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AVERAGING OF THE DIRICHLET PROBLEM FOR THE SPECIAL HYPERBOLIC QUASILINEAR EQUATION

In this paper we establish sufficient conditions on the data of the problem which guarantee a convergence of its solution to a limit solution. The domains where we consider the problem has a fine-grained structure. We use S.I.Pohozhaev's method for the proof of the unique solvability in entire and the D.Cioranescu-F.Murat hypothesis for the description of the domain milling.

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The hyperbolic Kirchhoff equation is named the following one

$$u_{tt}(x,t) - a\left(\|\nabla u(\cdot,t)\|^2_{L^2(\Omega)}\right)\triangle u(x,t) = f(x,t),$$

$$x \in \Omega \subset \mathbb{R}^n,\ t \in \mathbb{R}, \tag{1}$$

where $x = (x_1, \ldots, x_n)$, $\nabla = (\partial/\partial x_1, \ldots, \partial/\partial x_n)$, $\triangle = \sum\limits_{i=1}^{n} \partial^2/\partial x_i^2$, with positive continuous function $a : \bar{\mathbb{R}}^+ \to \mathbb{R}^+$. The initial boundary value Dirichlet problem was considered to this equation [1],[2] in a cylinder $Q = \Omega \times (0,T)$ where Ω is a bounded domain in $\mathbb{R}^n$, $n \geqslant 3$, with boundary $\partial\Omega$, T is an arbitrary fixed positive number, with boundary conditions

$$u(\cdot,t)\,|_{\partial\Omega} = 0\ ,\ t \in (0,T),$$

$$u(x,0) = \varphi(x),\ u_t(x,0) = \psi(x),\ x \in \Omega. \tag{2}$$

For an arbitrary function $a(t)$ such that

$$a(t) \in C^1(\bar{\mathbb{R}}^+),\ a(t) \geqslant \alpha_0 > 0\ \forall t \in \bar{\mathbb{R}}^+,\ \alpha_0 = const, \tag{3}$$

the solvability of the problem in entire was ascertained first in paper [1] for the special class of infinitely differentiable functions f, φ, ψ and $\partial\Omega$ of class C^∞.

For the concrete function

$$a(t) = a_0(t) := (C_1 t + C_2)^{-2},\ C_i = const > 0,\ i = 1, 2, \qquad (4)$$

the solvability of the problem in entire was ascertained [2] for the class of data having only second order derivatives summable in second power and a boundary $\partial\Omega$ of class C^2. It was showing [3],that function (4) is the unique one in the class

$$a \in C^2(\bar{\mathbb{R}}^+), \quad a \geqslant 0, \qquad (5)$$

for which the problem (1), (2) is solvable in entire in the set of data having only second order derivatives summable in square.

We consider a bounded domain $\Omega \subset \mathbb{R}^n,\ n \geqslant 3$, and domain $\Omega^{(s)} \subset \Omega$ which has a multiply connected boundary

$$\partial\Omega^{(s)} = \left(\bigcup_{i=1}^{N^{(s)}} \partial\Omega_i^{(s)} \right) \cup \partial\Omega,$$

where $s \in \mathbb{N}$ is a parameter, $N^{(s)}$ is a varying number of sets $\Omega_i^{(s)} \subset \Omega$ being closures of domains $\dot{\Omega}_i^{(s)}$ with smooth singly connected boundaries $\partial\Omega_i^{(s)}$ of dimensions $n-1,\ \Omega_i^{(s)} \cap \Omega_j^{(s)} = \phi (i \neq j),\ \Omega^{(s)} \cap \partial\Omega = \phi (i = 1, \ldots, N^{(s)})$,

$$F^{(s)} = \bigcup_{i=1}^{N^{(s)}} \Omega_i^{(s)}, \quad \Omega^{(s)} = \Omega \setminus F^{(s)}.$$

By notation $Q^{(s)} = \Omega^{(s)} \times (0, T), T = const > 0,\ \|v^{(s)}\| = \|v^{(s)}\|_{L^2(\Omega^{(s)})}$, we study the following boundary value problem on $\overline{Q^{(s)}}$ relatively to a real function $u^{(s)}(x, t)$:

$$u_{tt}^{(s)}(x, t) - a_0 \left(\|\nabla u^{(s)}(\cdot, t)\|^2 \right) \triangle u^{(s)}(x, t) = f^{(s)}(x, t),$$

$$(x, t) \in Q^{(s)}, \qquad (6)$$

$$u^{(s)}(\cdot, t)|_{\partial\Omega^{(s)}} = 0,\ t \in (0, T),\ u^{(s)}(x, 0) = \varphi^{(s)}(x),$$

$$u_t^{(s)}(x, 0) = \psi^{(s)}(x),\ x \in \Omega^{(s)}.$$

We show conditions on $\Omega^{(s)}, f^{(s)}, \varphi^{(s)}, \psi^{(s)}$ supplying a convergence of $u^{(s)}$ as $s \to \infty$ to the limit function $u(x, t)$ defined on $\bar{Q}$ when a perforation becomes very small.

We suppose that a boundary $\partial\Omega^{(s)}$ is Lip–continuous and following conditions are fulfilled

$$\varphi^{(s)} \in D\left(\triangle, L^2(\Omega^{(s)})\right) \cap \overset{\circ}{H}{}^1(\Omega^{(s)}),\ \psi^{(s)} \in \overset{\circ}{H}{}^1(\Omega^{(s)}),$$

$$f^{(s)} \in L^2\left(0,T; D(\triangle, L^2(\Omega^{(s)})\right) \cap \overset{\circ}{H}{}^1(\Omega^{(s)}). \tag{7}$$

Moreover, let the norms

$$\|f^{(s)}\|_{L^2(Q^{(s)})} + \|\triangle f^{(s)}\|_{L^2(Q^{(s)})},$$

$$\|\nabla\varphi^{(s)}\| + \|\triangle\varphi^{(s)}\|,\ \|\nabla\psi^{(s)}\| < K_0 \quad \forall s \tag{8}$$

to be bounded relatively to s. Then problem (6), as in [2], has the unique solution such that

$$u^{(s)} \in C\left([0,T]; \overset{\circ}{H}{}^1(\Omega^{(s)})\right),\ u_t^{(s)} \in L^\infty\left(0,T; \overset{\circ}{H}{}^1(\Omega^{(s)})\right),$$

$$\triangle u^{(s)} \in L^\infty\left(0,T; L^2(\Omega^{(s)})\right),\ u_{tt}^{(s)} \in L^2(Q^{(s)}), \tag{9}$$

moreover, following estimates are valid:

$$\max_{t\in[0,T]} \|u_t^{(s)}(t)\|^2 \leqslant K_1, \quad \max_{t\in[0,T]} \|\nabla u^{(s)}(t)\|^2 \leqslant K_2,$$

$$\underset{t\in(0,T)}{esssup}\, \|\nabla u_t^{(s)}(t)\|^2 \leqslant C_2^{-1}K_2, \tag{10}$$

$$\underset{t\in(0,T)}{esssup}\, \|\triangle u^{(s)}(t)\|^2 \leqslant K_2(C_1K_2 + C_2),$$

where constants K_1, K_2 do not depend on $s \in \mathbb{N}$ but depend only on C_1, C_2, T, K_0.

We describe the regularity of behaviour of domains $\Omega^{(s)}$ when $s \to \infty$ by the hypothesis of D.Cioranesku–F.Murat [4], [5]:

Hypothesis (A): let exists a sequence of real functions $w_s(x),\ s \in \mathbb{N}$, with following properties:

1) $w_s(x) \in H^1(\Omega) \cap L^\infty(\Omega)$,

2) $w_s(x) = 0,\ x \in F^{(s)}$,

3) $w_s(x) \to 1$ weakly in $H^1(\Omega)$, weakly* in $L^\infty(\Omega)$ and for almost every $x \in \Omega$ when $s \to \infty$,

4) $-\triangle w_s(x) = \mu_s(x) - \gamma_s(x)$ where $\mu_s, \gamma_s \in H^{-1}(\Omega)$, moreover $\mu_s \to \mu$ strongly in $H^{-1}(\Omega)$ when $s \to \infty$, $(\gamma_s, v)_\Omega = 0$ for all $v \in \overset{\circ}{H}{}^1(\Omega)$ such that $v = 0$ on $F^{(s)}$,

herein

$$(u, v)_\Omega = \int\limits_\Omega u(x)v(x)dx.$$

Corollary [5]: under conditions 1)–4), we have

5) $0 \leqslant \mu(x) \in H^{-1}(\Omega) \cap L^1(\Omega)$that is $\mu(x)$ to be a finite positive measure on Ω.

Remark 1. It is easy to prove [6] that conditions B_1, B_2, C in monograph [7, ch.9] and in the paper [8] with the measure density $\nu(x) \in L^r(\Omega),\ r > n/2$, formulated for the Laplace operator on $H^1(\Omega)$ are sufficient for validity of hypothesis (A) with a function $\mu \in C(\bar{\Omega})$ or $\mu \in L^r(\Omega),\ r > n/2$, accordingly and $w_s(x) \geqslant 0$.

We denote $\hat{v}^{(s)}(x)$ the prolongation on Ω of a function $v^{(s)}(x)$ defined on $\Omega^{(s)}$ by definition it by zero on $F^{(s)}$. We adopt some more notations for duality relations of vector functions
$\vec{u}(x,t) = (u_1(x,t), \ldots, u_n(x,t))$:

$$\langle \vec{u}, \vec{v} \rangle^{(s)} = \int\limits_{Q^{(s)}} \vec{u}(x,t) \cdot \vec{v}(x,t)dxdt, \langle \vec{u}, \vec{v} \rangle = \int\limits_{Q} \vec{u}(x,t) \cdot \vec{v}(x,t)dxdt,$$

and the same notations for scalar ones and denote

$$N_s(t) = \|\nabla u^{(s)}(\cdot, t)\|.$$

We multiply equation (6) by a function $w_s v, v \in C_0^\infty(Q)$, and integrate over $Q^{(s)}$. Then we obtain the integral identity for prolongations

$$\langle \hat{f}^{(s)}, w_s v \rangle = \langle \hat{u}_{tt}^{(s)}, w_s v \rangle - \langle a_0(N_s^2)\hat{u}^{(s)}, w_s \triangle v \rangle -$$
$$2\langle a_0(N_s^2)\hat{u}^{(s)} \nabla v, \nabla w_s \rangle + \langle a_0(N_s^2)\hat{u}^{(s)} v, \mu_s \rangle. \qquad (11)$$

We suppose additionally to (8) in order to average (11) that following convergences as $s \to \infty$ take place

$$\hat{f}^{(s)} \longrightarrow f \text{ weakly in } L^2(Q),$$

$$\hat{\varphi}^{(s)} \longrightarrow \varphi,\ \hat{\psi}^{(s)} \longrightarrow \psi \text{ both weakly in } \overset{\circ}{H}{}^1(\Omega). \qquad (12)$$

We take into account as (8), (10) that for $\hat{u}^{(s)}$ such estimates are valid:

$$\max_{t\in[0,T]} \|\hat{u}_t^{(s)}(t)\|^2 \leqslant K_1, \quad \max_{t\in[0,T]} \|\nabla\hat{u}^{(s)}(t)\|^2 \leqslant K_2, \tag{10'}$$

$$\operatorname*{esssup}_{t\in(0,T)} \|\nabla\hat{u}_t^{(s)}(t)\|^2 \leqslant C_2^{-1}K_2,$$

$$\|\hat{u}_{tt}^{(s)}\|_{L^2(Q)} \leqslant C_2^{-2}[TK_2(C_1K_2+C_2)]^{1/2} + K_0.$$

Moreover, some more estimate follows from (8), (10)

$$\max_{t\in[0,T]} \|\hat{u}^{(s)}(t)\|^2 \leqslant K_3, \tag{13}$$

where K_3 depends only on C_1, C_2, T, K_0. Therefore we can choose from $\mathbb{N}$ such sequence denoted $\{s\}$ that following convergences take place:

$$\begin{gathered} \hat{u}^{(s)} \longrightarrow u \text{ weakly in } H^1(Q) \text{ and strongly in } L^2(Q), \\ \nabla\hat{u}_t^{(s)} \longrightarrow \nabla u_t \text{ weakly* in } L^\infty(0,T;L^2(\Omega)), \\ \hat{u}_{tt}^{(s)} \longrightarrow u_{tt} \text{ weakly in } L^2(Q). \end{gathered} \tag{14}$$

We have here some more convergence

$$\hat{u}^{(s)} \longrightarrow u \text{ weakly* in } W^1_\infty(0,T;\overset{\circ}{H}{}^1(\Omega)). \tag{15}$$

It follows from (15), (14) the convergence [5]

$$\hat{u}^{(s)} \longrightarrow u \text{ in } C^1_{sc}([0,T];\overset{\circ}{H}{}^1(\Omega)), \tag{16}$$

where we denote $C_{sc}([0,T];V)$ the space of scalar continuous functions from $[0,T]$ into Banach space V [9, ch.3, 8.4].
Hence a limit function $u(x,t)$ satisfies initial conditions

$$u\,|_{t=0} = \varphi, \quad u_t|_{t=0} = \psi. \tag{17}$$

Let to ascertain (this is only a hypothesis for the present which has to be proved) that the presence of (14) imply convergence

$$N_s(t) \longrightarrow N(t) \text{ strongly in } C([0,T]). \tag{18}$$

Then using (14), (18) and properties 3), 4) of functions w_s we pass to the limit in integral identity (11) over the chosen sequence $\{s\}$. As the result we obtain identity

$$\langle f,v\rangle = \langle u_{tt},v\rangle - \langle a_0(N^2)u,\triangle v\rangle + \langle a_0(N^2)uv,\mu\rangle,$$

or the same in differential form taking into account (17):

$$u_{tt}(x,t) - a_0(N^2(t))\triangle u(x,t) + a_0(N^2(t))\mu(x)u(x,t) = f(x,t),$$
$$(x,t) \in Q,$$
$$u(\cdot,t)|_{\partial\Omega} = 0,\ t \in (0,T),\ u(x,0) = \varphi(x),\ u_t(x,0) = \psi(x), \qquad (19)$$
$$x \in \Omega.$$

It is clear from equation (19) that the following definition is probable

$$N^2(t) = \|u(\cdot,t)\|_V^2 := \|\nabla u(\cdot,t)\|^2 + \|u(\cdot,t)\|^2_{L^2(\Omega;\mu dx)},$$
$$V = \overset{\circ}{H}{}^1(\Omega) \cap L^2(\Omega;\mu dx). \qquad (20)$$

Remark 2. In the case of conditions of remark 1 we have either $\mu \in C(\bar{\Omega})$ or $\mu \in L^\infty(\Omega)$ if $\nu(x) \in L^\infty(\Omega)$. Then
$L^2(\Omega) \subset L^2(\Omega;\mu dx),\ V = \overset{\circ}{H}{}^1(\Omega)$.

In this paper, we prove that assumptions (18), (20) are right and problem (19) really is the limit one for problem (6) in the sense that for solutions (6) convergences (14), (15) are valid to a limit function $u(x,t)$ being a solution of problem (19), (20). More precisely, the following assertion is valid.

Theorem 1. *Let us suppose that the boundary $\partial\Omega^{(s)}$ is Lip-continuous and hypothesis (A) with a limit function $\mu(x) \in L^2(\Omega)$ and conditions (7), (8) are fulfiled. Let convergences (12) and following*

$$\|\hat{f}^{(s)} - f\|_{L^1(0,T;L^2(\Omega))} \longrightarrow 0, \quad \|\nabla(\hat{\varphi}^{(s)} - w_s\varphi)\| \longrightarrow 0 \qquad (21)$$

have place as $s \to \infty$. Let also suppose to exist a solution of problem (19), (20) having following properties

$$u \in C\left([0,T]; \overset{\circ}{W}{}^1_n(\Omega) \cap L^\infty(\Omega)\right),$$
$$\nabla u \in L^1(0,T;L^\infty(\Omega)),\ \triangle u \in L^\infty(0,T;L^2(\Omega)),$$
$$\partial^2 u = \left(\partial^2 u/\partial x_i \partial x_j : i,j = \overline{1,n}\right) \in L^1\left(0,T;L^n(\Omega)\right), \qquad (22)$$
$$u_t \in L^\infty(0,T;\overset{\circ}{H}{}^1(\Omega) \cap L^\infty(\Omega)),\ \nabla u_t \in L^1(0,T;L^n(\Omega)).$$

Then for the complete sequence $s \in \mathbb{N}$ convergences (14)–(16), (18), (20) and additional following ones

$$\nabla \hat{u}^{(s)} \longrightarrow \nabla u \ \ in\ C_{sc}([0,T];L^2(\Omega)),$$

$$\hat{u}_t^{(s)} \longrightarrow u_t \text{ strongly in } C([0,T];L^2(\Omega)) \tag{23}$$

have place to the solution of problem (19), (20) which is unique in class (22).

Remark 3. In order to fulfil convergence condition (21) to $\hat{\varphi}^{(s)}$ it is sufficient [5] by hypothesis (A) function $\varphi^{(s)}$ to be the solution of problem

$$-\triangle\varphi^{(s)}(x) = g^{(s)}(x),\ x \in \Omega^{(s)},\ \varphi^{(s)} \in \overset{\circ}{H}{}^1(\Omega^{(s)}),\ g^{(s)} \in H^{-1}(\Omega^{(s)}),$$

moreover, to exist a limit

$$\lim_{s\to\infty} \|\hat{g}^{(s)} - g\|_{H^{-1}(\Omega)} = 0$$

but the limit in the sense of (12) function φ being then the unique solution of problem

$$-\triangle\varphi(x) + \mu(x)\varphi(x) = g(x),\ x \in \Omega,\ \varphi \in V,$$

to belong still to $C(\bar{\Omega})$.

Proof of this theorem has been published [10].

Thus, validity of the assertion of theorem 1 depends on existence of a solution to problem (19), (20) having properties (22). We consider this problem separately denoting

$$Ju(t) = -C_1\left[(\nabla u(\cdot,t), \nabla u_t(\cdot,t))_\Omega + (u(\cdot,t), u_t(\cdot,t))_{\Omega,\mu}\right]^2 +$$

$$+(C_1N^2(t) + C_2)\left(\|\nabla u_t(\cdot,t)\|^2 + \|u_t(\cdot,t)\|_\mu^2\right) + \tag{24}$$

$$(C_1N^2(t) + C_2)^{-1}\|\triangle u(\cdot,t) - \mu(\cdot)u(\cdot,t)\|^2,$$

$$(u,v)_{\Omega,\mu} = \int\limits_\Omega u(x)v(x)\mu(x)dx, \quad \|u\|_\mu^2 = (u,u)_{\Omega,\mu},$$

$$Mu(x,t) = u_{tt}(x,t) - a_0^2(N^2(t))(\triangle u(x,t) - \mu(x)u(x,t)).$$

Lemma 1. *Let a function $u(x,t)$ be such that*

$$u \in C^1([0,T];V),\ u_{tt} \in C([0,T];L^2(\Omega)),$$

$$\triangle u - \mu u \in C^1([0,T]; L^2(\Omega)), \tag{25}$$

then we have $Ju \in C^1([0,T])$ *and*

$$\frac{dJu(t)}{dt} = C_1 \frac{dN^2(t)}{dt} (Mu(\cdot,t), \triangle u(\cdot,t) - \mu(\cdot)u(\cdot,t))_\Omega -$$

$$-2(C_1N^2(t) + C_2)(Mu(\cdot,t), \triangle u_t(\cdot,t) - \mu(\cdot)u_t(\cdot,t))_\Omega, \tag{26}$$

moreover,

$$Ju(t) \geqslant C_2\|u_t(\cdot,t)\|_V^2 + (C_1N^2(t) + C_2)^{-1}\|\triangle u(\cdot,t) - \mu(\cdot,t)\|^2,$$

$$t \in [0,T]. \tag{27}$$

Lemma 2. *Let* $u(x,t)$ *be a solution satisfying inclusions (25) of problem (19), (20) with right hand sides such that*

$$f \in L^2(0,T;V),\ \triangle f - \mu f \in L^2(Q), \tag{28}$$

$$\psi \in V,\ \varphi \in V,\ \triangle\varphi - \mu\varphi \in L^2(\Omega).$$

Then we have the estimate

$$\max_{t\in[0,T]} Ju(t) \leqslant K_4 \tag{29}$$

and hence following estimates

$$\max_{t\in[0,T]} N^2(t) \leqslant K_5,\ \max_{t\in[0,T]} \|u_t(\cdot,t)\|_V \leqslant K_6,$$

$$\max_{t\in[0,T]} \|\triangle u(\cdot,t) - \mu(\cdot)u(\cdot,t)\| \leqslant K_7, \tag{30}$$

where $K_i, i = \overline{4,7}$ *are known constants depending on (28).*

The proofs of these lemmas in their basic features are similar to Pohozhaev's ones [2].

Next, we consider the spectral problem

$$-\triangle\varphi_k(x) + \mu(x)\varphi_k(x) = \lambda_k\varphi_k(x),\ x \in \Omega,\ \varphi_k \in V,\ \|\varphi_k\| = 1,$$

which is solvable because the energy space $V \subset L^2(\Omega)$ compactly, and consider the finite dimensional problem of the type (19), (20)

$$Mu^n = f^n \text{ in } Q, u^n(\cdot,t) \in V^n =< \varphi_1, \ldots, \varphi_n >\subset V,$$

$$u^n(x,0) = \varphi^n(x),\ u_t^n(x,0) = \psi^n(x),\ x \in \Omega,\ n \in \mathbb{N}, \tag{31}$$

where $f^n(\cdot,t), \varphi^n, \psi^n \in V^n$ and are such that following convergences have place as $n \to \infty$:

$$\|f^n - f\|_{L^2(Q)} \longrightarrow 0,\ \|\varphi^n - \varphi\|_V \longrightarrow 0,\ \|\psi^n - \psi\| \longrightarrow 0, \tag{32}$$

and by conditions (28), such estimates are valid uniformly on n:

$$\|\triangle f^n - \mu f^n\|_{L^2(Q)} \leqslant K_8,\ \|\psi^n\|_V \leqslant K_9,\ \|\triangle\varphi^n - \mu\varphi^n\| \leqslant K_{10}\ \forall n. \tag{33}$$

Problem (31) has a solution $u^n \in H^2(0,T;V^n)$.

Then all conditions of lemma 2 are fulfiled for problem (31) and hence the estimates (30) are right for $u^n(x,t)$ uniformly on $n \in \mathbb{N}$. So we prove

Theorem 2. *If $\partial\Omega$ is Lip–continuous, $\mu \in L^2(\Omega)$ and conditions (28) are fulfiled, then problem (19), (20) has a solution $u(x,t)$ with such properties*

$$u \in C([0,T];V),\ u_t \in C([0,T];L^2(\Omega)) \cap L^\infty(0,T;V),$$

$$\triangle u - \mu u \in L^\infty(0,T;L^2(\Omega)),\ u_{tt} \in L^2(Q), \tag{34}$$

moreover, such convergence has place as $n \in \mathbb{N}, n \to \infty$

$$\max_{t\in[0,T]} \|u^n(\cdot,t) - u(\cdot,t)\|_V + \max_{t\in[0,T]} \|u_t^n(\cdot,t) - u_t(\cdot,t)\| \longrightarrow 0.$$

Now we are able to formulate some conditions to raise smoothness of a solution to problem (19), (20).

Theorem 3. *Let $k = 1,2,\dots, n = dim\Omega$ and*

1) $n = 4k-2, 4k-1$ and such additional conditions are fulfiled:

$$\partial\Omega \in C^{2m+1},\ \mu \in W_\infty^{2m-1}(\Omega),\ m = 1,2,\dots,$$

$$\varphi \in H^{2m+1}(\Omega),\ \psi \in H^{2m}(\Omega),\ f \in L^2(0,T;H^{2m}(\Omega)),$$

and boundary ones for them:

$$(-\triangle+\mu I)^\ell\varphi \in \overset{\circ}{H}{}^1(\Omega)(\ell = 0,1,\dots,m),\ (-\triangle+\mu I)^r\psi \in \overset{\circ}{H}{}^1(\Omega),$$

$$(-\triangle+\mu I)^r f \in L^2(0,T;\overset{\circ}{H}{}^1(\Omega))(r = 0,1,\dots,m-1),$$

then problem (19),(20) has a solution $u(x,t)$ such that following inclusions take place:

$$u \in L^{\infty}(0,T;H^{2m+1}(\Omega)),\ u_t \in L^{\infty}(0,T;H^{2m}(\Omega)),$$

$$u_{tt} \in L^{2}(0,T;H^{2m-1}(\Omega)),$$

which are sufficient for all inclusions (22) by

$$m > \frac{n}{4};$$

2) $n = 4k, 4k+1$ and following conditions are fulfiled:

$$\partial\Omega \in C^{2(m+1)},\ \mu \in W^{2m}_{\infty}(\Omega),\ m = 0,1,2,\ldots,$$

$$\varphi \in H^{2(m+1)}(\Omega),\ \psi \in H^{2m+1}(\Omega),\ f \in L^{2}(0,T;H^{2m+1}(\Omega)),$$

$$(-\triangle + \mu I)^{\ell}\varphi \in \overset{\circ}{H}{}^{1}(\Omega),$$

$$(-\triangle + \mu I)^{\ell}\psi \in \overset{\circ}{H}{}^{1}(\Omega), (-\triangle + \mu I)^{\ell} f \in L^{2}(0,T;\overset{\circ}{H}{}^{1}(\Omega)),$$

$$\ell = 0,1,\ldots,m,$$

then a solution $u(x,t)$ exists to problem (19), (20) which

$$u \in L^{\infty}(0,T;H^{2(m+1)}(\Omega)), u_t \in L^{\infty}(0,T;H^{2m+1}(\Omega)),$$

$$u_{tt} \in L^{2}(0,T;H^{2m}(\Omega)),$$

and these inclusions are sufficient for all ones of (22) by

$$2m+1 > \frac{n}{2}.$$

1. 1*Похожаев С.И.* Об одном классе квазилинейных гиперболических уравнений // Мат.сб. — 1975. — Т. 96(138), №1. —- С. 152–166.
2. 2*Похожаев С.И.* Об одном квазалинейном гиперболическом уравнении Кирхгофа // Дифференц. уравнения. — 1985. — Т. 21,№1. — С. 101–108.
3. 3*Похожаев С.И.* Квазилинейные гиперболические уравнения Кирхгофа и законы сохранения // Тр.Моск.энерг. ин-та. — 1974. — Вып.201. — С. 118–126.
4. 4 *Cioranesku D., Murat F.* Un terme étrange venu d'ailleurs // Res. Notes Math. — 1982. — Vol. 60. — P. 93–138.
5. 5 *Cioranesku D., Donato P., Murat F., Zuazua E.* Homogenization and corrector for the wave equation in domains with small holes // Ann. della Scuola Norm. Super. di Pica, Sei.fis. e matem. — 1991. — Ser.4, vol.18, F.2. — P. 251–293.

6. 6 *Скрыпник И.В., Сиденко Н.Р.* Усреднение начально–краевой задачи для нелинейного волнового уравнения в перфорированной области // Нелінійний аналіз: Праці Українського математичного конгресу — 2001. —
Ін–т математики НАН України, Київ, 2005, 98–112.
7. 7 *Скрыпник И.В.* Методы исследования нелинейных эллиптических граничных задач. — М.:Наука — 1990. — 448 с.
8. 8 *Dal Maso G., Skrypnik I.V.* Asymptotic behaviour of nonlinear Dirichlet problems in perforated domains. // — Ref. SISSA 162 |95| M (December 95). — Trieste, 1995. — 65 p.
9. 9 *Лионс Ж.-Л., Мадженес Э.* Неоднородные граничные задачи и их приложения. — М.:Мир, 1971. — 372 с.
10. 10 *Сиденко Н.Р.* Усреднение задачи Дирихле для специального гиперболического уравнения Кирхгофа // Укр. матем. ж., 2006, т.58, №2, 236–249.

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