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Radiation power spectral distribution for two electrons moving in magnetic fields

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Abstract. Integral expressions for spectral distributions of the radiation power for systems of non-interacting point charged particles moving on arbitrary trajectory in electromagnetic fields in isotropic transparent media and in vacuum are investigated using the Lorentz self-interaction method. Special attention is given to the research of the fine structure of the synchrotron radiation spectral distribution of two electrons spiraling in vacuum in a relativistic case. The spectra of synchrotron, Cherenkov and synchrotron-Cherenkov radiations for a single electron are analyzed.

Keywords: Cherenkov radiation, synchrotron radiation, synchrotron-Cherenkov radiation, Lorentz self-interaction.

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1. Introduction

Investigations of the radiation spectra of charged particles moving in magnetic fields in transparent isotropic media and vacuum are important from the viewpoint of their applications in electronics, astrophysics, plasma physics, physics of storage rings, *etc.* [1–3]. When charged particles move in magnetic field, three kinds of radiation are possible in a medium [4-5]: synchrotron, Cherenkov, and synchrotron-Cherenkov ones whereas in vacuum only synchrotron radiation takes place.

A question calling for further investigations is the coherence of synchrotron radiation [6-7]. Investigations of the fine structure of synchrotron, Cherenkov, and synchrotron-Cherenkov radiation spectra in vacuum and transparent media for the low-frequency spectral range are of great interest, too [4-7].

Using the exact integral relationships for the spectral distribution of radiation power of two electrons spiraling one after another in vacuum, the fine structure of the synchrotron radiation spectrum in relativistic case was investigated by analytical and numerical methods. The Doppler effect influence on peculiarities of the radiation spectrum of a single electron spiraling in transparent media and vacuum is investigated.

2. Instantaneous and time-averaged radiation powers of charged particles

The instantaneous radiation power of charged particles $P^{rad}(t)$ in an isotropic transparent medium and in vacuum is expressed in [8, 9] as

$$P^{rad}(t)= =\int_{\tau}\left(\vec{j}(\vec{r},t)\frac{1}{c}\frac{\partial\vec{A}^{Dir}(\vec{r},t)}{\partial t}-\rho(\vec{r},t)\frac{\partial\varphi^{Dir}(\vec{r},t)}{\partial t}\right)d\vec{r}\,. \qquad (1)$$

Here $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [8-10], the scalar $\varphi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$\varphi^{Dir}=\frac{1}{2}\left(\varphi^{ret}-\varphi^{adv}\right),\ \vec{A}^{Dir}=\frac{1}{2}\left(\vec{A}^{ret}-\vec{A}^{adv}\right). \qquad (2)$$

After substituting (2) into (1) we obtain the relationship for instantaneous radiation power of charged particles moving in isotropic transparent media as a function of spectral distribution

$$P^{rad}(t)=\int_0^{\infty}d\omega W(t,\omega), \qquad (3)$$

$$W(t,\omega)=\frac{1}{\pi c^2}\int_{-\infty}^{\infty}d\vec{r}\int_{-\infty}^{\infty}d\vec{r}\,'\int_{-\infty}^{\infty}dt'\omega\mu(\omega)\times$$

$$\times\frac{\sin\left[\frac{n(\omega)\omega}{c}|\vec{r}-\vec{r}\,'|\right]}{|\vec{r}-\vec{r}\,'|}\cos\omega(t-t')\times \qquad (4)$$

$$\times\left\{\vec{j}(\vec{r},t)\vec{j}(\vec{r}\,',t')-\frac{c^2}{n^2(\omega)}\rho(\vec{r},t)\rho(\vec{r}\,',t')\right\},$$

where $\mu(\omega)$ is the magnetic permeability, $n(\omega)$ is the refraction index, ω is the cyclic frequency, and c is the velocity of light in vacuum.

The time-averaged radiation power of charged particles is defined by the expression

$$\bar{P}^{rad} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} P^{rad}(t)dt \,. \tag{5}$$

It can be obtained after substitution of the instantaneous radiation power expressed by relationships (3) and (4) into (5).

3. Systems of non-interacting point charged particles

According to [4, 5], the source functions of N charged point particles are defined as

$$\vec{j}(\vec{r},t) = \sum_{l=1}^{N} \vec{V}_l(t)\rho_l(\vec{r},t), \;\; \rho(\vec{r},t) = \sum_{l=1}^{N} \rho_l(\vec{r},t),$$

$$\rho_l(\vec{r},\, t) = e\;\delta(\vec{r} - \vec{r}_l(t)), \tag{6}$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively.

Let us consider a system of point non-interacting charged particles ($q_l = e$, $m_{0l} = m_0$) moving one by one along an arbitrary defined trajectory. Then the motion law and the velocity of the j^{th} particle of this system are determined by the relationships

$$\vec{r}_j(t) = \vec{r}_p(t + \Delta t_j), \;\; \vec{V}_j(t) = \vec{V}(t + \Delta t_j). \tag{7}$$

Substituting the relationships (6) and (7) into (3) and (5) we obtain the expression for the averaged of charged particles system in transparent media (magnetic permeability $\mu(\omega)$ and dielectric permittivity $\varepsilon(\omega)$ are real):

$$\bar{P}^{rad} = \frac{e^2}{\pi c^2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \int_{-\infty}^{\infty} dt' \int_{0}^{\infty} d\omega \mu(\omega) \omega S_N(\omega) \times$$

$$\times \frac{\sin\left\{\frac{n(\omega)}{c}\omega\left|\vec{r}_p(t) - \vec{r}_p(t')\right|\right\}}{\left|\vec{r}_p(t) - \vec{r}_p(t')\right|} \cos\omega(t - t') \times \tag{8}$$

$$\times \left[\vec{V}(t)\,\vec{V}(t') - \frac{c^2}{n^2(\omega)}\right],$$

$$S_N(\omega) = \sum_{l,j=1}^{N} \cos\{\omega(\Delta t_l - \Delta t_j)\}. \tag{9}$$

The coherence factor $S_N(\omega)$ determines a redistribution of the charged particles radiation power between harmonics.

We study N electrons spiraling one by one in magnetic fields in transparent media. The law of motion and the velocity of the j-th electron are given by the expressions

$$\vec{r}_j(t) = r_0 \cos\{\omega_0(t + \Delta t_j)\}\vec{i} \, +$$

$$+ r_0 \sin\{\omega_0(t + \Delta t_j)\}\vec{j} + V_{\|}(t + \Delta t_j)\,, \vec{V}_j(t) = \frac{d\vec{r}_j(t)}{dt}. \tag{10}$$

Here $r_0 = V_{\perp}\omega_0^{-1}$, $\omega_0 = ceB^{ext}\tilde{E}^{-1}$, $\tilde{E} = c\sqrt{p^2 + m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\,\|0Z$, $V_{\perp}$ and $V_{\|}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass.

We obtain the time-averaged radiation power of N electrons on substitution of the expressions (10) into (8). Then

$$\bar{P}^{rad} = \int_0^{\infty} W(\omega)d\omega\,, \tag{11}$$

$$W(\omega) = \frac{2e^2}{\pi c^2} \int_0^{\infty} dx \mu(\omega) \omega S_N(\omega) \frac{\sin\left\{\frac{n(\omega)}{c}\omega\eta(x)\right\}}{\eta(x)} \times$$

$$\times \cos\omega x \left[V_{\perp}^2 \cos(\omega_0 x) + V_{\|}^2 - \frac{c^2}{n^2(\omega)}\right], \tag{12}$$

where $\eta(x) = \sqrt{V_{\|}^2 x^2 + 4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_N(\omega)$ of N electrons is defined as (9).

4. Fine structure of the radiation spectra of two electrons moving along a spiral in vacuum

Peculiarities of the radiation spectra of two electrons moving one by one in a spiral in vacuum can be investtigated combining analytical and numerical methods.

The time-averaged radiation power of two electrons we can obtain from expressions (11) and (12). Then

$$\bar{P}^{rad} = \int_0^{\infty} W(\omega)d\omega\,, \tag{13}$$

$$W(\omega) = \frac{2e^2}{\pi c^2} \int_0^{\infty} dx \omega S_2(\omega) \frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)} \times$$

$$\times \cos\omega x \left[V_{\perp}^2 \cos(\omega_0 x) + V_{\|}^2 - c^2\right], \tag{14}$$

where $\eta(x) = \sqrt{V_{\|}^2 x^2 + 4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

The coherence factor $S_2(\omega)$ of two electrons is defined as

$$S_2(\omega) = 2 + 2\cos(\omega\Delta t). \tag{15}$$

Here $\Delta t = \Delta t_2 - \Delta t_1$ is the time shift of the electrons moving along a spiral. The analogous expression for the coherence factor was investigated by Bolotovskii [11].

From relationships (13) and (14) on some transformations the contributions of separate harmonics to the averaged radiation power can be written as

$$\overline{P}^{rad} = \frac{e^2}{c^3}\sum_{m=1}^{\infty}\int_0^{\infty} d\omega\omega^2 \int_0^{\pi} \sin\theta d\theta 2\left[1+\cos\{\omega(\Delta t)\}\right]\times$$

$$\times\,\delta\left\{\omega\left(1-\frac{1}{c}V_{\parallel}\cos\theta\right)-m\omega_0\right\}\times \tag{16}$$

$$\times\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_{\parallel}^2-c^2\right)J_m^2(q)\right\},$$

where $q=\frac{V_{\perp}}{c}\frac{\omega}{\omega_0}\sin\theta$, $J_m(q)$ and $J'_m(q)$ are the Bessel function with integer index and its derivative, respectively.

Each harmonic is a set of the frequencies, which are the solution of the equation

$$\omega\left(1-\frac{1}{c}V_{\parallel}\cos\theta\right)-m\omega_0=0\,. \tag{17}$$

The limits of the m^{th} harmonic are determined by the frequencies

$$\omega_m^{\min}=\frac{m\omega_0}{1+\frac{V_{\parallel}}{c}},\quad \omega_m^{\max}=\frac{m\omega_0}{1-\frac{V_{\parallel}}{c}}, \tag{18}$$

and the total radiation power emitted by a separate electron is determined according to [12] as

$$P_{vac}^{tot}=\frac{2}{3}\frac{e^2}{c^3}\omega_0^2V_{\perp}^2\left(1-\frac{V^2}{c^2}\right)^{-2}, \tag{19}$$

where $\omega_0=\frac{eB^{ext}}{m_0c}\sqrt{1-\frac{V^2}{c^2}}$.

Our numerical calculations of the radiation power spectral distribution were performed at $B^{\text{ext}}=1$ Gs.

For the velocities components $V_{\perp\text{vac}}=0.4713c$ and $V_{\parallel\text{vac}}=0.3333c$ the radiation power spectral distributions of two electrons in dependence of their location along a spiral are shown in Fig. 1 (curves 1–3) and magnitudes of radiation power are presented in Table 1.

It is interesting to compare the radiation power spectral distribution for two electrons with that of a single electron (curve 0 in Fig. 1a). The radiation power of a single electron in vacuum $P_{\text{vac0}}^{\text{tot}}=0.529\cdot10^{-15}$ erg/s calculated according to the relationship (19) is in good agreement with the power $P_{\text{vac0}}^{\text{int}}=0.526\cdot10^{-15}$ erg/s xetermined on integration of the relationships (13) and (14). For the time difference $\Delta t_1=0.001\pi/\omega_{01}$ (curve 1 in Fig. 1a) the coherence factor $S_2(\omega)=4$, and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e., by a factor of four more intensively than the single electron.

For the time difference $\Delta t_2=\pi/\omega_{02}$ (curve 2 in Fig. 1b), we have found the peaks of the spectral distribution function appoximately at the frequencies $2i\omega_{02}$, i = 1, 2, 3, 4 whereas the radiation was absent at the frequencies $(2i-1)\,\omega_{02}$, i = 1, 2, 3, 4.

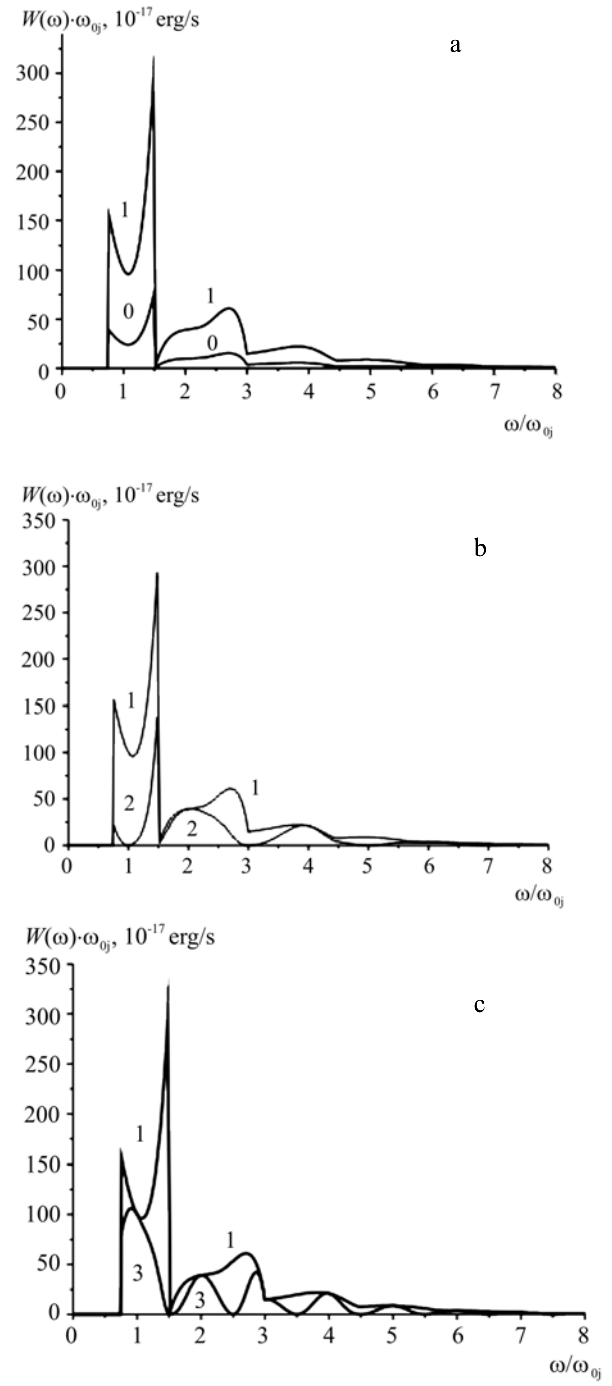

Fig. 1. Spectral distribution of radiation power for a single electron (curve 0) and two electrons spiraling one by one at: a) $\Delta t_1=0.001\pi/\omega_{01}$ (curve 1); b) $\Delta t_2=\pi/\omega_{02}$ (2); c) $\Delta t_3=$ $=2\pi/\omega_{03}$ (3).

Table 1. Radiation power for two electrons moving one by one in a spiral in relativistic case (B^{ext} = 1 Gs, ω_{0j} = 14.36·10^6 rad/s, r_{0j} = 984 cm, j = 1, 2, 3, c = 2.997925·10^{10} cm/s).

Curve j	Δt_j	$V_{\perp vac}$	$V_{\parallel vac}$	$P_{vacj}^{\rm int}$, 10^{-15} erg/s
1	$\frac{0.001\pi}{\omega_{01}}$	$0.47\cdot c$	$0.33\cdot c$	2.113
2	π/ω_{02}	$0.47\cdot c$	$0.33\cdot c$	0.7742
3	$2\pi/\omega_{03}$	$0.47\cdot c$	$0.33\cdot c$	1.046

For the time difference $\Delta t_3 = 2\pi/\omega_{03}$ (curve 3 in Fig. 1c), we have found the peaks of the spectral distribution function appoximatelly at the frequencies $i\,\omega_{03}$, i = 1, 2, …, 8, and the radiation was absent at the frequencies $\left(\frac{1}{2}+i\right)\omega_{03}$, i = 1, 2, …, 8.

The dependence of the radiation power magnitude for two electrons moving one by one in dependence of their location in a spiral is presented in Fig. 2. With increasing Δt, the radiation power of the system of two charges tends to double radiation power of a single charge.

5. Spectral distribution of synchrotron-Cherenkov radiation power in low-frequency range

Let us consider the Doppler effect influence on synchrotron-Cherenkov radiation in transparent media. The expressions for the synchrotron-Cherenkov radiation power in such a medium can be obtained starting from (13). Then for the single electron spiraling we have found [3, 5]

$$\overline{P}^{\rm rad} = \int_0^\infty W(\omega)d\omega\,, \tag{20}$$

$$W(\omega) = \frac{2e^2}{\pi c^2}\int_0^\infty dx\,\mu(\omega)\omega\frac{\sin\left\{\frac{n(\omega)\omega}{c}\eta(x)\right\}}{\eta(x)}\times$$
$$\times\cos(\omega x)\left[V_\perp^2\cos(\omega_0 x)+V_\parallel^2-\frac{c^2}{n^2(\omega)}\right], \tag{21}$$

where $\eta(x) = \sqrt{V_\parallel^2 x^2 + 4\frac{V_\perp^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$.

For the case of low-frequency spectral range, i.e., at $\varepsilon = \text{const}$ and $\mu = 1$, the power of the Cherenkov radiation at rectilinear motion in a medium (n is the constant) is determined as [5]:

$$P_{\rm ch}^{\rm tot} = \frac{e^2}{2c^2}V\,\omega_{\max}^2\left(1-\frac{c^2}{V^2n^2}\right). \tag{22}$$

For the refraction index n = 2 at the velocities $V_{\perp\rm med} = 0.15.\ 10^{10}$ cm/s = $0.05003c$ and $V_{\parallel\rm med}$ = $= 0.1493\cdot10^{11}$ cm/s = $0.4981c$ as well as $V_{\perp\rm med}$ = $= 0.9\cdot10^9$ cm/s $= 0.03002c$ and $V_{\parallel\rm med} = 0.1498\cdot10^{11}$ cm/s $= 0.4997c$ (curves 5 and 6 in Fig. 3, respectively) the conditions for the existence of synchrotron-Cherenkov radiation are fulfilled. The spectral distribution for these two cases is shown in Fig. 3. The upper boundary of the first harmonic band in curve 5 is located at the frequency $\omega_1^{\max 5} = 265\omega_{05}$ and for curve 6 it is at the frequency $\omega_1^{\max 6} = 1852\omega_{06}$. The values of the synchrotron-Cherenkov radiation power are listed in Table 2.

The power of the Cherenkov radiation at rectilinear motion $P_{\rm ch7}^{\rm tot} = 0.6979\cdot10^{-11}$ erg/s (relation (22)) is in good agreement to the synchrotron-Cherenkov radiation

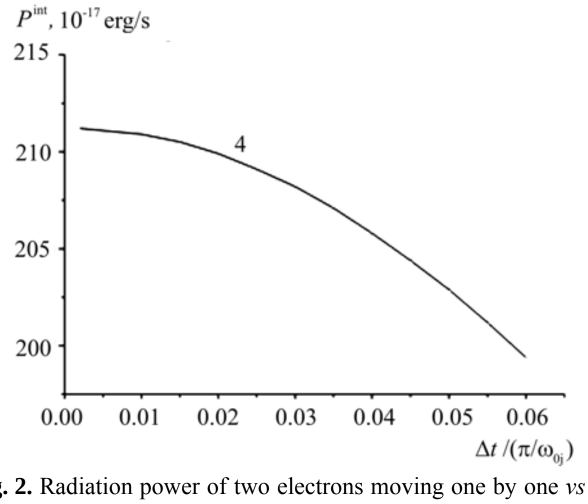

Fig. 2. Radiation power of two electrons moving one by one *vs* their location in a spiral for $V_{\perp\rm vac} = 0.4713c$ and $V_{\parallel\rm vac}$ = $= 0.3333c$.

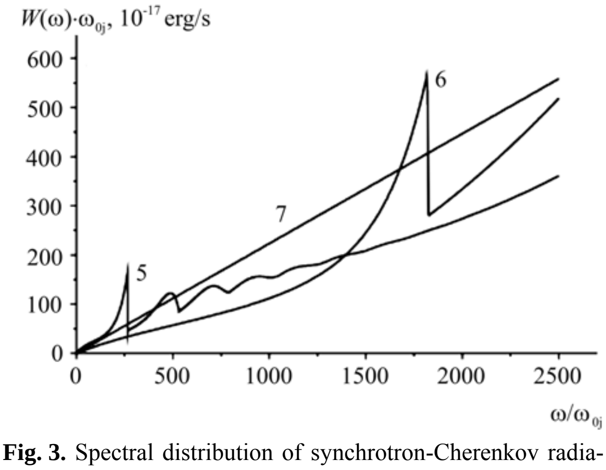

Fig. 3. Spectral distribution of synchrotron-Cherenkov radiation power at n = 2. Curve 5 – $V_{\perp\rm med} = 0.05003c$ and $V_{\parallel\rm med}$ = $= 0.4981c$, curve 6 – $V_{\perp\rm med} = 0.03002c$ and $V_{\parallel\rm med} = 0.4997c$, curve 7 – $V_{\perp\rm med} = 0.00033c$ and $V_{\parallel\rm med} = 0.5006c$.

Table 2. Synchrotron-Cherenkov radiation power of the electron at B^{ext} = 1 Gs, (n = 2, B^{ext} = 1 Gs, ω_{0j} = 15.23·10⁶ rad/s).

Curve j	$V_{\perp \mathrm{med}}$	$V_{\parallel \mathrm{med}}$	r_{0j}, cm	$P^{\mathrm{int}}_{\mathrm{med}j}$, 10^{-11} erg/s
5	0.05003 c	0.4981c	98.5	0.4688
6	0.03002c	0.4997c	59.1	0.5266
7	0.00033c	0.5006c	0.6	0.6992

power $P^{\mathrm{int}}_{\mathrm{med}7} = 0.6992 \cdot 10^{-11}$ erg/s calculated according to the relationships (20) and (21) at the motion of the charged particle having a small transverse velocity component (the absolute values of the velocities are the same).

The synchrotron-Cherenkov radiation is the unified process with interesting properties [2-7, 13]. The analytical and numerical calculations showed that the Doppler effect influence on the peculiarities of the radiation power spectral distribution of the electrons were essential near the Cherenkov threshold.

6. Conclusions

In the radiation spectrum of charged particles the Doppler effect establishes the limits between the bands of separate harmonics.

The synchrotron-Cherenkov radiation is the unified process with interesting properties. The influence of the Doppler effect on the peculiarities of the spectral distribution of the electron radiation power in a medium is significant only nearby the Cherenkov threshold.

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