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TRIBANDS OF SUBTRIOIDS

We introduce the notion of a triband of subtrioids and prove that every trioid with a commutative periodic semigroup is a semilattice of unipotent subtrioids. Also we give examples of trioids which are decomposed into a triband of subtrioids.

Key words: *trioid, dimonoid, semigroup, congruence, triband of subtrioids.*

1. Introduction. Jean-Louis Loday and María O. Ronco introduced the notion of a trioid and a trialgebra [1]. A trioid is a set equipped with three binary associative operations satisfying some axioms (see below). A trialgebra is just a linear analogue of a trioid. If the operations of a trioid coincide, then the trioid becomes a semigroup. The first result about trioids is the description of the free trioid generated by a given set [1]. If the operations $\vdash$ and $\perp$ of a trioid coincide, then it becomes a dimonoid [2]. Dimonoids were introduced by J.-L. Loday for studying of properties of Leibniz algebras. The first result about dimonoids is the description of the free dimonoid generated by a given set [2]. The structure of commutative dimonoids was described in [3]. In [4] author constructed a free commutative dimonoid and described the least idempotent congruence on this dimonoid. The notion of a semiretraction of a dimonoid was introduced and applied to studying of congruences in [5]. In [6] author described the structure of an arbitrary diband of subdimonoids.

In the work of L.M. Gluskin, B.M. Schein and L.N. Shevrin [7] it is stated: “The notion of a band, which emerged in 1950s in the semigroup theory and since then has been playing a very important role, can be naturally specified for any class of abstract algebras. In fact, bands are congruences, all the classes of which are the essence of subalgebras. Furthermore, any congruence on an algebra with idempotent operations (for example, on a structure) is a band... But this congruence was scrutinized in the semigroup theory. The special importance is laid on so-called commutative and matrix bands, mostly due to Clifford and McLean’s result about the decomposition of an arbitrary band into commutative and matrix bands; this result is still being specific exactly for the semigroup theory”. Based on the quotation by L.M. Gluskin, B.M. Schein and L.N. Shevrin, the question of generalization of a band of semigroups to other algebraic structures is natural. So, the notion of a diband for such a class of abstract algebras as dimonoids, was introduced in [3].

In this paper we introduce the notion of a triband of subtrioids to describe decompositions of trioids. This notion generalizes the notion of a diband of subdimonoids [3] and the notion of a band of semigroups [8]. In section 2 we give necessary definitions, an auxiliary result (Lemma 2), some properties of trioids (Lemmas 1 and 3) and two examples of commutative trioids (Propositions 4 and 5). Schwarz [9] proved that every commutative periodic semigroup is a semilattice of unipotent semigroups. In section 3 we extend Schwarz's theorem (Theorem 6): every trioid with a commutative periodic semigroup is a semilattice of unipotent subtrioids. In section 4 we construct examples of trioids which are decomposed into a triband of subtrioids (Lemmas 8 and 11).

2. Preliminaries. A set T equipped with three binary associative operations $\dashv$, $\vdash$ and $\perp$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \tag{T1}$$

$$(x \vdash y) \dashv z = x \vdash (y \dashv z), \tag{T2}$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), \tag{T3}$$

$$(x \dashv y) \dashv z = x \dashv (y \perp z), \tag{T4}$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), \tag{T5}$$

$$(x \dashv y) \perp z = x \perp (y \vdash z), \tag{T6}$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), \tag{T7}$$

$$(x \perp y) \vdash z = x \vdash (y \vdash z) \tag{T8}$$

for all $x, y, z \in T$, is called a trioid.

A map f from a trioid T_1 to a trioid T_2 is a homomorphism, if $(x \dashv y)f = xf \dashv yf$, $(x \vdash y)f = xf \vdash yf$, $(x \perp y)f = xf \perp yf$ for all $x, y \in T_1$. A subset A of a trioid $(T, \dashv, \vdash, \perp)$ is called a subtrioid, if for any $a, b \in T$, a, $b \in A$ implies $a \dashv b$, $a \vdash b$, $a \perp b \in A$.

We call a trioid $(T, \dashv, \vdash, \perp)$ commutative (respectively, idempotent or a triband), if the semigroups $(T, \dashv)$, $(T, \vdash)$ and $(T, \perp)$ are commutative (respectively, idempotent).

Define the notion of a triband of subtrioids.

If $\varphi : S \to M$ is a homomorphism of trioids, then the corresponding congruence on S will be denoted by Δ_φ.

Let S be an arbitrary trioid, J be some idempotent trioid. If there exists a homomorphism $\alpha : S \to J : x \mapsto x\alpha$, then every class of the congruence Δ_α is a subtrioid of the trioid S, and the trioid S itself is a union of such trioids S_ξ, $\xi \in J$ that

$$x\alpha = \xi \Leftrightarrow x \in S_\xi = \Delta_\alpha^x = \{t \in S \,|(x;t) \in \Delta_\alpha\},$$

$$S_\xi \dashv S_\varepsilon \subseteq S_{\xi \dashv \varepsilon}, \quad S_\xi \vdash S_\varepsilon \subseteq S_{\xi \vdash \varepsilon}, \quad S_\xi \perp S_\varepsilon \subseteq S_{\xi \perp \varepsilon},$$

$$\xi \neq \varepsilon \Rightarrow S_\xi \bigcap S_\varepsilon = \emptyset.$$

In this case we say that S is decomposable into a triband of subtrioids (or S is a triband J of subtrioids S_ξ $(\xi \in J)$). If J is an idempotent semigroup (band), then we say that S is a band J of subtrioids S_ξ $(\xi \in J)$. If J is a commutative band, then we say that S is a semilattice J of subtrioids S_ξ $(\xi \in J)$.

Observe that the notion of a triband of subtrioids generalizes the notion of a diband of subdimonoids [3] (see also [6]) and the notion of a band of semigroups [8].

A commutative idempotent semigroup is called a semilattice.

Lemma 1. *The operations of a trioid* $(T, \dashv, \vdash, \perp)$ *coincide, if* $(T, \dashv)$ *is a semilattice.*

Proof. For all $x, y, z \in T$ we have

$$(x \vdash y) \dashv z = z \dashv (x \vdash y) = (z \dashv x) \dashv y =$$
$$= x \dashv (y \dashv z) = x \vdash (y \dashv z)$$

according to the commutativity and the associativity of the operation $\dashv$ and the axioms $(T1), (T2)$. Substituting $z = y$ in the last equality and using the idempotent property of the operation $\dashv$, we obtain $x \dashv y = x \vdash y$.

For all $x, y, z \in T$ we have

$$(x \perp y) \dashv z = z \dashv (x \perp y) = (z \dashv x) \dashv y =$$
$$= x \dashv (y \dashv z) = x \perp (y \dashv z)$$

according to the commutativity and the associativity of the operation $\dashv$ and the axioms (T4), (T5). Substituting $z = y$ in the last equality and using the idempotent property of the operation $\dashv$, we obtain $x \dashv y = x \perp y$. $\square$

We denote by N the set of positive integers. Let $(T, \dashv, \vdash, \perp)$ be a trioid and $a \in T$, $n \in N$. Denote by a^n the degree n of an element a concerning the operation $\dashv$.

Recall that a set D equipped with two binary associative operations $\dashv$ and $\vdash$ satisfying the axioms $(T1)$-$(T3)$ is called a dimonoid (see [2]-[6], [10]).

Lemma 2. *([3], Lemma 1) Let* $(D, \dashv, \vdash)$ *be a dimonoid with a commutative operation* $\dashv$*. For all* $b, c \in D$*,* $m \in N$*,* $m > 1$*,*

$$(b \dashv c)^m = b^m \vdash c^m = (b \vdash c)^m.$$

Lemma 3. *Let* $(T, \dashv, \vdash, \perp)$ *be a trioid with a commutative operation* $\dashv$*. For all* $b, c \in T$*,* $m \in N$*,* $m > 1$*,*

$$(b \dashv c)^m = b^m \perp c^m = (b \perp c)^m.$$

Proof. For any $b, c \in T$ we have

$$(b \dashv c)^m = b^m \dashv c^m = b^m \dashv c^{m-1} \dashv c =$$
$$= (c \dashv b^m) \dashv c^{m-1} = c \dashv (b^m \perp c^{m-1}) =$$
$$= (b^m \perp c^{m-1}) \dashv c = b^m \perp (c^{m-1} \dashv c) = b^m \perp c^m$$

according to the commutativity and the associativity of the operation $\dashv$ and the axioms $(T4)$, $(T5)$.

We prove that $(b \dashv c)^m = (b \perp c)^m$ for $m > 1$ using an induction on m. For $m = 2$ we have

$$(b \dashv c)^2 = (b \dashv c) \dashv (b \dashv c) = (b \dashv c) \dashv (b \perp c) =$$

$$= (b \perp c) \dashv (b \dashv c) = (b \perp c) \dashv (b \perp c) = (b \perp c)^2$$

according to the associativity and the commutativity of the operation $\dashv$ and the axiom $(T4)$.

Let $(b \dashv c)^k = (b \perp c)^k$ for $m = k$. Then for $m = k + 1$ we obtain

$$(b \dashv c)^{k+1} = (b \dashv c)^k \dashv (b \dashv c) = (b \dashv c)^k \dashv (b \perp c) =$$
$$= (b \perp c)^k \dashv (b \perp c) = (b \perp c)^{k+1}$$

according to the associativity of the operation $\dashv$, the axiom $(T4)$ and the supposition.

Thus, $(b \dashv c)^m = (b \perp c)^m$ for $m > 1$. $\square$

Now we give examples of commutative trioids.

Let $2N$ be the set of even positive integers and $2N$-1 be the set of odd positive integers. Fix $t, t_1, t_2, t_3 \in 2N$-1 and define the operations $\dashv, \vdash$ and $\perp$ on N by

$$x \dashv y = \begin{cases} x + y + t_1, \ x, y \in 2N, \\ \quad t \quad \text{otherwise}, \end{cases}$$

$$x \vdash y = \begin{cases} x + y + t_2, \ x, y \in 2N, \\ \quad t \quad \text{otherwise}, \end{cases}$$

$$x \perp y = \begin{cases} x + y + t_3, \ x, y \in 2N, \\ \quad t \quad \text{otherwise} \end{cases}$$

for all $x, y \in N$.

Proposition 4. $(N, \dashv, \vdash, \perp)$ is a commutative trioid.

Proof. It is immediate to check that $(N, \dashv, \vdash, \perp)$ is a trioid. It is clear that the operations $\dashv, \vdash$ and $\perp$ are commutative. $\square$

Let X be an arbitrary set such that $0, \ a, \ b, \ c, \ d, \ e, \ f \in X$ and $a \neq b, \ b \neq c, \ c \neq d, \ d \neq a, \ f \neq a, \ b \neq e, \ d \neq e, \ f \neq c, \ e \neq f$. Define the operations $\dashv$, $\vdash$ and $\perp$ on X by

$$x \dashv y = \begin{cases} b, \ x = y = a, \\ 0 \qquad \text{otherwise}, \end{cases}$$

$$x \vdash y = \begin{cases} d, \ x = y = c, \\ 0 \qquad \text{otherwise}, \end{cases}$$

$$x \perp y = \begin{cases} f, \ x = y = e, \\ 0 \qquad \text{otherwise} \end{cases}$$

for all $x, y \in X$.

Proposition 5. $(X, \dashv, \vdash, \perp)$ is a commutative trioid.

Proof. It is immediate to check that $(X, \dashv, \vdash, \perp)$ is a trioid. It is clear that the operations $\dashv, \vdash$ and $\perp$ are commutative. $\square$

3. Trioids with a commutative perioidic semigroup. In this section we prove that every trioid $(T, \dashv, \vdash, \perp)$ with a commutative periodic semigroup $(T, \dashv)$ is a semilattice of unipotent subtrioids.

Recall that a semigroup S is called a periodic semigroup, if every element of S has a finite order, that is, if for every element a of S the subsemigroup $\langle a\rangle = \{a, a^2, ..., a^n, ...\}$ generated by a contains a finite number of different elements. A trioid $(T, \dashv, \vdash, \perp)$ will be called unipotent, if it contains exactly one element $x \in T$ such that $x \dashv x = x \vdash x = x \perp x = x$.

Theorem 6. *Every trioid $(T, \dashv, \vdash, \perp)$ with a commutative periodic semigroup $(T, \dashv)$ is a semilattice Y of unipotent subtrioids $T_i,\ i \in Y$.*

Proof. Define a relation γ on the trioid $(T, \dashv, \vdash, \perp)$ with a commutative periodic semigroup $(T, \dashv)$ by

$$\begin{gathered} a\gamma\, b \text{ if and only if there exists an} \\ \text{idempotent } \varepsilon \text{ of the semigroup } (T, \dashv) \text{ such} \\ \text{that } a^l = b^k = \varepsilon \text{ for some } l,\ k \in N. \end{gathered}$$

The fact that the relation γ is a semilattice congruence on the semigroup $(T, \dashv)$ has been proved by Schwarz [9]. Show that γ is compatible concerning the operation $\vdash$.

Let $a\gamma b,\ a, b, c \in T$. Then $a \dashv c\,\gamma b \dashv c$. It means that there exists an idempotent e of the semigroup $(T, \dashv)$ such that $(a \dashv c)^n = (b \dashv c)^m = e$ for some $n, m \in N$. Hence

$$(a \dashv c)^n \dashv (a \dashv c)^n = (a \dashv c)^{2n} = e, \tag{1}$$

$$(b \dashv c)^m \dashv (b \dashv c)^m = (b \dashv c)^{2m} = e. \tag{2}$$

By Lemma 2 from (1) and (2) it follows that $(a \vdash c)^{2n} = (b \vdash c)^{2m} = e$ and so, $a \vdash c\,\gamma\, b \vdash c$. Dually, the left compatibility of the relation γ concerning the operation $\vdash$ can be proved. So, γ is a congruence on $(T, \dashv, \vdash)$.

Now we show that γ is compatible concerning the operation $\perp$.

By Lemma 3 from (1) and (2) it follows that $(a \perp c)^{2n} = (b \perp c)^{2m} = e$ and so, $a \perp c\,\gamma\, b \perp c$. Dually, the left compatibility of the relation γ concerning the operation $\perp$ can be proved. So, γ is a congruence on $(T, \dashv, \vdash, \perp)$.

As ${}^{(T, \dashv)}/_{\gamma}$ is a semilattice, then by Lemma 1 the operations of ${}^{(T, \dashv, \vdash, \perp)}/_{\gamma}$ coincide and so, it is a semilattice. From [9] it follows that every class A of the congruence γ is a unipotent subsemigroup of the semigroup $(T, \dashv)$. Let $e \in A$ and $e \dashv e = e$. For an arbitrary element $a \in A$ there exists $p \in N,\ p > 1$ such that $a^p = e$. Hence

$$\begin{gathered} e \vdash e = a^p \vdash a^p = \\ = a^p \vdash (a^{p-1} \dashv a) = (a^p \vdash a^{p-1}) \dashv a = \\ = a \dashv (a^p \vdash a^{p-1}) = (a \dashv a^p) \dashv a^{p-1} = \\ = (a \dashv a^{p-1}) \dashv a^p = a^p \dashv a^p = e \dashv e = e \end{gathered}$$

according to the commutativity and the associativity of the operation $\dashv$ and the axioms (T1), (T2). Thus, e is an idempotent of the subsemigroup A of $(T, \vdash)$.

Show that e is an idempotent concerning the operation $\perp$. We have

$$\begin{gathered} e \perp e = a^p \perp a^p = \\ = a^p \perp (a^{p-1} \dashv a) = (a^p \perp a^{p-1}) \dashv a = \\ = a \dashv (a^p \perp a^{p-1}) = (a \dashv a^p) \dashv a^{p-1} = \\ = (a \dashv a^{p-1}) \dashv a^p = a^p \dashv a^p = e \dashv e = e \end{gathered}$$

according to the commutativity and the associativity of the operation $\dashv$ and the axioms (T4), (T5). So, A is a unipotent subtrioid of $(T, \dashv, \vdash, \perp)$. $\square$

This result extends Schwarz's theorem [9] about the decomposition of commutative periodic semigroups into semilattices of unipotent semigroups and the theorem [10] about the decomposition of dimonoids with a commutative periodic semigroup into semilattices of unipotent subdimonoids.

4. Examples of tribands of subtrioids. In this section we construct examples of trioids which are decomposed into a triband of subtrioids.

a) Let X^* be a set of finite nonempty words in the alphabet X. If $w \in X^*$, then the first (respectively, last) letter of a word w we denote by $w^{(0)}$ (respectively, by $w^{(1)}$).

Define the operations $\dashv, \vdash$ and $\perp$ on the set X^* by

$$w \dashv u = w^{(0)}w^{(1)}, \quad w \vdash u = u^{(0)}u^{(1)}, \quad w \perp u = w^{(0)}u^{(1)}$$

for all $w, u \in X^*$.

Proposition 7. $(X^*, \dashv, \vdash, \perp)$ is a trioid.

Proof. By Proposition 3 from [3] $(X^*, \dashv, \vdash)$ is a dimonoid. In order to complete the proof we must prove that the axioms $(T4)$-$(T8)$ of a trioid and the associativity of $\perp$ hold. For any $w, u, \omega \in X^*$ we obtain

$$\begin{aligned}
&(w \dashv u) \dashv \omega = w^{(0)}w^{(1)} \dashv \omega = \\
&= w^{(0)}w^{(1)} = w \dashv (u \perp \omega), \\
&(w \perp u) \dashv \omega = w^{(0)}u^{(1)} \dashv \omega = w^{(0)}u^{(1)} = \\
&= w \perp u^{(0)}u^{(1)} = w \perp (u \dashv \omega), \\
&(w \dashv u) \perp \omega = w^{(0)}w^{(1)} \perp \omega = w^{(0)}\omega^{(1)} = \\
&= w \perp \omega^{(0)}\omega^{(1)} = w \perp (u \vdash \omega), \\
&(w \vdash u) \perp \omega = u^{(0)}u^{(1)} \perp \omega = u^{(0)}\omega^{(1)} = \\
&= w \vdash u^{(0)}\omega^{(1)} = w \vdash (u \perp \omega), \\
&(w \perp u) \vdash \omega = w^{(0)}u^{(1)} \vdash \omega = \omega^{(0)}\omega^{(1)} = \\
&= w \vdash \omega^{(0)}\omega^{(1)} = w \vdash (u \vdash \omega).
\end{aligned}$$

Obviously, the operation $\perp$ is associative. $\square$

Let $(X \times X, \dashv', \vdash', \perp')$ be an idempotent trioid with the operations

$$(x, y) \dashv' (a, b) = (x, y), \ (x, y) \vdash' (a, b) = (a, b), \ (x, y) \perp' (a, b) = (x, b)$$

for all $(x, y), (a, b) \in X \times X$. Denote by $\bar{X}$ this trioid and for all $i, j \in X$ put

$$A_{(i,j)} = \{w \in X^* | (w^{(0)}, w^{(1)}) = (i, j)\}.$$

The next assertion describes the structure of the trioid $(X^*, \dashv, \vdash, \perp)$.

Lemma 8. *The trioid $(X^*, \dashv, \vdash, \perp)$ is a triband $\bar{X}$ of zero semigroups $A_{(i,j)}$, $(i,j) \in \bar{X}$.*

Proof. Define a map $\mu : (X^*, \dashv, \vdash, \perp) \to \bar{X} : w \mapsto (w^{(0)}, w^{(1)})$. The map μ is a homomorphism. Indeed, if $w, u \in X^*$, then

$$(w \perp u)\mu = (w^{(0)}u^{(1)})\mu = (w^{(0)}, u^{(1)}) =$$

$$= (w^{(0)}, w^{(1)}) \perp' (u^{(0)}, u^{(1)}) = w\mu \perp' u\mu.$$

The equalities

$$(w \dashv u)\mu = w\mu \dashv' u\mu, \quad (w \vdash u)\mu = w\mu \vdash' u\mu$$

for all $w, u \in X^*$ follow from Proposition 4 of [3].

It is clear that $A_{(i,j)}, (i,j) \in \bar{X}$ is an arbitrary class of the congruence $\triangle_\mu$. Moreover, if $w, u \in A_{(i,j)}$, then $w \dashv u = w \vdash u = w \perp u = ij$, hence $A_{(i,j)}$ is a zero semigroup with the zero ij. □

b) Let S be a semigroup and let f be its idempotent endomorphism. Define the operations $\dashv, \vdash$ and $\perp$ on S by

$$x \dashv y = x(yf),\ x \vdash y = (xf)y,\ x \perp y = (xy)f$$

for all $x, y \in S$.

Proposition 9. $(S, \dashv, \vdash, \perp)$ is a trioid.

Proof. By Proposition 1 from [3] $(S, \dashv, \vdash)$ is a dimonoid. In order to complete the proof we must prove that the axioms $(T4)$-$(T8)$ of a trioid and the associativity of $\perp$ hold. For any $x, y, z \in S$ we obtain

$$\begin{aligned}
&(x \dashv y) \dashv z = x(yf) \dashv z = x(yf)(zf) = x((yz)f),\\
&x \dashv (y \perp z) = x \dashv ((yz)f) = x((yz)f^2) = x((yz)f),\\
&(x \perp y) \dashv z = (xy)f \dashv z = (xy)f(zf) = (xyz)f,\\
&x \perp (y \dashv z) = x \perp (y(zf)) = (x(y(zf)))f =\\
&= (xf)(yf)(zf^2) = (xy)f(zf) = (xyz)f,\\
&(x \dashv y) \perp z = x(yf) \perp z = (x(yf)z)f =\\
&= (xf)(yf^2)(zf) = (xf)(yf)(zf) = (xyz)f,\\
&x \perp (y \vdash z) = x \perp ((yf)z) = (x((yf)z))f =\\
&= (xf)(yf^2)(zf) = (xf)(yf)(zf) = (xyz)f,\\
&(x \vdash y) \perp z = (xf)y \perp z = ((xf)yz)f =\\
&= (xf^2)(yf)(zf) = (xf)((yz)f) = (xyz)f,\\
&x \vdash (y \perp z) = x \vdash ((yz)f) =\\
&= (xf)((yz)f) = (xyz)f,\\
&(x \perp y) \vdash z = (xy)f \vdash z = (xy)f^2z = (xy)fz,
\end{aligned}$$

$$
\begin{aligned}
&x \vdash (y \vdash z) = x \vdash ((yf)z) = (xf)(yf)z = (xy)fz,\\
&(x \perp y) \perp z = (xy)f \perp z = ((xy)fz)f =\\
&= (xy)f^2(zf) = (xy)f(zf) = (xyz)f,\\
&x \perp (y \perp z) = x \perp ((yz)f) = (x((yz)f))f =\\
&= (xf)((yz)f^2) = (xf)((yz)f) = (xyz)f.
\end{aligned}
$$

Comparing these expressions, we conclude that $(S, \dashv, \vdash, \perp)$ is a trioid. □

The trioid obtained will be denoted by S^f.

Lemma 10. *There exists a homomorphism $S^f \to S$.*

Proof. Define a map $\alpha : S^f \to S : t \mapsto t\alpha = tf$. For all t, $s \in S^f$ we have

$$(t \dashv s)\alpha = (t(sf))\alpha = (t(sf))f = (tf)(sf^2) = (tf)(sf) = (t\alpha)(s\alpha),$$

$$(t \vdash s)\alpha = ((tf)s)\alpha = ((tf)s)f = (tf^2)(sf) = (tf)(sf) = (t\alpha)(s\alpha),$$

$$(t \perp s)\alpha = ((ts)f)\alpha = (ts)f^2 = (ts)f = (tf)(sf) = (t\alpha)(s\alpha),$$

hence α is a homomorphism. □

Let X be a nonempty set. If $\varphi : X \to X$ is a transformation, then $ker(\varphi) = \{(x, y) \in X \times X | x\varphi = y\varphi\}$.

Let E be an arbitrary idempotent semigroup, f its idempotent endomorphism and let T_x be an arbitrary equivalence class of $ker(f)$ with the representative $x \in Ef$.

Lemma 11. *The trioid E^f is a band Ef of subtrioids T_x, $x \in Ef$.*

Proof. From Lemma 10 it follows that there exists a homomorphism $\alpha : E^f \to E$ such that its image is an idempotent subsemigroup Ef of the semigroup E. It is clear that the classes of the congruence on E^f which corresponds to the homomorphism α are the sets T_x, $x \in Ef$ which are trioids with respect to the operations $\dashv$, $\vdash$ and $\perp$. □

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А.В. Жучок

Трисвязки подтриоидов.

В работе введено понятие трисвязки подтриоидов и доказано, что каждый триоид с коммутативной периодической полугруппой является полурешёткой унипотентных подтриоидов. Построены примеры триоидов, которые раскладываются в трисвязки подтриоидов.

Ключевые слова: *триоид, димоноид, полугруппа, конгруэнция, трисвязка подтриоидов.*

А.В. Жучок

Трисполуки підтріоїдів.

У роботі введено поняття трисполуки підтріоїдів та доведено, що кожний тріоїд з комутативною періодичною напівгрупою є напівструктурою уніпотентних підтріоїдів. Побудовано приклади тріоїдів, які розкладаються в трисполуки підтріоїдів.

Ключові слова: *тріоїд, дімоноїд, напівгрупа, конгруенція, трисполука підтріоїдів.*

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