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Nonredundant Hexagonal Grid Interferometer Configurations with Element-Free Central Domains

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Nonredundant hexagonal grid interferometers having element-free central domains are configured with the difference set procedure. The obtained configurations possess third-order symmetry that provides more uniform coverage of the spatial-frequency plane.

1. Introduction

The construction of radio interferometers on hexagonal grids has been suggested in [1], and the possibility of configuring nonredundant (i. e. having no repeated baselines) interferometers on such grids that have rather low sidelobe patterns has been shown in [2].

The investigation of nonredundant arrays (NRA) on hexagonal grids dates back to the paper by Golay [3] where arrays with a small number of elements were considered. The method of constructing a NRA maximized in terms of the number of its elements on the hexagon of a given radius was elaborated in [4] using cyclic difference sets. Owing to regularity, it is especially useful for large arrays, in which case the numerical methods, e. g. the random search, are unacceptably time-consuming and thus practically unrealizable. The procedure of obtaining NRAs by this method was also described in the monograph [5].

In [3-5] no limitation was imposed on the location of configuration elements, so they could be arranged all over the grid area. It may be of interest to study the case when the grid central domain is element-free (thus allowing to use this domain for other purposes). This issue has been touched upon in [2], where the nonredundant arrangement of elements within a hexagonal "ring" whose inner radius was nearly half an outer (grid) one, was examined.

The present paper studies the problem in more detail in the form equivalent to that considered in [4]: we construct the nonredundant interferometer configurations on hexagonal grids of minimal radii with the number of elements given and with no elements in their central domains.

2. On the method of constructing nonredundant arrays

The method used here for constructing the optimal nonredundant configurations employs the two ideas, viz. the cyclic difference sets (CDS) and the folding procedure. The description of CDSs and their properties were given in [5, 6]; in particular, in [5] it has been noted that a k-element CDS with the parameters V and $\Lambda = 1$ represents the NRA on linear segment $[0, V-1]$, where $V = k^2 - k + 1$. The CDSs with $\Lambda = 1$ exist for all values $k = p^s + 1$, where p means prime, s – natural, and each of these cases has an ensemble of such sets obtainable from any of them by shifting modulo V or by multiplying it by a number coprime with V.

A linear segment, together with a CDS placed on it, can be folded onto a square (recall that folding a segment onto a square is the procedure inverse to scanning). The set thus obtained on the square is also nonredundant as can be proven by contradiction. Similarly, the segment can be folded onto a rhomb. If the rhomb with a sidelength n (n odd) and acute angle 60° is taken and two of its corners are then cut off, as is shown in Fig. 1, the remaining part will represent the rectilinear hexagon of radius $r = (n-1)/2$. Thus, folding segment $[0,\ V-1]$, together with a k-element CDS with the parameters V and $\Lambda = 1$ placed on it, onto a rhomb yields a NRA on this interior hexagon.

Calculating through all sets of such an ensemble at a given k yields a CDS giving maximum number of elements in the hexagon while keeping the condition of leaving the central domain of the hexagon element-free.

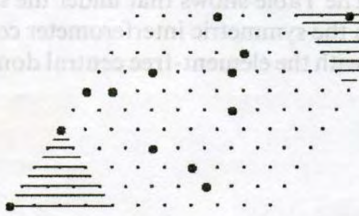

Fig. 1. *Nonredundant array on a rhomb. The unhatched part of the rhomb represents a rectilinear hexagon*

3. Configurations possesing third-order symmetry

As follows from [4] the optimal NRAs on hexagons in most cases possess the third-order symmetry. The latter means that they are invariant to rotation by 120° about a certain centre of symmetry. An example of such an array is shown in Fig. 2, where an oblique coordinate system (x, y) with the axes being directed along the hexagon diagonals is introduced. Third-order symmetry implies that the relation between the element (x_1, y_1) of the array and the elements (x_2, y_2) and (x_3, y_3) symmetric to it at the rotations by 120 and 240° respectively, are described by the expressions

$$x_2 = -(x_1 + y_1), \qquad y_2 = x_1 - d;$$
$$x_3 = y_1 + d, \qquad y_3 = x_2 - d. \tag{1}$$

Here the quantity d characterizes the symmetry centre displacement with respect to the hexagon centre. When these two coincide, $d = 0$.

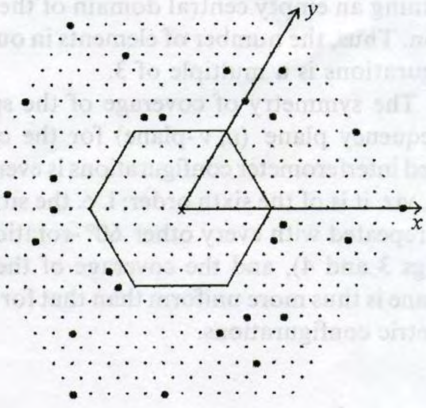

Fig. 2. *Example of a nonredundant configuration possessing third-order symmetry on a hexagonal grid. Here the number of elements $m = 27$, the hexagon radius $r = 12$; the inner, element-free hexagon of radius $r_0 = 5$ is singled out. The oblique coordinate system x, y is shown as a guide to build the configurations from Table 2*

Fig. 3. *24-element nonredundant configuration on the hexagonal grid of radius $r = 10$ and the coverage of the u, v-plane. The large dots represent the configuration elements while the small ones – the baselines in the u, v-plane. The inner, element-free domain of radius $r_0 = 4$ is singled out*

Some examples of the arrays corresponding to the case $d = 1$ were shown in [4]; though the case $d = 0$ predominates. Here, if the NRA possesses the central element (e. g. when $d = 0$), this latter has to be excluded in obtaining an empty central domain of the hexagon. Thus, the number of elements in our configurations is a multiple of 3.

The symmetry of coverage of the spatial-frequency plane (u, v-plane) for the considered interferometer configurations is even higher, viz. it is of the sixth order; i. e. the situation is repeated with every other $60°$-rotation (see Figs 3 and 4), and the coverage of the u, v-plane is thus more uniform than that for asymmetric configurations.

4. The calculation results

The NRAs with the number of elements $m = 15$, 18, ..., and 60 arranged on hexagons of minimal radii r, provided the element-free central domain of the hexagon is of the radius r_0 no less than a prescribed value, have been searched for. As a rule, they possess the third-order symmetry. The characteristics of such symmetric NRAs are shown in Table 1 (for all these arrays the symmetry centres coincide with the hexagon ones). The hexagon radii are taken to be minimal, provided $r_0 \geq 4$ (note that in the range $r \leq 10$ $r_0 = 4$ is the maximal value obtained). The Table shows that under the stated condition the symmetric interferometer configurations with the element-free central domains

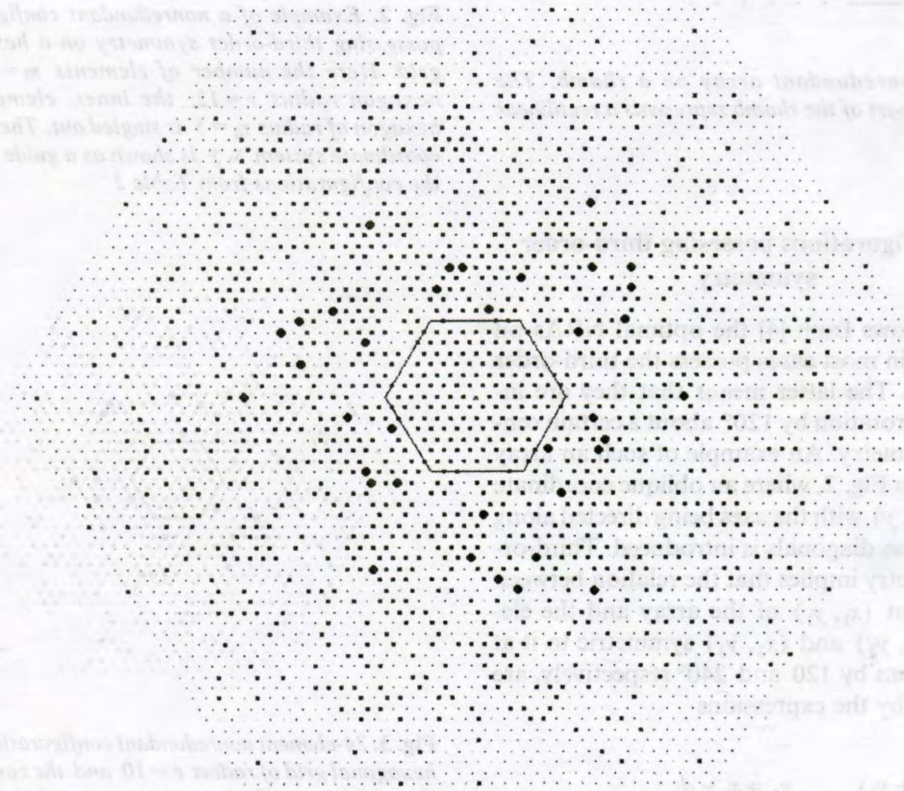

Fig. 4. *36-element nonredundant configuration on the hexagonal grid of radius $r = 18$ and the coverage of the u, v-plane. Here $r_0 = 7$ (the notations are the same as in Fig. 3.)*

Table 1. *Characteristics of symmetric nonredundant arrays on hexagons with the element-free central domains (the notations are given in the text)*

m	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
r	6	8	9	10	12	14	16	18	19	21	22	24	25	27	28	30
r_0	4	4	4	4	5	5	6	7	7	5	5	6	6	7	6	7

of radii r_0 and with the number of elements $m \geq 2r$ can be built. The comparison with the data in [4] shows that the discrepancy between m and maximum number m_{max} of the elements that can be arranged nonredundantly on a hexagon of radius r never exceeds 3 within $r \leq 25$.

As can be seen from Table 1, the values of r_0 are relatively small. These can be increased by increasing the radius r of the hexagon on which m elements are placed. Though the difference between m and m_{max} also grows in this case.

The interferometer configurations having the parameters from Table 1 represented as sets $\{(x_i, y_i)\}$, where x_i, y_i are the coordinates in the introduced oblique coordinate system, are shown in Table 2. It will be observed that when these are compared with those obtained in [4], the domain of the complete coverage in the u, v-plane shows some decrease in our case. Thus the 24-element configuration provides complete coverage of the domain of radius $r_c = 4$ in the u, v-plane (see Fig. 3), whereas such a radius provided by the 25-element configuration in [4], which differs from the former only by the central element presence, is $r_c = 6$, as can be seen in Fig. 3. To this end, however, the number of baselines decreases slightly, and the coverage, as well as the interferometer sidelobe pattern, are nearly the same.

Table 2. *Obtained symmetric nonredundant interferometer configurations*

m	elements $\{x_i, y_i\}$, $i = 1,...,m$
15	(0, 6); (-2, 5); (1, 5); (-5, 4); (2, 3); (-5, 2); (-6, 1); (4, 1); (-6, 0); (-3, -2); (5 ,-3); (1, -5); (3, -5); (5,-6); (6, -6)
18	(-6, 8); (-5, 7); (-1, 7); (-2, 5); (1, 5); (2, 3); (-5, 2); (-6, 1); (-6, -1); (-3, -2);(7, -2); (8, -2); (5 ,-3); (-2, -5); (3, -5); (-2, -6); (5 ,-6); (7, -6)
21	(-5, 9); (0, 9); (3, 6); (-7, 5); (-1, 5); (-6, 4); (-9, 3); (2, 3); (-5, 2); (4, 2);(5, 2); (-9, 0); (-4, -1); (5, -4); (9, -4); (-4, -5); (3, -5); (2, -6); (2, -7); (6, -9); (9, -9)
24	(-9, 9); (-4, 9); (-6, 8); (2, 8); (-7, 7); (3, 7); (-3, 5); (2, 4); (-10, 3); (-10, 2); (-6 2); (7, 0); (9, 0); (5, -2); (8, -2); (-2, -3); (-5, -4); (9, -5); (-2, -6); (4, -6); (0, -7);(0, -9); (7, -10); (8, -10)
27	(-12, 12); (-8, 10); (-2, 9); (1, 9); (-12, 7); (0, 7); (-5, 6); (-4, 6); (2, 6); (7, 5); (-8, 2); (-10, 1); (-7, 0); (12, 0); (6 ,-1); (-7, -2); (6,-2); (10, -2); (-2, -4); (-1, -5); (7, -7); (9, -7); (-2, -8); (6, -8); (9, -10); (0, -12); (5, -12)
30	(-13, 14); (-12, 14); (-8, 14); (-12, 12); (2, 10); (5, 9); (-9, 7); (-2, 6); (5, 6); (-14, 5); (-11, 5); (3, 5); (-8, 3); (-12, 2); (7, 2); (12, 0); (14, -1); (-4, -2); (14, -2); (6, -4); (14, -6); (-6, -8); (5, -8); (2, -9); (6, -11); (-2, -12); (0, -12); (10, -12); (-1, -13); (9, -14)

Table 2 (Continued)

33	(-15, 15); (-2, 15); (-11, 12); (-9, 12); (-16, 11); (-5, 10); (5, 9); (4, 8); (-6, 7); (0, 7); (4, 7) (-14, 5); (11, 5); (-12, 4); (-11, 4); (-7, 0); (15, 0); (7, -1); (12, -1); (-13, -2); (12, -3); (-5, -5); (10, -5); (-1, -6); (7, -7); (-3, -9); (-1, -11);(7, -11); (8, -12); (15, -13); (9, -14); (0, -15); (5, -16)
36	(0, 18); (-16, 16); (-8, 12); (-7, 12); (3, 12); (6, 12); (-2, 11); (-8, 10); (7, 10);(-15, 8); (-3, 8); (-17, 7); (8, 7); (-18, 6); (5, 6); (-11, 5); (-15, 3); (-18, 0); (16, 0);(-9, -2); (10, -2); (-5, -3); (12, -4); (8, -5); (12, -5); (-5, -7); (-4, -8); (-2, -8); (11, -9); (6, -11); (7, -15); (12, -15); (0, -16); (10, -17); (12, -18); (18, -18)
39	the NRA can be obtained from the preceeding array by increasing its hexagon radius by 1 and adding the elements (-12, 19), (-7, -12), (19, -7))
42	(-12, 21); (-11, 20); (-21, 19); (-12, 19); (1, 19); (-8, 18); (-1, 14); (4, 14); (-14, 11); (-10, 11); (-7, 11); (-4, 10); (-6, 6); (5, 6); (-11, 5); (-18, 4); (11, 3); (19, 2); (-20, 1); (6, 0); (-13, -1); (11, -1); (-6, -4); (11, -4); (0, -6); (10, -6); (-4, -7); (19, -7); (-10, -8); (20, -9); (21, -9); (-1, -10); (18, -10); (-9, -11); (6, -11); (-9, -12); (-7, -12); (14, -13); (3, -14); (14, -18); (19, -20); (2, -21)
45	the NRA can be obtained from the preceeding array by increasing its hexagon radius by 1 and adding the elements (6, 16), (-22, 6), (16, -22)
48	(-13, 24); (-15, 23); (-11, 23); (-4, 23); (-16, 20); (-3, 19); (-19, 18); (-19, 17); (10, 12); (-18, 11); (-22, 10); (-13, 10); (1, 9); (6, 9); (-9, 8); (-6, 8); (11, 7); (-15, 6); (-7, 6); (10, 3); (17, 2); (-10, 1); (6, 1); (8, 1); (18, 1); (8, -2); (-16, -3); (-19, -4); (20, -4); (-2, -6); (1, -7); (23, -8); (1, -9); (9, -10); (-12, -11); (24, -11); (23, -12); (-11, -13); (3, -13); (-8, -15); (9, -15); (-4, -16); (19, -16); (7, -18); (1, -19); (2, -19); (23, -19); (12, -22)
51	(-16, 25); (-10, 24); (-22, 23); (-12, 23); (-11, 21); (-25, 20); (-4, 20); (-2, 20); (-17, 18); (-24, 16); (-21, 15); (0, 13); (-6, 12); (-21, 11); (11, 10); (-6, 9); (-16, 8); (8, 8); (16, 8); (0, 7); (15, 6); (20, 5); (-13, 0); (-7, 0); (18, -1); (23, -1); (-18, -2); (9, -3); (-16, -4); (-6, -6); (-3, -6); (12, -6); (7, -7); (25, -9); (-14, -10); (21, -10); (-10, -11); (23, -11); (-11, -12); (13, -13); (24, -14); (-9, -16); (8, -16); (20, -16); (-1, -17); (20, -18); (6, -21); (10, -21); (-1, -22); (8, -24); (5, -25)
54	(-25, 27); (-23, 27); (-22, 26); (-4, 26); (-26, 23); (-17, 23); (0, 23); (3, 20); (-20, 18); (-8, 18); (-27, 17); (8, 13); (-15, 12); (-2, 12); (3, 10); (17, 10); (-1, 9); (-21, 8); (1, 8); (-4, 6); (-23, 3); (-13, 3); (12, 3); (23, 3); (18, 2); (-9, 1); (-23, 0); (-8, -1); (-10, -2); (6, -2); (27, -2); (-22, -4); (-2, -4); (26, -4); (27, -4); (23, -6); (-10, -8); (9, -8); (8, -9); (12, -10); (18, -10); (10, -13); (3, -15); (-6, -17); (2, -20); (13, -21); (-4, -22); (26, -22); (-4, -23); (20, -23); (23, -23); (-2, -25); (3, -26); (10, -27)
57	(-10, 28); (-15, 27); (-11, 27); (-21, 26); (-1, 26); (-9, 24); (-28, 22); (-18, 19); (-7, 18); (-21, 17); (-20, 17); (-3, 17); (2, 15); (-20, 14); (-12, 13); (-12, 11); (1, 11); (-7, 7); (-2, 7); (14, 6); (22, 6); (17, 4); (17, 3); (-17, 2); (-12, 1); (11, 1); (7, 0); (-25, -1); (13, -1); (19, -1); (-5, -2); (-14, -3); (7, -5); (26, -5); (-11, -7); (0, -7); (-15, -9); (-18, -10); (-16, -11); (18, -11); (-1, -12); (1, -12); (11, -12); (27, -12); (17, -14); (-12, -15); (24, -15); (27, -16); (15, -17); (-1, -18); (28, -18); (3, -20); (6, -20); (-5, -21); (4, -21); (26, –25); (6, -28)
60	(-11, 30); (-7, 30); (-6, 30); (-1, 28); (-14, 23); (-26, 22); (-15, 21); (9, 21); (-5, 20); (-13, 19); (-30, 18); (4, 18); (-14, 16); (-6, 16); (-25, 15); (-7, 15); (-3, 13); (18, 12); (0, 11); (9, 10); (15, 10); (-30, 9); (-19, 9); (3, 5); (-22, 4); (22, 4); (-8, 3); (-11, 0); (-27, -1); (16, -2); (-10, -3); (-15, -5); (-24, -6); (-10, -6); (19, -6); (21, -6); (-23, -7); (-8, -7); (5, -8); (15, -8); (23, -9); (13, -10); (16, -10); (-19, -11); (11, -11); (-6, -13); (-9, -14); (-2, -14); (-6, -15); (20, -15); (10, -19); (30, -19); (18, -22); (30, -23); (30, -24); (10, -25); (4, -26);(28, -27); (12, -30); (21, -30)

5. Conclusion

The approach used here for building the nonredundant multielement interferometer configurations on hexagonal grids having no elements in the central domains can equally be applied for the other element arrangement conditions, too. For a large number of elements, it offers an advantage over other accepted methods of solving similar problems.

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Безызбыточные конфигурации интерферометров на гексагональных решетках со свободной центральной областью

Л. Е. Копилович

С помощью метода, основанного на использовании разностных множеств, строятся безызбыточные конфигурации интерферометров со свободной центральной областью на гексагональных решетках. Эти конфигурации обладают симметрией 3-го порядка, что улучшает равномерность покрытия плоскости пространственных частот.

Безнадлишкові конфігурації інтерферометрів на гексагональних решітках з вільною центральною частиною

Л. Ю. Копилович

За методом, що грунтується на використанні різницевих множин, побудовано безнадлишкові конфігурації інтерферометрів з вільною від елементів центральною частиною на гексагональних решітках. Ці конфігурації мають симетрію 3-го порядку, що поліпшує рівномірність покриття площини просторових частот.