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PULSE SCATTERING ON OBJECTS IN THE INHOMOGENEOUS CONDUCTING MEDIUM

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The objective of the work is to present the results of computer simulation of the problem of pulse scattering on cylinder objects buried in the inhomogeneous conducting medium. The finite difference time domain method (FD-TD) is used for solving the problem. The grid discretization in space and time is carried out taking into account the required stability of the realized method. The elementary absorbing boundary conditions (ABC), such as the perfect matched layer, Bayliss-Turkel annihilation operators and the Mur finite-difference scheme have been realized. Visualization of the results obtained has been carried out, and scattered field images have been constructed.

1. Introduction

Ground penetrating radars (GPR) attract more and more attention and are of great interest in recent years. The theoretical works in this area are in two directions: solution of direct problems of pulse diffraction on subsurface objects and inverse problems of object reshaping and relocation by the present echo-response.

The direct problem solution is very important because during investigations one can obtain a set of "pictures" of fields reflected from different objects. Comparing them with the available echo-response permits to detect a required object without solving inverse problems in some cases. The direct diffraction problems are reduced to solving non-stationary Maxwell's equations with initial and boundary conditions. Up-to-date computers solve this sort of problems by direct methods. One of such methods is the finite-difference time-domain method (FD-TD method), on the basis of which it is possible to solve vector problems of electromagnetic pulse diffraction.

The given work is aimed at the FD-TD method

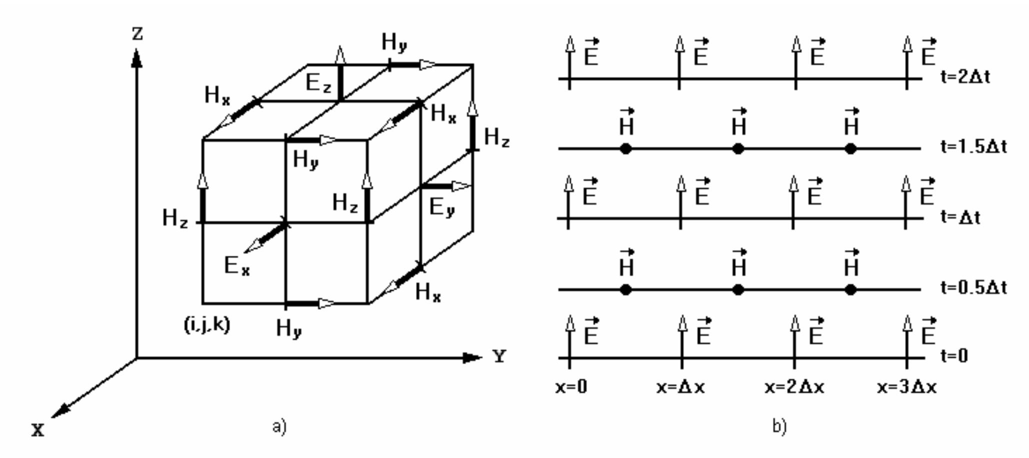

Fig. 1.
a) Position of the electric and magnetic field vector components about a cubic unit cell of the Yee space lattice;
b) Space-time chart of the Yee algorithm for a one-dimensional wave propagation

application to the problems of pulse diffraction on cylinder objects in dispersion and absorption media and investigation of scattered field on this basis.

2. General Characteristics of the Given Method

The finite-difference time-domain method [1,2] is the direct solution method for Maxwell's time-dependent curl equations. It employs no potentials. Rather, it is based upon volumetric sampling of the unknown near-field distribution ($\vec{E}$ and $\vec{H}$) within and surrounding the structure of interest, and over a period of time. The sampling in space depends on pulse duration and is drawn by the user. Typically, 10 to 20 samples per wavelength are needed. The sampling in time is selected to ensure numerical stability of the algorithm.

FD-TD is a procedure that simulates the continuous actual electromagnetic waves in a finite spatial region by sampled-data numerical analogs propagating in a computer data space. For simulations where the modeled region must extend to infinity, absorbing boundary conditions (ABC) are employed at the outer grid truncation planes which ideally permit all outgoing wave analogs to exit the region with negligible reflection. Phenomena such as induction of surface currents, scattering and multiple scattering, aperture penetration, and cavity excitation are modeled time-step by time-step by the action of the numerical analog to the curl equations.

Time-stepping is continued until the desired late-time pulse response is observed at the field point of interest.

Yee Algorithm

1. As it is shown in Fig. 1(a), the Yee algorithm [3] centers its components E and H in the tree-dimensional space in such a way that each component E is surrounded by four circulating components H, and each component H is surrounded by four circulating components E.
   - Resulting finite-difference expressions for space derivatives used in curl operators are central by their nature and have the second order of accuracy.
   - The continuity of tangential components E and H remains naturally the same when passing across the boundary in case that the boundary is parallel to one of the coordinate axes of the grid.
   - The location of components E and H in the Yee-grid and central-difference operations with these components implicitly realize two relations on the Gaussian law.
2. As it is shown in Fig. 1(b), the Yee algorithm centers its components E and H in time in the so-called "leapfrog" (lf) order. All calculations of E in the interesting for us three-dimensional space are made and stored in memory for a single time point, using the data of H pre-stored in the computer memory. Then all calculations of H in the formed space are made and stored in memory, using recently-calculated data of E. This process is circular and continues till finishing the time-stepping (ts).
   - This lf-ts process is all-explicit and therefore there are no problems related with solution of combined equations and matrix inversion.
   - Resulting finite-difference expressions for time derivatives used in curl operators are central by their nature and have the second order of accuracy.

The resulting ts algorithm is nondissipative; i.e. oscillations of the numerical wave propagating in the grid do not falsely decay.

Finite Differences

By introducing the following designation $u(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = u^n_{i,j,k}$ for field components and using finite-difference expressions for space and time derivatives

$$\frac{\partial u}{\partial x}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u^n_{i+1/2,j,k} - u^n_{i-1/2,j,k}}{\Delta x} + \mathrm{O}[(\Delta x)^2],$$

$$\frac{\partial u}{\partial t}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{u^{n+1/2}_{i,j,k} - u^{n-1/2}_{i,j,k}}{\Delta t} + \mathrm{O}[(\Delta x)^2]$$

we obtain the numerical approximation of three-dimensional numerical Maxwell's equations.

3. Computer Realization of the Method

For the two-dimensional region with a finite number of media having different electrical properties we can determine the $MEDIA(i,j,k)$ structure for each component of the vector field with information about dielectric properties of a medium at the given point. Maxwell's curl equations are reduced to the following finite-difference system by the described above finite differences:

TM mode:

$$m = MEDIA_{H_x}\big|_{i,j},$$

$$H_x\big|^{n+1/2}_{i,j} = D_a(m) H_x\big|^{n-1/2}_{i,j} + D_b(m)\left(E_z\big|^{n}_{i,j-1/2} - E_z\big|^{n}_{i,j+1/2}\right),$$

$$m = MEDIA_{H_y}\big|_{i,j},$$

$$H_y\Big|_{i,j}^{n+1/2} = D_a(m)H_y\Big|_{i,j}^{n-1/2} + D_b(m)\Big(E_z\Big|_{i+1/2,j}^{n} - E_z\Big|_{i-1/2,j}^{n}\Big),$$

$$m = MEDIA_{E_z}\big|_{i,j},$$

$$E_z\Big|_{i,j}^{n+1} = C_a(m)E_z\Big|_{i,j}^{n} + C_b(m)\Big(H_y\Big|_{i+1/2,j}^{n+1/2} - -H_y\Big|_{i-1/2,j}^{n+1/2} + H_x\Big|_{i,j-1/2}^{n+1/2} - H_x\Big|_{i,j+1/2}^{n+1/2}\Big).$$

TE mode:

$$m = MEDIA_{E_x}\big|_{i,j},$$

$$E_x\Big|_{i,j}^{n+1} = C_a(m)E_x\Big|_{i,j}^{n} + C_b(m)\Big(H_z\Big|_{i,j+1/2}^{n+1/2} - -H_z\Big|_{i,j-1/2}^{n+1/2}\Big),$$

$$m = MEDIA_{E_y}\big|_{i,j},$$

$$E_y\Big|_{i,j}^{n+1} = C_a(m)E_y\Big|_{i,j}^{n} + C_b(m)\Big(H_z\Big|_{i-1/2,j}^{n+1/2} - -H_z\Big|_{i+1/2,j}^{n+1/2}\Big),$$

$$m = MEDIA_{E_y}\big|_{i,j},$$

$$H_z\Big|_{i,j}^{n+1/2} = D_a(m)H_z\Big|_{i,j}^{n-1/2} + D_b(m)\times \Big(E_x\Big|_{i,j+1/2}^{n} - E_x\Big|_{i,j-1/2}^{n} + E_y\Big|_{i-1/2,j}^{n} - E_y\Big|_{i+1/2,j}^{n}\Big)$$

where updating coefficients are:

$$C_a|_{i,j} = \left(1 - \frac{\sigma_{i,j}\Delta t}{2\varepsilon_{i,j}}\right)\Bigg/\left(1 + \frac{\sigma_{i,j}\Delta t}{2\varepsilon_{i,j}}\right),$$

$$C_b|_{i,j} = \left(\frac{\Delta t}{\varepsilon_{i,j}\Delta}\right)\Bigg/\left(1 + \frac{\sigma_{i,j}\Delta t}{2\varepsilon_{i,j}}\right),$$

$$D_a|_{i,j} = \left(1 - \frac{\rho'_{i,j}\Delta t}{2\mu_{i,j}}\right)\Bigg/\left(1 + \frac{\rho'_{i,j}\Delta t}{2\mu_{i,j}}\right),$$

$$D_b|_{i,j} = \left(\frac{\Delta t}{\mu_{i,j}\Delta}\right)\Bigg/\left(1 + \frac{\rho'_{i,j}\Delta t}{2\mu_{i,j}}\right).$$

The Source

The hard source is set up simply by specifying a desired time function for definite components of the electric and magnetic fields in the spatial FD-TD grid. For example, in the one-dimensional TM grid the following hard source on E_z should be set up at the source point i_s for generating a continuous sinusoidal wave with the frequency f_0:

$$E_z|_{i_s}^{n} = E_0 \sin(2\pi f_0 n\Delta t).$$

Another common hard source is the wideband Gaussian pulse with the finite dc. The pulse is centered in the time step n_0 and has the $1/e$ characteristic decay n_{decay} of time steps:

$$E_z|_{i_s}^{n} = E_0 e^{-[(n-n_0)/n_{decay}]^2}.$$

There is a simple method to avoid the reflexive action of the hard pulse source – to remove it from the algorithm after the pulse will decrease essentially to 0 and apply an updated field instead of the normal Yee field. In the source context we will program an equivalent of the following updating relation for the electric field at i_s:

if $((n+1-n_0)/n_{decay} \le 3.0)$

$$E_z|_{i_s}^{n+1} = E_0 e^{-[(n+1-n_0)/n_{decay}]^2}$$

else

$$E_z|_{i_s}^{n+1} = C_a(m)E_z|_{i_s}^{n} + C_b(m)\Big(H_y|_{i_s+1/2}^{n+1/2} - H_y|_{i_s-1/2}^{n+1/2}\Big)$$

Absorbing Boundary Conditions

The main problem related with the FD-TD-approach to solving the problems of electromagnetic wave interaction is the fact that many interesting geometries are determined as open regions in which the computed field spatial region is unbounded in one or more coordinate directions. Clearly there is no computer capable to store an unbounded quantity of data and therefore the region of field calculation should be limited in size. The calculation region should be sufficiently large in order to surround the interesting structure, and around the external perimeter it is necessary to apply appropriate boundary conditions for simulating the wave propagation to infinity – so-called absorbing boundary conditions (ABC).

Bayliss-Turkel operators of scattered wave annihilation

For cylindrical coordinates the Bayliss-Turkel annihilation operator of n-order is defined as [4]:

$$B_n = \prod_{k=1}^{n}\left(L + \frac{4k-3}{2r}\right),$$

where the operator $L \equiv \dfrac{1}{c}\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial R}$.

The given differential operator systematically "destroys" or "annihilates" arbitrary outgoing scattered waves and leaves the remainder term, which is the difference process error. At any point of the grid external boundary the application of this differential operator to the local field allows us to estimate the field space derivative in the direction of outgoing wave propagation in terms of transverse space and time derivatives by using the data at points that are entirely inside the grid. The knowledge of the field space derivative in the direction of outgoing wave propagation permits to close the calculation region.

The Finite-Difference Mur Scheme

Let $W|_{0,j}^{n}$ be the Cartesian component $\vec{E}$ or $\vec{H}$ located on the boundary $x = 0$ of the Yee grid. Mur realized the space derivatives as central differences decomposed at the auxiliary point $(1/2,\ j)$ and obtained [5]:

$$
\begin{aligned}
&W|_{0,j}^{n+1} = \\
&\quad - W|_{1,j}^{n-1} + \frac{c\Delta t - \Delta x}{c\Delta t + \Delta x}\left(W|_{1,j}^{n+1} + W|_{0,j}^{n-1}\right) + \\
&\quad \frac{2\Delta x}{c\Delta t + \Delta x}\left(W|_{0,j}^{n} + W|_{1,j}^{n}\right) + \\
&\quad \frac{(c\Delta t)^2 \Delta x}{2(\Delta y)^2 (c\Delta t + \Delta x)}\left(W|_{0,j+1}^{n} - 2\,W|_{0,j}^{n} + \right. \\
&\quad \left. W|_{0,j-1}^{n} + W|_{1,j+1}^{n} - 2\,W|_{1,j}^{n} + W|_{1,j-1}^{n}\right) \ -
\end{aligned}
$$

the ts algorithm for W components along the boundary $x = 0$. Similarly it is possible to obtain analogous finite-difference expressions for Mur absorbing boundary conditions on every other boundary of the grid $x = h$, $y = 0$ and $y = h$.

Berenger Perfectly Matched Layer (PML)

Berenger obtained effective reflection coefficients for his absorbing boundary condition constituting 1/3000 of reflection coefficients of considered above standard analytic absorbing boundary conditions of second and third order. The approach, which he called "the perfectly matched layer (PML) for electromagnetic wave absorption" [6], is based on splitting the electric and magnetic field components in the absorbing boundary region into single subcomponents.

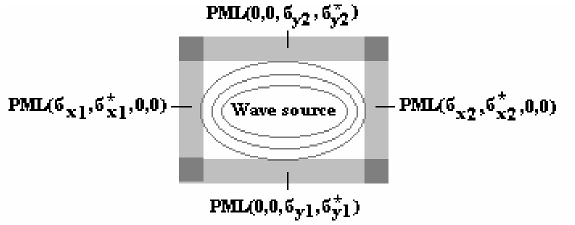

Fig. 2. *Structure of a two-dimensional FD-TD grid having the Berenger PML ABC*

The TE Case

Let's consider two-dimensional Maxwell's equations for the TE polarization case. After splitting H_z into two components H_{zx} and H_{zy} we obtain four (earlier we obtained three components) components for the TE case, connected by the following equations:

$$\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma_y E_x = \frac{\partial (H_{zx} + H_{zy})}{\partial y},$$

$$\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma_x E_y = -\frac{\partial (H_{zx} + H_{zy})}{\partial x},$$

$$\mu_0 \frac{\partial H_{zx}}{\partial t} + \rho_x' H_{zx} = -\frac{\partial E_y}{\partial x},$$

$$\mu_0 \frac{\partial H_{zy}}{\partial t} + \rho_y' H_{zy} = \frac{\partial E_x}{\partial y},$$

where σ_x and σ_y denote electric conductivity, and ρ_x' and ρ_y' denote magnetic loss.

When $\sigma_y = \rho_y'$ =0, the PML-medium can absorb plain waves with field components (E_y, H_{zz}) propagating along x, but it does not absorb waves with field components (E_x, H_{zy}) propagating along y. We have the opposite situation when $\sigma_x = \rho_x' = 0$.

4. The Program Grider1. Example

The program Grider1 uses the described above method of calculation of fields scattered in dispersion and absorbing media.

Let's consider the spatial region with length $x = 4$ m and depth $y = 4$ m, consisting of three dielectric layers with parameters ($\varepsilon = 1$, $\mu = 1$, $\sigma = 0$, $\rho' = 0$), ($\varepsilon = 5$, $\mu = 1$, $\sigma = 0.000001$, $\rho' = 0$), ($\varepsilon = 10$, $\mu = 1$, $\sigma = 0.00001$, $\rho' = 0$) and a submerged in the last layer object with dielectric parameters ($\varepsilon = 1$, $\mu = 1$, $\sigma = 100000000$, $\rho' = 0$).

We specify the grid 200×200, $dx = 0.002$ m, $dy = 0.002$ m. To ensure solution stability the time-step should not exceed 0.04714045 ns.

The following hard source on E_x is set up at the point (100, 100): it is a wideband Gaussian pulse with finite dc with the central time step equal to 20, characteristic decay of 4 time-steps and the amplitude of 1 V/m.

Fig. 3(a) represents distribution of the field component E_x in space at the time $t = 150$ time-steps:

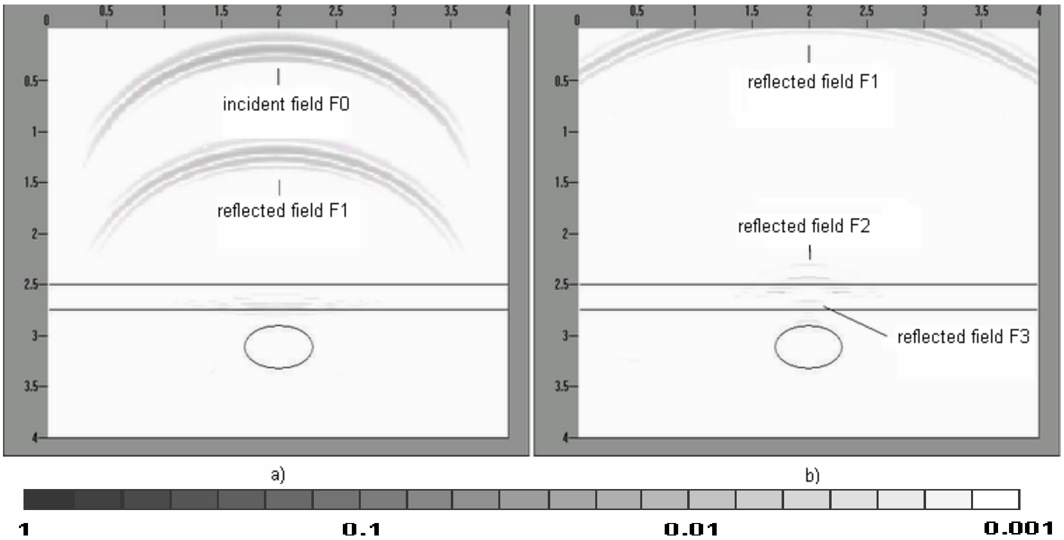

Fig. 3. *Distribution of the field component* E_x *in space at the time*
a) $t = 150$ *time-steps;*
b) $t = 250$ *time-steps*

The reflection (field F1) from the boundary between first two media is clearly seen in Fig. 3(a). But at the present time the signal hasn't reached yet the third medium and the object.

Fig. 3(b) represents distribution of the field component E_x in space at the time $t = 150$ time-steps.

In the given figure (Fig. 3(b)) one can see reflections (field F1, field F2) from both boundaries between dielectric media and the signal (field F3) reflected from the conducting object.

Fig. 4 represents distribution of the field component E_x at the point (100, 110) during 300 time-steps.

The echo-response of the transmitted signal is also easily observable in the given figure (Fig. 4).

For comparison let's consider the same spatial region with only parameters of layers ($\varepsilon = 1$, $\mu = 1$, $\sigma = 0$, $\rho' = 0$), ($\varepsilon = 8$, $\mu = 1$, $\sigma = 0.000005$, $\rho' = 0$), ($\varepsilon = 15$, $\mu = 1$, $\sigma = 0.00005$, $\rho' = 0$) that correspond to moisture saturation. For this case we obtain following distribution of the field component E_x at the point (100, 110) during 300 time-steps:

We can see a typical modification of pulse shape for this case on Fig. 5.

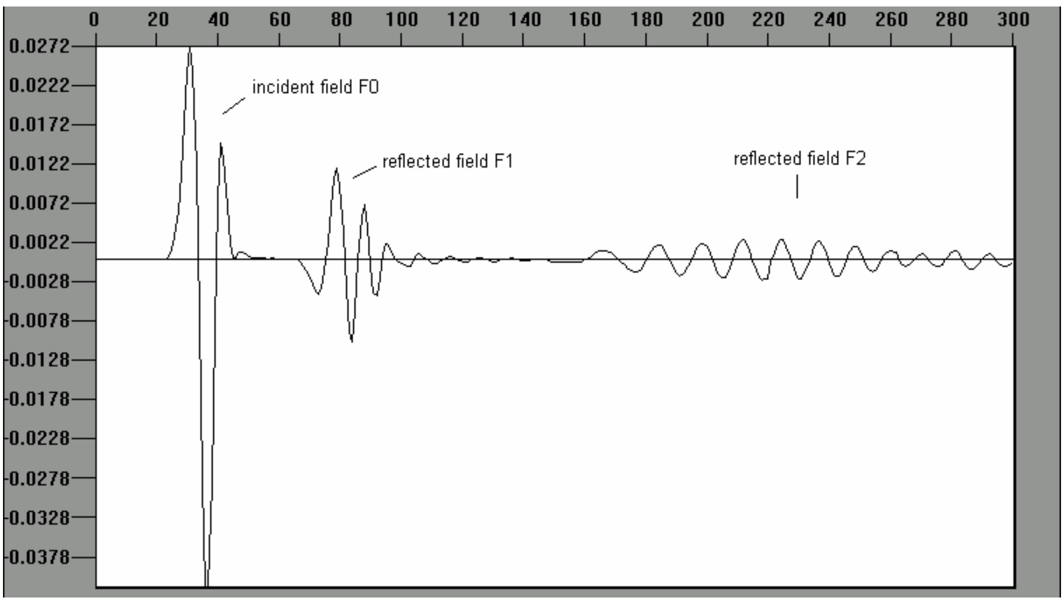

Fig. 4. *Distribution of the field component* E_x *at the point (100, 110) during 300 time-steps*

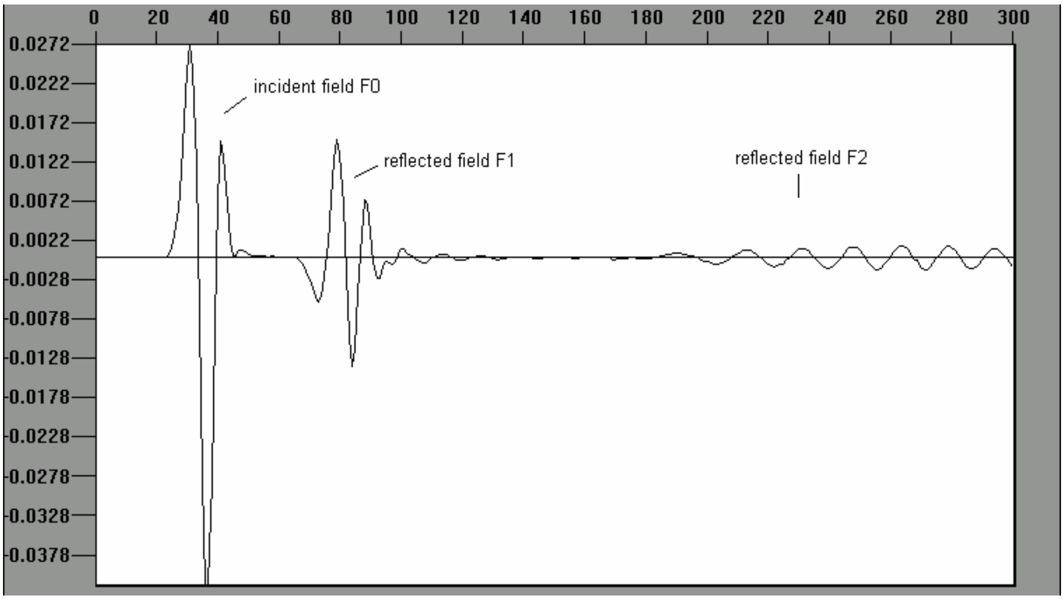

Fig. 5. *Distribution of the field component E_x at the point (100, 110) during 300 time-steps*

5. Conclusion

As it is seen from the above-said material the finite difference time domain method is very useful for realization on computer and gives totally valid results at sufficiently small spatial and time domain discretization. Thus, using the given method and Grider1 program realizing it one can investigate in future scattered fields on various objects in different media.

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РАССЕЯНИЕ ИМПУЛЬСОВ НА ОБЪЕКТАХ В НЕОДНОРОДНОЙ ПРОВОДЯЩЕЙ СРЕДЕ

Л.А. Варяница-Рощупкина, В.О. Коваленко

Цель настоящей работы – представить результаты компьютерного моделирования рассеяния импульсов на цилиндрических объектах, погруженных в неоднородную проводящую среду. Для регуляризации задачи используется метод конечных разностей во временной области (FD-TD). Дискретизация сетки в пространстве и времени проводится с учетом обеспечения требования устойчивости реализуемого метода. Реализованы простейшие поглощающие граничные условия (ABC), такие как PML (идеально согласованный слой), операторы аннигиляции Байлисса-Туркела, конечно-разностная схема Мура. Проведена визуализация полученных результатов, построены изображения рассеянных полей.

РОЗСІЯННЯ ІМПУЛЬСІВ НА ОБ'ЄКТАХ В НЕОДНОРІДНОМУ ПРОВІДНОМУ СЕРЕДОВИЩІ

Л.А. Варяниця-Рощупкіна, В.О. Коваленко

Мета цієї праці – представити результати комп'ютерного моделювання розсіювання імпульсів на циліндричних об'єктах, занурених у неоднорідне провідне середовище. Для регуляризації задачі використовується метод скінчених різниць у часовій області (FD-TD). Дискретизація сітки в просторі і часі проводиться з урахуванням забезпечення вимоги стійкості реалізованого методу. Реалізовано найпростіші поглинаючі граничні умови (ABC), такі як PML (ідеально погоджений шар), оператори анігіляції Байлісса-Туркела, скінченно-різницева схема Мура. Проведено візуалізацію отриманих результатів, побудовано зображення розсіяних полів.