinverse (generalized inverse) model approach first proposed in the paper [10] can be exploited to cope with the noninevitability of nonsquare system. Recently, this approach was extended in [18–20] for controlling a wide class of discrete-time interconnected systems.

The purpose of the paper is to generalize the results obtained by the authors in their last works which are related to the asymptotic properties of the pseudoinverse model-based method for designing an efficient steady-state control of interconnected systems with uncertainties and arbitrary bounded disturbances and also to present some new results.

THE DESCRIPTION OF CONTROL SYSTEM AND PROBLEM STATEMENT

Basic assumptions. Suppose the plant to be regulated is a nonlinear interconnected time-invariant system whose static characteristic is

$$y = \varphi(u) \tag{1}$$

where $y = [y^{(1)},...,y^{(m)}]^T$ denotes the m-dimensional output vector, $u = [u^{(1)},...,u^{(r)}]^T$ denotes the r-dimensional input (control) vector, and $\varphi(\cdot): \mathbf{R}^r \to \mathbf{R}^m$ represents some nonlinear vector-valued function given by

$$\varphi(u) = [\varphi^{(1)}(u),...,\varphi^{(m)}(u)]^T. \tag{2}$$

Consider a class of systems in which the number of inputs is not more than the number of outputs:

$$r \le m.$$

The following assumption with respect to the nonlinearity $\varphi(u)$ will be required.

Assumption 1. The components $\varphi^{(1)}(u),...,\varphi^{(m)}(u)$ of $\varphi(u)$ in (2) are all the continuously differentiable functions of the variables $u^{(1)},...,u^{(r)}$.

In order to implement the discrete-time control, the signals $y^{(1)}(t),...,y^{(m)}(t)$ given in the continuous time t need to be sampled with a sampling period T_0 to yield the sequences $\{y^{(i)}(nT_0)\}$, whereas the control signals are of zero-order sampled-hold type, i.e.,

$$u^{(i)}(t) = u^{(i)}(nT_0) \text{ for } nT_0 \le t < (n+1)T_0, \; i = 1,\dots, r.$$

Assumption 2. As in [14] and [16], suppose that the sampling period T_0 is large enough so that the transient stage caused by stepwise changes of inputs $u^{(1)}(t),...,u^{(r)}(t)$ at each (n–1)th time instant $t = (n-1)T_0$ may practically be completed during the time interval $[(n-1)T_0, nT_0)$. In view of (1), this narrative description of the discrete-time steady-state control gives that the steady state of this interconnected system can be mathematically modeled by the first-order nonlinear difference equation

$$y_n = \varphi(u_{n-1}) \tag{3}$$

similar to that in [16], if any disturbances are absent. In this equation, the notations $y_n := y(nT_0)$ and $u_n := u(nT_0)$ are introduced (for the simplicity of exposition).

In practical applications, the outputs $y^{(1)}(t),...,y^{(m)}(t)$ are usually influenced by certain classes of persistent external disturbances $d^{(1)}(t),...,d^{(m)}(t)$, respectively. Then, instead of (3), another equation

$$y_n = \varphi(u_{n-1}) + d_{n-1} \tag{4}$$

with the disturbance vector $d_n := [d_n^{(1)},...,d_n^{(m)}]^T$ as a steady-state model of system will be further considered. Now, the following assumption about $\{d_n\}$ is introduced.

Assumption 3. The components of d_n are upper bounded in modulus by an ε_i for all $n = 1, 2, \ldots,$ i.e.,

$$| d_n^{(i)} | \le \varepsilon_i < \infty \quad (i = 1, \ldots, m). \tag{5}$$

Let $y^* := [y^{*(1)}, \ldots, y^{*(m)}]^T$ $(y^{*(i)} \equiv \text{const})$ be some vector defining the desired output vector (a given set-point). The following assumption with respect to this vector is made.

Assumption 4. y^* is not the m-dimensional zero-vector $0_m := [0,...,0]^T$, i.e., $\| y^* \| \ne 0$ implying that

$$| y^{*(1)} | + \ldots + | y^{*(m)} | \ne 0. \tag{6}$$

Regulation strategy using pseudoinverse model-based control approach. Let

$$B_0 = \begin{pmatrix} b_0^{(11)} & \cdots & b_0^{(1r)} \\ & \vdots & \\ b_0^{(m1)} & \cdots & b_0^{(mr)} \end{pmatrix}$$

be a fixed $m \times r$ matrix chosen further by the designer to deal with some linear model of (1). Define the so-called pseudoinverse (generalized inverse) $r \times m$ matrix $B_0^+ = (\beta_0^{(ij)})$ specified as

$$B_0^+ = \lim_{\delta \to 0} (B_0^T B_0 + \delta^2 I_r)^{-1} B_0^T, \tag{7}$$

where I_N denotes the identity $N \times N$ matrix. (Note that the limit (7) exist for any $B_0 \in \mathbf{R}^{m \times r}$ [21].)

According to [19], [20] the control law utilizing the pseudoinverse model-based control strategy to regulate y_n around y^* is given by

$$u_n = u_{n-1} + B_0^+ e_n, \tag{8}$$

where e_n represents the output error vector at nth time instant $t = nT_0$ specified as

$$e_n = y^* - y_n, \tag{9}$$

The equations (8), (9) describe the some linear interconnected controller of the integral action. Namely, to implement the control law (8), one needs the discrete integrator whose output is

$$u_n = \sum_{k=1}^{n} \Delta u_k, \tag{10}$$

where

$$\Delta u_n = B_0^+ e_n. \tag{11}$$

Due to (11) together with (10), this controller plays the role of an I-type interconnected discrete-time controller with a matrix gain B_0^+ (Fig. 1).

Regulation problems. To formulate the goals of the regulation, we before need the following definition.

Definition 1 [22]. The closed-loop control system containing the plant described by (4) and the feedback (8), (9) is said to be BIBS (bounded-input bounded-state) stable if there exist some nonnegative numbers C_u, C_y, C_d such that

$$\limsup_{n\to\infty} \| y_n \| \le C_u \sup_{n\ge 0} \| u_n \| + C_d \sup_{n\ge 0} \| d_n \|, \tag{12}$$

$$\limsup_{n\to\infty} \| u_n \| \le C_y \sup_{n\ge 0} \| y_n \| + C_d \sup_{n\ge 0} \| d_n \| \tag{13}$$

are satisfied.

Now, introduce the performance index

$$J := \limsup_{n\to\infty} \| e_n \| \tag{14}$$

evaluating the asymptotic behavior of the control system (4), (8), (9). Then, one of the following control objectives may be stated [22].

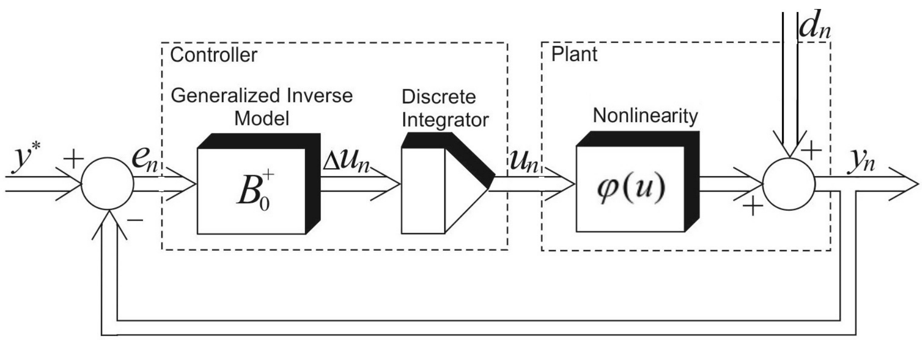

Fig. 1. Configuration of the regulation system (4), (9), (10), (11)

• The optimization: it is required to minimize J defined by (14) in the sense that

$$\limsup_{n\to\infty} \| e_n \| = \inf_{\{u_n\}} \tag{15}$$

must be achieved.

• Quasi-optimization: it is necessary to minimize an upper bound of J given in the inequality

$$\limsup_{n\to\infty} \| e_n \| \le \bar{J}. \tag{16}$$

• Stability (robust stability): the closed-loop system (4), (8), (9) must be stable (in the sense of Definition 1) by suitable choice of B_0.

LINEAR CASE

Regulation without parameter uncertainty. In the linear case, $\varphi(u)$ in (1) is defined as $\varphi(u) = Bu$, where $B = (b^{(ij)})$ represents some numerical $m \times r$ matrix with the elements $b^{(ij)}$ whose rank satisfies $1 \le \text{rank } B \le r.$ In this case, the equation (4) becomes

$$y_n = Bu_{n-1} + d_{n-1}. \tag{17}$$

Let $r = m$ and $\text{rank } B = r.$ Clearly, it implies that B is non-singular. Then the inverse matrix B^{-1} exists and $B^{+} = B^{-1}.$ Assume that there is no parameter uncertainty, i.e., B is known *a priori*. We can derive immediately the inverse-model based control law

$$u_n = u_{n-1} + B^{-1} e_n, \tag{18}$$

followed from (8) after setting $B_0 = B.$

It turns out the control law above guarantees the optimality of the closed-loop system (17), (18), (9) (in the sense of (15)). This fact is established in the theorem below.

Theorem 1 [22]. *Let the plant to be regulated be described by* (17). *Suppose B is the known non-singular square matrix* $(\det B \ne 0)$. *Then, the controller* (18), (9) *when applied to* (17) *achieves the regulation objective* (15). *Furthermore, subject to Assumptions* 4, *it yields*

$$\begin{gathered} \limsup_{n\to\infty} \| u_n \| \le \| B^{-1} \| \| y^* \| + \| B^{-1} \| \sup_{0\le n<\infty} \| d_n \| < \infty, \\ J \le \sup_{0\le n<\infty} \| d_n - d_{n-1} \| < \infty \end{gathered} \tag{19}$$

for any initial $\| u_0 \| < \infty.$

Corollary. Under the conditions of Theorem 1, in the terms of the Euclidean norm $\|\cdot\|_2$, the asymptotic properties of the controller (18), (9) are given by

$$\begin{gathered}\limsup_{n\to\infty}\| u_n \|_2 \le \| B^{-1} \|_2 \| y^* \|_2 + \| B^{-1} \|_2\, \varepsilon,\\ \limsup_{n\to\infty}\| e_n \|_2 \le 2\varepsilon\end{gathered} \tag{20}$$

with $\| y^* \|_2 = [| y^{*(1)} |^2 + \ldots + | y^{*(M)} |^2]^{1/2}$ and $\varepsilon = [\varepsilon_1^2 + \ldots + \varepsilon_m^2]^{1/2}$.

Proof. Immediate from (19) together with (5) and from the definition of y^* taking into account the definitions of the Euclidean vector and matrix norms [23].□

Let B be a known nonsquare matrix $(r < m)$. In this case, instead of (18),

$$u_n = u_{n-1} + B^+ e_n \tag{21}$$

is chosen as the control law. The equation (21) together with (9) describes the pseudoinverse model-based controller.

The following result can be shown to be valid.

Theorem 2 [20]. *The controller* (21), (9) *applied to* (17) *leads to a stable closed-loop system* (*in the sense of Definition* 1). *Moreover, subject to Assumption* 4, *it gives that quasi-optimality property of the form* (16) *is ensured with the minimal* $\bar{J}$ *such that*

$$\begin{gathered}\limsup_{n\to\infty}\| u_n - u^e \|_2 \le \| I_r - B^+ B \|_2 \| u_0 - u^e \|_2 + \| B^+ \|_2\, \varepsilon < \infty,\\ \limsup_{n\to\infty}\| e_n \|_2 \le \| I_m - BB^+ \|_2 (\| y^* \|_2 + \varepsilon) + 2\varepsilon < \infty.\end{gathered} \tag{22}$$

Remark 1. Note that if $r=m$ and $\det B \ne 0$ yielding $B^+ = B^{-1}$, then the inequalities (22) finally leads to (20), respectively.

Regulation in the presence of parameter uncertainty. Consider the steady-state model of the plant given in the form (17) with an arbitrary nonzero matrix $B = (b^{(ij)})$. Assume that $b^{(ij)}$s are unknown but the bounds, $b_{\min}^{(ij)}$, $b_{\max}^{(ij)}$ of the intervals

$$b_{\min}^{(ij)} \le b^{(ij)} \le b_{\max}^{(ij)} \quad (i = 1, \ldots, m;\ j = 1, \ldots, r) \tag{23}$$

to which they belong are known. Additionally, let

$$0 < b_{\min}^{(ij)} b_{\max}^{(ij)} < \infty. \tag{24}$$

Denote by Ξ the set of possible $\hat{B}$s whose elements, $\hat{b}^{(ij)}$ satisfy $\hat{b}^{(ij)} \in [b_{\min}^{(ij)}, b_{\max}^{(ij)}]$. This means that

$$\Xi = \{(b^{(ij)}) : b^{(ij)}_{\min} \le \hat{b}^{(ij)} \le b^{(ij)}_{\max} \quad i = 1, \ldots, m,\ j = 1, \ldots, r\}. \tag{25}$$

Further, choose a matrix B_0 from the set Ξ provided $\det B_0 = 0$ if this set contains at least one singular matrix $\hat{B}$. Thus,

$$b^{(ij)}_{\min} \le b^{(ij)}_0 \le b^{(ij)}_{\max} \quad (i = 1, \ldots, m;\ j = 1, \ldots, r)$$

has to be met.

The sufficient condition guaranteeing the boundedness of $\{y_n\}$ and $\{u_n\}$ is established in the following theorem.

Theorem 3. *Consider the feedback system* (17), (8), (9). *Let the requirements* (23), (24) *hold and the requirements on the choice of* B_0 *above mentioned be met. Assume that the equilibrium state of the feedback system* (17), (8), (9) *defined by the pair* (u^e, y^e) *which is the solution of the equation*

$$B_0^+ B u^e = B_0^+ y^*$$

together with $y^e = Bu^e$ *exists. Introduce the matrix* $\Delta = B_0 - B$. *If the condition*

$$q < 1 \tag{26}$$

with

$$q = \max_{\Delta : (B_0 - \Delta) \in \Xi} \| B_0^+ \Delta \| \tag{27}$$

is satisfied, then the closed-loop control system containing the plant (17) *and the pseudoinverse model-based controller*

$$u_n = u_{n-1} + B_0^+ (y^* - y_n) \tag{28}$$

will be the robust BIBS stable. Moreover, subject to Assumption 3, *this controller makes it possible to achieve*

$$\begin{aligned} &\limsup_{n\to\infty} \| u_n - u^e \|_2 \le (1-q)^{-1} \| I_r - B_0^+ B_0 \|_2 \| u_0 - u^e \|_2 + \varepsilon (1-q)^{-1} \| B_0^+ \|_2 < \infty, \\ &\limsup_{n\to\infty} \| e_n \|_2 \le \| I_m - B_0 B_0^+ \|_2 [\| e_0 \|_2 + 2\varepsilon] + 2\varepsilon (1-q)^{-1} < \infty. \end{aligned} \tag{29}$$

Proof. Due to space limitation, details are omitted. □

By virtue of (12), (13), the condition (26) together with the expression (27) guarantee the boundedness of $\{y_n\}$ and $\{u_n\}$ as $n \to \infty$ (according to (29)). Note that this condition can simply be verified by setting $q = \max_{\Delta : (B_0 - \Delta) \in \Xi} \| B_0^+ \Delta \|_1$ and by using the linear programming technique ($\| P \|_1$ denotes here the 1-norm of arbitrary matrix P; the definition of $\| P \|_1$ can be found in [23]).

A numerical example and simulation. To illustrate the robust stability properties derived from Theorem 3, a numerical example was considered setting $r = m = 2$ and $\Xi = \{b^{(ij)} : 0.4 \le b^{(11)} \le 1.4,\ -1.2 \le b^{(12)} \le -0.5,\ 0.8 \le b^{(21)} \le 2.8,$ $-2.7 \le b^{(22)} \le -0.7\}$ Such set was chosen to ensure the singularity of some $\hat{B}$s belonging to Ξ. In this example,

$$B_0 = \begin{pmatrix} 0.9 & -0.85 \\ 1.8 & -1.70 \end{pmatrix}$$

was put. Such a choice of B_0 gives $B_0 \in \Xi$ and $\det B_0 = 0$. Using the formula (7),

$$B_0^+ = \begin{pmatrix} 72/613 & 144/613 \\ -68/613 & -136/613 \end{pmatrix}$$

was found. By exploiting the linear programming technique, it was established that $q = \max_{\Delta:(B_0-\Delta)\in\Xi} \| B_0^+ \Delta \|_1 \approx 0{,}572 < 1$. Thus, requirement (26) together with (27) will be satisfied.

Next, taking $b^{(11)} = 0.878$, $b^{(12)} = -0.864$, $b^{(21)} = 1.082$, $b^{(22)} = -1.096$ under which $B = (b^{(ij)})$ will satisfy $B \in \Xi$, a simulation experiment with the closed-loop control system described by (17), (8), (9) was conducted. In this experiment, $d_n^{(1)}, d_n^{(2)}$ were simulated as the pseudo-random variables within $[-0.07, 0.07]$. The components of y^* were chosen as follows:

$$y^{*(1)} = \begin{cases} 0.4 & \text{if } 0 \le n \le 50, \\ 0.2 & \text{if } 50 < n \le 100 \end{cases} \quad \text{and} \quad y^{*(2)} = \begin{cases} 0.6 & \text{if } 0 \le n \le 50, \\ 0.8 & \text{if } 50 < n \le 100. \end{cases}$$

Results of the simulation experiment are depicted in Figs. 2 and 3.

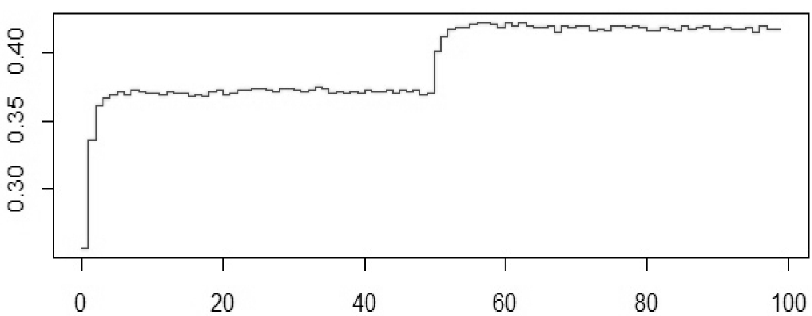

Fig. 2. The norm of control input vector

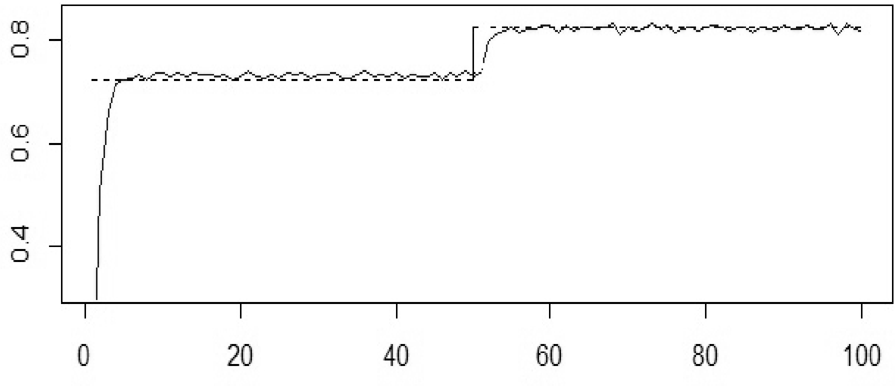

Fig. 3. The norms of output vector (solid line) and of set-point vector (dashed line)

We observe that system behavior is successful while B and B_0 are different.

REGULATION OF UNKNOWN NONLINEAR SYSTEM

Case 1. Now, consider the nonlinear interconnected system described by (4). Recalling Assumption 1 and denoting $b^{(ij)}(u) := \partial\varphi^{(i)}(u) / \partial u^{(j)}$, introduce the matrix

$$B(u) = \begin{pmatrix} b^{(11)}(u) & \ldots & b^{(1r)}(u) \\ & \vdots & \\ b^{(m1)}(u) & \ldots & b^{(mr)}(u) \end{pmatrix} \tag{30}$$

which represents the $m \times r$ Jacobian matrix whose elements $b^{(ij)}(u)$ play a role of some "dynamical" gains from the jth input, $u^{(j)}$ to the ith output, $y^{(i)}$ for each fixed $u \in \mathbf{R}^r$. Next, the following two additional assumptions regarding the nonlinearity $\varphi(u)$ will be required.

Assumption 5. $b^{(ij)}(u)$s in (30) do not change its sign and remain uniformly bounded for all u from $\mathbf{R}^r$ according to (24) and to

$$b_{\min}^{(ij)} \le b^{(ij)}(u) \le b_{\max}^{(ij)}, \quad (i = 1, \ldots, m; \; j = 1, \ldots, r). \tag{31}$$

Assumption 6. In case 1 to be studied, Ξ represents the set of matrices having the full rank: rank $\bar{B} = r$.

Under these assumptions we first choose a $B_0 \in \Xi$ and design again the pseudoinverse model-based controller of the form (28). The asymptotic properties of this controller are formulated in the theorem below.

Theorem 4 [19]. *Consider the feedback control system described by* (4), (28). *Let the equilibrium state defined by*

$$B_0^+\varphi(u^e) = B_0^+ y^*, \quad y^e = \varphi(u^e)$$

exist. Suppose that Assumptions 1 *and* 3 *to* 6 *are valid. Then this system will be robust BIBS stable for any nonlinearity* $\varphi(u)$ *satisfying* (31) *together with* (24) *if the requirement* (26) *in which*

$$q = \max_{1 \le k \le r} \max_{\delta^{(ij)} \in [\underline{\delta}^{(ij)}, \overline{\delta}^{(ij)}]} \sum_{i=1}^{r} \left| \sum_{j=1}^{m} \beta_0^{(kj)} \delta^{(ji)} \right|, \tag{32}$$

where $\underline{\delta}^{(ij)} = b_{\min}^{(ij)} - b_0^{(ij)}$, $\overline{\delta}^{(ij)} = b_{\max}^{(ij)} - b_0^{(ij)}$ *is met. Furthermore,*

$$\limsup_{n \to \infty} \| u_n \|_\infty \le (1-q)^{-1} \| B_0^+ \|_1 \max\{\varepsilon_1, \ldots, \varepsilon_m\}$$

will take place, where $\| x \|_\infty$ *denotes the* ∞*-norm of a vector x.*

As in the linear case before studied, the condition (26) but with q given by (32) can be verified via the linear programming tool.

Case 2. In this case, instead of Assumptions 1, 5 and 6, another assumption with respect to $\varphi(u)$ is introduced.

Assumption 7. The nonlinearity $\varphi(u)$ can be represented as the sum

$$\varphi(u) = Bu + g(u), \tag{33}$$

in which $B = (b^{(ij)})$ is a numerical $m \times r$ matrix and $g(u)$ is a nonlinear vector-valued function satisfying

$$\sup_{u \in \mathbf{R}^r} \| g(u) \| \le C < \infty \tag{34}$$

with some *C*.

Due to the expression (33) given in Assumption 7, the system equation becomes

$$y_n = Bu_{n-1} + g(u_{n-1}) + d_{n-1}. \tag{35}$$

As in the linear case with unknown *B*, it is assumed that $B \in \Xi$, where Ξ is given by (25). Similarity to this case, we choose $B_0 \in \Xi$ so that $\det B_0 = 0$ if *r=m* and there is at least a singular matrix $\hat{B} \in \Xi$. Next, the pseudoinverse model-based controller of the form (28) is designed to regulate the plant (35).

The following theorem establishes stability results of the closed-loop system (35), (28).

Theorem 5. *Under the conditions of Theorem* 3 *added by Assumption* 7, *the closed-loop system containing the controller* (28) *and the plant* (35) *will be robust BIBS stable.*

Proof. Proceeds along the lines of the proof of Theorem 3 after replacing $\sup_{0 \le n < \infty} \| d_n \| < \infty$ by $\sup_{0 \le n < \infty} \| d_n \| + C < \infty$. □

Remark 2. In contrast with [19], it is not required that $\varphi^{(1)}(u), \ldots, \varphi^{(m)}(u)$ in (2) to be smooth functions of *u*.

Remark 3. Note that $g(u)$ may not be the Lipchitz function, i.e.,

$$\| g(u') - g(u'') \| \le L \| u' - u'' \| \quad \forall u', u'' \in \mathbf{R}^r \quad (0 < L < \infty)$$

is not necessary. However, due to (34) it has to be bounded as $\| u \| \to \infty$.

Comment. Contrary to the case 1, the set Ξ may contain singular $\hat{B}$s and it is essential.

CONCLUSION

In this paper, the main effort has been focused on analyzing the asymptotic properties of the closed-loop systems containing the pseudoinverse model-based controllers. We have established that the pseudoinverse model-based concept can be used as a unified concept to deal with the steady-state regulation of the linear interconnected discrete-time systems and of some classes of nonlinear interconnected systems with possible uncertainties in the presence of arbitrary unmeasured but bounded disturbances. In the framework of this concept, new theoretical results related to the asymptotic behavior of these systems have been presented.

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ДИСКРЕТНЕ КЕРУВАННЯ УСТАЛЕНИМИ СТАНАМИ БАГАТОЗВ'ЯЗНИХ СИСТЕМ НА ОСНОВІ КОНЦЕПЦІЇ ПСЕВДООБЕРНЕННЯ

Розглянуто концепцію псевдообернення як деяку уніфіковану концепцію керування усталеними станами багатозв'язних систем за наявності невимірюваних обмежених збурень з повною і неповною інформацією про параметри лінійної номінальної моделі, по якій будується зворотний зв'язок. Припускається, що ранг матриці коефіцієнтів підсилення цієї моделі може бути довільним. Встановлено достатні умови граничної обмеженості всіх сигналів у замкнених системах керування, що реалізують запропоновану концепцію. Наведено результати моделювання.

Ключові слова: дискретний час, зворотний зв'язок, псевдообернення, багатозв'язні системи, оптимальність, стійкість, невизначеність.

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ДИСКРЕТНОЕ УПРАВЛЕНИЕ УСТАНОВИВШИМИСЯ СОСТОЯНИЯМИ МНОГОСВЯЗНЫХ СИСТЕМ НА ОСНОВЕ КОНЦЕПЦИИ ПСЕВДООБРАЩЕНИЯ

Рассмотрена концепция псевдообращения как некоторая унифицированная концепция управления установившимися состояниями многосвязных систем при наличии неизмеряемых ограниченных возмущений с полной и неполной информацией о параметрах линейной номинальной модели, по которой строится обратная связь. Предполагается, что ранг матрицы коэффициентов усиления этой модели может быть произвольным. Установлены достаточные условия предельной ограниченности всех сигналов в замкнутых системах управления, реализующих предлагаемую концепцию. Приведены результаты моделирования.

Ключевые слова: дискретное время, обратная связь, псевдообращение, многосвязные системы, оптимальность, устойчивость, неопределенность.