RELATIVISTIC NEOCLASSICAL TRANSPORT COEFFICIENTS WITH MOMENTUM CORRECTION

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The parallel momentum correction technique is generalized for relativistic approach. It is required for proper calculation of the parallel neoclassical flows and, in particular, for the bootstrap current at fusion temperatures. It is shown that the obtained system of linear algebraic equations for parallel fluxes can be solved directly without calculation of the distribution function if the relativistic mono-energetic transport coefficients are already known. The first relativistic correction terms for Braginskii matrix coefficients are calculated.

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INTRODUCTION

It was shown recently [1] that relativistic effects in collisional neoclassical electron transport in hot plasmas appear even for electron temperatures about few tens kiloelektronvoltage, typical for D-T fusion reactors, and surely are non-negligible for future aneutronic fusion reactors with temperatures about 70 keV. However, all transport codes developed to date and applied for simulations of the reactor scenarios are still based on the non-relativistic approach and the validity of the model for hot electrons is not justified. Fully relativistic description of neoclassical transport processes based on a general relativistic kinetic theory requires a development of new transport codes from scratch. In contrast, the main advantage of the approach proposed in [1] is the possibility to take the relativistic effects into account without making any significant changes in transport solvers.

As it was suggested in [1], it is possible to reformulate the non-relativistic transport model to take into account the relativistic effects by modification of the energy-dependent part of the transport coefficients, while the pitch-dependent part can be calculated from the mono-energetic solver of the non-relativistic drift-kinetic equation (DKE). With a proper choice of parameters and the right-hand-side of DKE, the output parameters of such solver can be re-interpreted as relativistic ones and the transport fluxes can be calculated according to relativistic definitions [1, 2].

Alternatively, there is the way (see [3] and the references therein) to calculate transport fluxes without solving the DKE. The key point in this method is the representation of both distribution function and fluxes as an expansion in Sonine polynomials $L_n^{(3/2)}(x)$ (here, x is the normalized energy) followed by the calculation of the first moments of the DKE. As was shown in our previous publication [2], this method can also be effectively modified to take the relativistic effects into account.

In this paper, the electron fluxes of particles and heat are derived from the relativistic DKE and represented as an expansion in $L_1^{(\alpha)}(x)$ polynomials with $\alpha = 3/2 + \mathcal{R}$, where $\mathcal{R}$, the non-linear function of $T_e/m_e c^2$ (in classical limit $c \to \infty$ tends to zero), is the measure of relativistic effects. Finally, applying the same algorithm as in [3], the corresponding Braginskii matrix coefficients are calculated and the first relativistic correction terms are obtained.

1. MOMENT EQUATIONS FOR PARALLEL FLUXES CALCULATION

In the conventional moment-equation method for the parallel neoclassical fluxes calculation [3-5], the non-relativistic distribution function was expanded by Sonine polynomials $L_n^{(3/2)}(x)$, and its zeroth and first moments give the parallel fluxes of particles, $\boldsymbol{\Gamma}_{e\parallel}$, and heat, $\boldsymbol{q}_{e\parallel}$, respectively. In relativistic approach for electrons, the relation between relativistic fluxes of a heat, energy and particles differs from the classical one [6] by the additional relativistic term [1, 7],

$$\boldsymbol{q}_e = \boldsymbol{Q}_e - \left(\frac{5}{2} + \mathcal{R}\right) T_e \boldsymbol{\Gamma}_e, \qquad (1)$$

where $\boldsymbol{\Gamma}_e = \int d^3u\, \boldsymbol{v} f_e$ is the electron flux of particles and $\boldsymbol{Q}_e = \int d^3u\, \boldsymbol{v}\, m_e c^2(\gamma - 1) f_e$ is the electron flux of energy, respectively, $\boldsymbol{u} = \boldsymbol{v}\gamma$ is the momentum per unit mass with relativistic factor γ, and

$$\mathcal{R}(\mu_r) = \mu_r\left(\frac{K_3(\mu_r)}{K_2(\mu_r)} - 1\right) - \frac{5}{2} = \frac{15}{8\mu_r} + \cdots (\mu_r \gg 1), \qquad (2)$$

where $K_n(x)$ is the modified Bessel function of n-th order, and $\mu_r = m_e c^2/T_e$. In this case, the moment-equation method can be directly applied with expansion over generalized Laguerre polynomials $L_n^{(\alpha)}(\kappa)$ with $\alpha = 3/2 + \mathcal{R}$ and $\kappa = \mu_r(\gamma - 1)$. Here, $L_0^{(\alpha)}(x) = 1$, $L_1^{(\alpha)}(x) = \alpha + 1 - x$, etc. [2].

Following [3], the parallel flow moments can be introduced by

$$n_e V_{\parallel i}^e = \int d^3u\, v_\parallel L_i^{(\alpha)}(\kappa) f_e. \qquad (3)$$

Then the moment $i = 0$ can be identified with the flux of particles,

$$n_e V_{\parallel 0}^e = \int d^3u\, v_\parallel f_e = \Gamma_\parallel^e,$$

with the parallel flow velocity $V_{\parallel 0}^e$, while $i = 1$ corresponds to the parallel heat flux,

$$n_e V_{\parallel 1}^e = -\int d^3u\, (\kappa - 5/2 - \mathcal{R})\, v_\parallel f_e = -q_\parallel^e/T_e.$$

Representing f_e as the Legendre polynomials series and taking into account that only the 1st Legendre harmonic contributes in parallel fluxes, one can replace f_e with ξf_{e1}, where $f_{e1} = (3/2)\int_{-1}^{+1} d\xi\, \xi f_e$ and $\xi = u_\parallel/u$. In order to make it consistent with Eq. (3), it is convenient to represent f_{e1} as a series

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$$f_{e1} = \frac{m_e u}{T_e} w(\kappa) F_{eMJ}(\kappa) \sum_i a_i V^e_{\parallel i} L^{(\alpha)}_i(\kappa), \quad (4)$$

where $F_{eMJ}(\kappa)$ is the electron relativistic Maxwell-Jüttner equilibrium distribution function,

$$F_{eMJ}(u) = \frac{n_e}{\pi^{3/2} u_{te}^3} C_{eMJ}(\mu_r) e^{-\mu_r(\gamma-1)}, \quad (5)$$

with

$$C_{eMJ}(\mu_r) = \sqrt{\frac{\pi}{2\mu_r}} \frac{e^{-\mu_r}}{K_2(\mu_r)} = 1 - \frac{15}{8\mu_r} + \cdots \ (\mu_r \gg 1), \quad (6)$$

and the weight function

$$w(\kappa) = \kappa^{\mathcal{R}} \left(\frac{2}{\gamma+1}\right)^{3/2} C^{-1}_{eMJ}(\kappa). \quad (7)$$

The series coefficient,

$$a_i = a_i^0 \frac{\Gamma(i+5/2)}{\Gamma(i+5/2+\mathcal{R})} \text{ with } a_i^0 = \frac{3(2i)!!}{(2i+3)!!}, \quad (8)$$

can be found from the orthogonality of $L^{(\alpha)}_i(\kappa)$ [2]. One can check that in the non-relativistic limit $c \to \infty$, i.e. with $w = 1$ and $\mathcal{R} = 0$, the expressions in Eqs. (4)-(8) perfectly fit the non-relativistic formulas given in [3].

Applying for linearized collision operator with parallel momentum conservation the Taguchi approach [4], which perfectly works in weakly collisional plasmas,

$$C^{ab}(f_a) = \nu^{ab}_D(u) L(f_a) + \xi\left(C^{ab}_1(f_{a1}) + \nu^{ab}_D(u) f_{a1}\right), \quad (9)$$

one can represent $C^e_1(f_{a1}) = C^{ee}_1(f_{e1}) + C^{ei}_1(f_{e1})$ as

$$C^e_1(f_{a1}) = \frac{u}{T_e}\frac{w(\kappa)}{\gamma} F_{eMJ}(\kappa) \ \sum_j a_j F^e_{\parallel j} L^{(\alpha)}_j(\kappa), \quad (10)$$

where the relativistic neoclassical parallel collisional friction forces have been introduced:

$$n_e F^e_{\parallel j} = \int d^3u \, m_e u_\parallel L^{(\alpha)}_j(\kappa) \times \left(C^{ee}\left[\xi f_{e1}, F_{eMJ}\right] + C^{ee}\left[F_{eMJ}, \xi f_{e1}\right] + C^{ei}\left[\xi f_{e1}, F_{iMJ}\right]\right). \quad (11)$$

The approximate collision operator in Eq. (9) with f_{e1} from Eq. (4) and C^e_1 from Eq. (10) preserves the property of the momentum conservation.

Using f_{e1} defined in Eq. (4), one can find that the integrals in Eq. (10) are well defined and can be represented as a series through the parallel fluxes,

$$n_e F^e_{\parallel i} = \sum_j a_j V^e_{\parallel j}\left[\frac{n_e}{\tau_{ee}}\left(M^{ee}_{ij} + N^{ee}_{ij}\right) + \frac{n_e}{\tau_{ei}} M^{ei}_{ij}\right], \quad (12)$$

with the transport matrix coefficients

$$M^{ee}_{ij} = \frac{\tau_{ee}}{n_e}\int d^3u \, u_\parallel L^{(\alpha)}_i C^{ee}\left[\frac{m_e u_\parallel}{T_e} L^{(\alpha)}_j w F_{eMJ}, F_{eMJ}\right],$$
$$N^{ee}_{ij} = \frac{\tau_{ee}}{n_e}\int d^3u \, u_\parallel L^{(\alpha)}_i C^{ee}\left[F_{eMJ}, \frac{m_e u_\parallel}{T_e} L^{(\alpha)}_j w F_{eMJ}\right],$$
$$M^{ei}_{ij} = \frac{\tau_{ei}}{n_e}\int d^3u \, u_\parallel L^{(\alpha)}_i C^{ei}\left[\frac{m_e u_\parallel}{T_e} L^{(\alpha)}_j w F_{eMJ}, F_{iM}\right], \quad (13)$$

where τ_{ab} is the collision time for corresponding particles.

From the momentum conservation these matrix coefficients satisfy the relation $M^{ee}_{i0} + N^{ee}_{i0} = 0$. The electron-ion collisions are considered in $m_e/m_i \to 0$ limit. Again, in the non-relativistic limit these matrix coefficients turn into the well-known Braginskii matrix coefficients (see, for example, [6]).

As far as the matrix elements M^{ab}_{ij} and N^{ab}_{ij} can be directly calculated, one can turn the adjoint monoenergetic rDKE into the set of algebraic equations with respect to the parallel fluxes [2]. Let us introduce adjoint mono-energetic rDKE as

$$V(g_e) + \nu^e_D(u) L(g_e) = b v_\parallel F_{eMJ}. \quad (14)$$

Here, $V(g_e)$ is the relativistic Vlasov operator [1], $L(g_e)$ is the Lorentz operator, $b = B/B_0$ is the normalized magnetic field and $\nu^e_D(u) = \nu^{ee}_D(u) + \nu^{ei}_D(u)$ is the relativistic frequency of pitch angle scattering with $\nu^{ab}_D(u)$ given in [9]. Solution of this equation is determined by the relativistic mono-energetic transport coefficients, calculated in [2]. Multiplying Eq. (13) by f_e/F_{eMJ}, integrating in momentum space and then averaging over the magnetic flux surface, we can derive the expression:

$$\sum_j \left\{\langle b V^e_{\parallel j}\rangle \left(\delta_{ij} - \frac{2a_j}{\langle b^2\rangle u^2_{th}} \times \left[\left[\!\left[\gamma w \nu^e_D L^{(\alpha)}_i D^e_{33}\right]\!\right]_i + \sum_l c_{lj}\left[\!\left[\gamma w \nu^e_D L^{(\alpha)}_i D^e_{33}\right]\!\right]_i\right]\right)\right\} = -\left[\!\left[D^e_{31}\right]\!\right]_i A^e_1 + \left[\!\left[L^{(\alpha)}_i(\kappa) D^e_{33}\right]\!\right]_i A^e_2 - \left[\!\left[D^e_{31}\right]\!\right]_i A^e_3, \quad (15)$$

where $\langle \ldots \rangle$ is the averaging over magnetic flux surface, D^e_{mn} are the relativistic mono-energetic transport coefficients, which are depended on the collisionality, $\nu^* = R_0 \nu^e_D(u)/v$ (see [8]), A^e_n are the thermodynamic forces,

$$A^e_1 = \frac{p'_e}{p_e} - \frac{eE_r}{T_e}, \quad A^e_2 = \frac{T'_e}{T_e}, \quad A^e_3 = -\frac{e}{T_e}\frac{\langle E_\parallel B\rangle}{\langle B^2\rangle},$$

with electron pressure $p_e = n_e T_e$, electron density n_e, temperature of electrons T_e, radial and parallel electric fields, E_r and $E_\parallel$, respectively; the prime means the radial derivative and the angle brackets mean the averaging over the magnetic flux surface.

The coefficients c_{lj} are defined as

$$c_{lj} = a_j n_e \left[\frac{M^{ee}_{lj} + N^{ee}_{lj}}{\tau_{ee}} + \frac{M^{ei}_{lj}}{\tau_{ei}}\right]. \quad (16)$$

Here it is accounted that the mono-energetic transport coefficients satisfy the Onsager symmetry relations, $D_{31} = -D_{13}$. The operation of energy convolution with the relativistic Maxwellian [8] is defined as

$$\left[\!\left[\varphi(\kappa)\right]\!\right]_i = \frac{2}{\sqrt{\pi}} C_{MJ} \int d\kappa \, \kappa^{1/2} e^{-\kappa\gamma}\left(\frac{\gamma+1}{2}\right)^{1/2} L^{(\alpha)}_i \varphi(\kappa). \quad (17)$$

If the series in Eq. (4) are truncated after the second term ($i = 0, 1$), the heat and particles fluxes are directly defined.

2. CALCULATION OF MATRIX ELEMENTS

The differential part of relativistic collisional operator which describes the electron-electron collisions for the 1st Legendre harmonic is taken in the form:

$$C^{ee}\left[\xi f_{e1}, F_{eMJ}\right] = \frac{\xi}{u^2}\frac{\partial}{\partial u} u^2\left[D^{ee}_{uu}(u)\frac{\partial f_{e1}}{\partial u} - F^{ee}_u(u) f_{e1}\right] - \nu^{ee}_D(u)\xi f_{e1}, \quad (18)$$

where $D^{ee}_{uu}(u)$ and $F^{ee}_u(u)$ are diffusion and friction coefficients respectively, and $\nu^{ee}_D(u)$ is the pitch angle scattering frequency, which all together describe the corresponding processes of the test electrons over the background Maxwellian. The complete expressions for those one can find in [9]. This operator is necessary for matrix coefficients M_{ij}^{ee} in Eq. (13).

The part of collisional operator responsible for the collisions between electrons and ions can be taken in the Lorentz limit as:

$$C^{ei}[f_e, F_{iM}] = -\nu_D^{ei}(u)\xi f_{e1}, \quad (19)$$

where the pitch-angle scattering frequency $\nu_D^{ei}(u)$ is taken in $m_e/m_i \to 0$ limit,

$$\nu_D^{ei}(u) = \nu_{e0} Z_{eff} \frac{\gamma u_{te}^3}{u^3},$$

with $Z_{eff} = \sum_i n_i Z_i^2 / n_e$. The integral part of the relativistic collisional operator, deviation from equilibrium, f_e, is needed to account the momentum correction for electrons and used in matrix coefficient N_{ij}^{ee}. The complete expression as well can be found in [9].

Since $\mu_r = m_e c^2 / T_e$ is large, the weakly relativistic limit can be applied when the integrands in Eq. (13) are expanded in series over $1/\mu_r$. In this approach, the M_{ij}^{ab} and N_{ij}^{ab} matrix elements can be represented as a sum of the well-known non-relativistic part (not shown here; for definition, see, for example, [6]) plus the first order relativistic correction terms,

$$M_{ij}^{ab} = M_{ij}^{ab(0)} + \frac{1}{\mu_r}\delta M_{ij}^{ab},$$

$$N_{ij}^{ab} = N_{ij}^{ab(0)} + \frac{1}{\mu_r}\delta N_{ij}^{ab}. \quad (20)$$

For integration in Eqs. (13), the method of generating function (similar to [10]) was applied. In order to reduce the number of integrals, the generating function,

$$g(p,\kappa) = \frac{1}{(1-p)^{1+\alpha}}\exp\left(-\frac{p\kappa}{(1-p)}\right) = \sum_{n=0}^{\infty} p^n L_n^{(\alpha)},$$

has been used [11]. Replacing in Eq. (13) the polynomials $L_i^{(\alpha)}(\kappa)$ by $g(p,\kappa)$ and $L_j^{(\alpha)}(\kappa)$ by $g(q,\kappa)$, the results can be expanded then in series of $p^i q^j$ with $i, j = 0, 1, 2$, and the coefficients of this series corresponds to the desired transport coefficients. The integration is performed by the Mathematica package.

Finally, the relativistic corrections for electron-electron transport coefficients M_{ij}^{ee} have been calculated. Since the final expressions for the transport coefficients are bulky, only numerical evaluation of the first relativistic correction in shown:

$$\delta M_{00}^{ee} \approx 6.55;\ \delta M_{01}^{ee} \approx -6.85;\ \delta M_{02}^{ee} \approx -0.43;$$
$$\delta M_{10}^{ee} \approx 2.96;\ \delta M_{11}^{ee} \approx 19.56;\ \delta M_{12}^{ee} \approx -17.80;$$
$$\delta M_{20}^{ee} \approx 2.22;\ \delta M_{21}^{ee} \approx 12.23;\ \delta M_{22}^{ee} \approx 33.93.$$

Similarly, one can calculate the transport coefficients N_{ij}^{ee}, which are related to the integral part of for electron-electron collisions operator:

$$\delta N_{00}^{ee} \approx -5.76;\ \delta N_{01}^{ee} \approx 1.17;\ \delta N_{02}^{ee} \approx 2.92;$$
$$\delta N_{10}^{ee} \approx 1.52;\ \delta N_{11}^{ee} \approx -8.43;\ \delta N_{12}^{ee} \approx -4.89;$$
$$\delta N_{20}^{ee} \approx 5.06;\ \delta N_{21}^{ee} \approx -4.33;\ \delta N_{22}^{ee} \approx -8.37.$$

Finally, the transport coefficients for electron-ion collisions are calculated:

$$\delta M_{00}^{ei} \approx 4.83;\ \delta M_{01}^{ei} \approx 3.50;\ \delta M_{02}^{ei} \approx 3.44;$$
$$\delta M_{10}^{ei} \approx 3.50;\ \delta M_{11}^{ei} \approx 10.08;\ \delta M_{12}^{ei} \approx 6.31;$$
$$\delta M_{20}^{ei} \approx 3.44;\ \delta M_{21}^{ei} \approx 6.31;\ \delta M_{22}^{ei} \approx 13.94;$$

A practical importance of the relativistic corrections for transport coefficients in fusion plasmas can be demonstrated straightforwardly. Let us estimate the corrections for T_e = 25 keV, i.e. $\mu_r \approx 20$, expected as the typical value for thermonuclear reactors. Then, the relativistic correction terms in electron-electron transport coefficients differs from the classical ones by amount:

$$\frac{1}{\mu_r}\delta M_{00}^{ee} \approx 0.45\, M_{00}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{00}^{ee} \approx 0.40\, N_{00}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{01}^{ee} \approx 0.63\, M_{01}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{01}^{ee} \approx 0.11\, N_{01}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{10}^{ee} \approx 0.27\, M_{10}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{10}^{ee} \approx 0.14\, N_{10}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{02}^{ee} \approx 0.06\, M_{02}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{02}^{ee} \approx 0.43\, N_{02}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{20}^{ee} \approx 0.33\, M_{20}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{20}^{ee} \approx 0.75\, N_{20}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{11}^{ee} \approx 0.37\, M_{11}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{11}^{ee} \approx 0.35\, N_{11}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{12}^{ee} \approx 0.38\, M_{12}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{12}^{ee} \approx 0.19\, N_{12}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{21}^{ee} \approx 0.26\, M_{21}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{21}^{ee} \approx 0.17\, N_{21}^{ee(0)};$$
$$\frac{1}{\mu_r}\delta M_{22}^{ee} \approx 0.29\, M_{22}^{ee(0)};\quad \frac{1}{\mu_r}\delta N_{22}^{ee} \approx 0.23\, N_{22}^{ee(0)};$$

i.e. typical relativistic correction term for matrix coefficients is found to be noticeable compare to the respective non-relativistic value.

The same procedure applied for the electron-ion transport coefficients leads to the following relativistic correction terms:

$$\frac{1}{\mu_r}\delta M_{00}^{ei} \approx 0.24\, M_{00}^{ei(0)},$$
$$\frac{1}{\mu_r}\delta M_{01}^{ei} \approx 0.11\, M_{01}^{ei(0)},$$
$$\frac{1}{\mu_r}\delta M_{02}^{ei} \approx 0.09\, M_{02}^{ei(0)},$$
$$\frac{1}{\mu_r}\delta M_{11}^{ei} \approx 0.15\, M_{11}^{ei(0)},$$
$$\frac{1}{\mu_r}\delta M_{12}^{ei} \approx 0.07\, M_{12}^{ei(0)},$$
$$\frac{1}{\mu_r}\delta M_{22}^{ei} \approx 0.10\, M_{22}^{ei(0)},$$

i.e. for T_e = 25 keV the relativistic corrections for electron-ion collisions are approximately in the 10…20% range. Note, that in electron-ion collisions only the electrons are relativistic particles.

CONCLUSIONS

In this paper, the moment-equation technique, previously developed for non-relativistic plasmas [3-5], was adapted to use in the relativistic approach. This technique extends a range of applicability of the neoclassical transport theory for correct calculation of the electron parallel fluxes in high temperature plasmas. The weakly relativistic limit seems to be sufficient for applications in reactor plasmas with the electron temperature about several tens keV. It is shown in the paper that the system of linear equations obtained for the parallel fluxes of particles, $\langle bV_{\parallel 0}^e \rangle = \Gamma_\parallel^e / n_e$, and heat, $\langle bV_{\parallel 1}^e \rangle = -q_\parallel^e / n_e T_e$, can be solved directly, without calculation of the distribution function, using only the mono-energetic transport coefficients. Note, that relativistic mono-energetic transport coefficients can be pre-calculated by any non-relativistic solver [8]. As the main result of the paper, the first order relativistic correction terms for Braginskii matrix elements have been calculated.

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РЕЛЯТИВИСТСКИЕ НЕОКЛАССИЧЕСКИЕ МАТРИЧНЫЕ КОЭФФИЦИЕНТЫ С СОХРАНЕНИЕМ ИМПУЛЬСОВ

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Метод сохранения продольных импульсов обобщён для описания слаборелятивистских электронов. Это необходимо для правильного вычисления продольных неоклассических потоков и, в частности, для бутстреп-тока при термоядерных температурах. Показано, что полученная система линейных алгебраических уравнений для продольных потоков может быть решена непосредственно, без вычисления функции распределения, если релятивистские моноэнергетические коэффициенты уже известны. Получены численные значения для первой релятивистской поправки для вычисления матричных коэффициентов Брагинского.

РЕЛЯТИВІСТСЬКІ НЕОКЛАСИЧНІ МАТРИЧНІ КОЕФІЦІЄНТИ ЗІ ЗБЕРІГАННЯМ ІМПУЛЬСІВ

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Метод зберігання поздовжніх імпульсів узагальнено для опису слаборелятивістських електронів. Це необхідно для правильного обчислення поздовжніх неокласичних потоків і, зокрема, для бутстреп-струму при термоядерних температурах. Показано, що отримана система лінійних алгебраїчних рівнянь для поздовжніх потоків може бути вирішена безпосередньо, без обчислення функції розподілу, якщо релятивістські моноенергетичні коефіцієнти вже відомі. Отримано числові значення для першої релятивістської поправки для обчислення матричних коефіцієнтів Брагінського.