**CONDENSED
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Quantum statistical mechanics of electron gas in magnetic field

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Electron eigenstates in a magnetic field are considered. Density of the electrical current and an averaged magnetic moment are obtained. Density of states is investigated for two-dimensional electron in a circle that is bounded by the infinite potential barrier. The present study shows that the common quantum statistical mechanics of electron gas in a magnetic field leads to incorrect results. The magnetic moment of electron gas can be computed as the sum of averaged moments of the occupied states. The computations lead to the results that differ from the ones obtained as the derivative of the thermodynamical potential with respect to the magnetic field. Other contradictions in common statistical thermodynamics of electron gas in a magnetic field are pointed out. The conclusion is done that these contradictions arise from using the incorrect statistical operator. A new quantum function of distribution is derived from the basic principles, taking into account the law of conservation of an angular momentum. These results are in accord with the theory that has been obtained within the framework of classical statistical thermodynamics in the previous work.

Key words: *electron states, magnetic field, angular momentum, averaged magnetic moment, quantum function of distribution, quantum statistical thermodynamics*

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1. Introduction

It is generally agreed that the task mentioned in the title had been resolved by L.D. Landau in work [1], the result of which is known as "the Landau diamagnetism". The energy spectrum discreteness was originally obtained in this work. The following consideration was based on quasi-classical conceptions. It is assumed that an electron is localized in an orbit of classical circular motion. The eigenvalue of the circle area is proportional to the energy eigenvalue. One from the orbit center coordinates also has definite value in the eigenstate. An electron in an orbit creates a localized magnetic moment. The orbit centers are homogeneously distributed over the area bound by infinite potential barrier. These conceptions form the basis of numerous investigations concerning electron theory of conductors (see [2–4], and the works cited there). The thermodynamical characteristics of an electron gas in a magnetic field were also computed using these conceptions and the common quantum function of distribution. The latter follows from the quantum statistical operator that depends only on the Hamiltonian. Therefore the main term of thermodynamical potential does not depend on a magnetic field, as it is observed in the classical Bohr – van Leeuwen theorem. The averaged magnetic moment was computed as the derivative of thermodynamical potential with respect to the magnetic field. It was obtained from a small term generated due to the energy spectrum discreteness.

It was shown in work [5] that the classical Bohr – van Leeuwen theorem is incorrect because the common density of distribution that depends only on the Hamiltonian is inappropriate in the case of a charge particle gas in a magnetic field. In this case the density of distribution also depends on the component of the full angular momentum that is parallel to the magnetic field. Then the considered system should be inhomogeneous. The gas density should decrease with the distance from the mass center. There are diamagnetic currents that envelop the mass center and generate the diamagnetic moment.

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The first purpose of the present study is to show that the quasi-classical conceptions and the common quantum function of distribution lead to inconsistent results when applied to an electron gas in a magnetic field. It is known that the wave functions localized near the classical orbits (the coherent states described in work [6]) are not the eigenfunctions of the Hamiltonian. In the second section of this paper the eigenstates of one-particle Hamiltonian are under investigation. It is shown that these states give the probability current lines that envelop the zero of the coordinate system. The averaged value of the magnetic moment is always non-zero and negative in these states. The energy spectrum is obtained with zero boundary condition on a large-radius circle. This could not be done in work [1] because some theorems concerning zeros of the degenerate hypergeometric function were obtained only in the forties. A complementary formulae about these zeros are obtained in this work (section 5). In the third section it is shown that an electron gas in a magnetic field is a non-homogeneous system. Then there is no sense passing to the thermodynamical limit and to the densities of the extensive quantities. The magnetic moment of a finitesimal system is computed by the conventional method with the application of the results obtained in the second section. It is very different from the result of [1] and from the experimental data. Usually the averaged magnetic moment was computed as the derivative of the averaged energy or the thermodynamical potential. It is proved that this formula is incorrect in general case. Other contradictions in common statistical thermodynamics of an electron gas in a magnetic field are also pointed out.

The second purpose of the present study is to derive the quantum function of distribution for an electron gas in a magnetic field from the basics taking into account the conservation of the angular momentum. This is done in section four. The averaged magnetic moment of the ideal gas in a magnetic field without potential fields is computed. The qualitative investigation of the effect of an external field is made.

2. Electron eigenstates in circle perpendicular to magnetic field

The probability current in a magnetic field is proportional to $[\nabla\Phi - (e/c\hbar)\mathbf{A}]$ where $\Phi(x,y,z)$ is the phase of the wave function. A vector potential of the magnetic field $\mathbf{A}$ cannot be equal to gradient of a single-valued function. Consequently, the eigenfunctions cannot be taken so that the density of the probability current is identically equal to zero. Then every eigenstate must satisfy the requirement of closure of all lines of the probability current within the considered volume. That is why the choice of vector potential of a magnetic field is not arbitrary. In work [1] the vector potential was chosen so that the lines of the probability current are parallel to the axis of the Cartesian coordinate system that is perpendicular to the magnetic field. The choice of the vector potential is appropriate for the case at which the considered system has a transport current. In work [1] eigenstates were obtained by imposing the cyclic boundary condition upon the planes perpendicular to this axis. This is the same as to obtain the approximate eigenfunctions in a circular ring with a large radius and a small width. The computation of the average magnetic moment can be incorrect in this approximation. An axially symmetrical vector potential is only appropriate in the case of an isolated system. Then the probability current lines of the eigenstates are circles. Such current cannot lead to alternation of the probability density with time.

The electron Hamiltonian with this vector potential is as follows:

$$\widehat{\mathcal{H}} = \frac{1}{2m}\left[\left(\widehat{p_x} - \frac{eH}{2c}y\right)^2 + \left(\widehat{p_y} + \frac{eH}{2c}x\right)^2\right] + u(|\mathbf{r}|) = \widehat{\mathcal{H}}_e + \beta H\widehat{L_z}. \qquad (1)$$

Here

$$\widehat{\mathcal{H}}_e = \frac{1}{2m}\left(\widehat{p_x}^2 + \widehat{p_y}^2\right) + \frac{m}{2}\left(\frac{\beta H}{\hbar}\right)^2|\mathbf{r}|^2 + u(|\mathbf{r}|), \qquad \hbar\widehat{L_z} = x\widehat{p_y} - y\widehat{p_x}. \qquad (2)$$

The operators $\widehat{\mathcal{H}}_e$ and $\widehat{L_z}$ commute with each other and each of them with the Hamiltonian. For simplicity we shall restrict ourselves to the case of two degrees of freedom on a $\mathbf{XY}$ plane perpendicular to the uniform magnetic field $\mathbf{H} = (0,0,H)$. The charge of an electron is $-e$, the mass of one is m, c is the speed of light in a vacuum, β is the Bohr magneton. The potential

energy $u(|\mathbf{r}|)$ is created by the interaction with other electrons and with a neutralizing background and involves the boundary barrier. In what follows it is assumed that the potential energy of the Coulomb interaction can be described by the self-consistent field in the electron gas that is in the equilibrium state. In this section we assume that the self-consistent field is exactly equal to zero, and the boundary barrier is infinite just like in the conventional theory. These assumptions will be discussed in section four.

The solutions of the time-independent two-dimensional Schrödinger equation for an electron in a magnetic field are (see [7]):

$$\psi_{\varepsilon,l}(r,\varphi) = \frac{A(\varepsilon,l)}{\lambda} \exp(\mathrm{i}l\varphi) \exp\left(-\frac{r^2}{2\lambda^2}\right) \left(\frac{r}{\lambda}\right)^{|l|} \Phi\left(-\frac{\varepsilon}{2\beta H} + \frac{l+|l|+1}{2}, |l|+1; \frac{r^2}{\lambda^2}\right). \qquad (3)$$

We use the polar coordinates (r,φ), ε is the energy eigenvalue. The quantum number of an angular momentum l is the eigenvalue of the operator $\widehat{L_z}$. It takes any integer value. A normalizing factor is denoted as $A(\varepsilon,l)/\lambda$. The square of the magnetic length is described by the formula $\lambda^2 = 2c\hbar/|e|H$. The degenerate hypergeometric function (DHF) is written, as the convention, $\Phi(a,c;z)$. (In the monograph [7] the number l has an opposite sign in the first parameter of the DHF, which is a mistake.)

Let the wave functions be equal to zero beyond the circle with the radius R_0. The boundary condition $\psi(R_0,\varphi) \equiv 0$ can be satisfied only if the first parameter of the DHF labelled as a is negative $a < 0$. Then:

$$-\frac{\varepsilon}{2\beta H} + \frac{l+|l|+1}{2} = -N - \gamma(N,|l|), \qquad (4)$$

where $N \geqslant 0$ is integer, and $0 \leqslant \gamma < 1$. (On an infinite plane the functions can be normalized only when $\gamma = 0$).

The eigenvalues of the operator

$$\widehat{R^2} = \widehat{X_{\mathrm{c}}}^2 + \widehat{Y_{\mathrm{c}}}^2 = \frac{\lambda^2}{2\beta H}(\widehat{\mathcal{H}} - 2\beta H \widehat{L_z}) \qquad (5)$$

give the spectrum of the square distance of the classical orbit center from the zero of the coordinate system. The operators $\widehat{X_{\mathrm{c}}}$ and $\widehat{Y_{\mathrm{c}}}$ are the Cartesian coordinates of the classical orbit center. The last equality in (5) is obtained when these operators are expressed as the functions of the coordinates and the momenta of an electron (see [4,8]). All these operators commutate with Hamiltonian $\widehat{\mathcal{H}}$. The operators $\widehat{X_{\mathrm{c}}}$ and $\widehat{Y_{\mathrm{c}}}$ do not commutate with each other and $[\widehat{X_{\mathrm{c}}},\widehat{Y_{\mathrm{c}}}] = \mathrm{i}\lambda^2/2$. Hence, if one of the two coordinates of the orbit center is determined then the other one is undetermined. The fundamental point of the problem investigated lies in the fact that an electron cannot be localized in the determined classical orbit. From (5) it follows:

$$\widehat{L_z} = \frac{1}{\lambda^2}\left(\widehat{\rho^2} - \widehat{R^2}\right), \qquad (6)$$

where $\widehat{\rho^2} = (\lambda^2/2\beta H)\widehat{\mathcal{H}}$ is the operator of the square radius of the classical orbit. The spectrum of the operator $\widehat{R^2}$ differs from the spectrum of the operator $\widehat{\mathcal{H}}$ only by the same coefficient $\lambda^2/2\beta H$. The quantum number of the angular momentum can be positive only when $\rho > R$, i. e., for orbits that envelop the zero of the coordinate system. The angular momentum has the same property in classical mechanics.

In a state $\psi_{\varepsilon,l}(r,\varphi)$ only the φ-component of the density of the electrical current is non-zero and is described by formula:

$$j_\varphi = -\frac{e\hbar}{mr}|\psi_{\varepsilon,l}(r,\varphi)|^2 \left[l + \frac{r^2}{\lambda^2}\right]. \qquad (7)$$

The current lines are concentric circles. Their center and the symmetry point of the probability density is zero of coordinate system. This symmetry does not depend on the distance of the classical orbit center from zero of coordinate system.

We introduce the new quantum numbers n and k as was done in works [4] and [8].

$$N = \frac{n+k-|n-k|}{2} = \min(n,k), \quad l = n - k; \quad n = N + \frac{l+|l|}{2}, \quad k = N - \frac{l-|l|}{2}. \quad (8)$$

The numbers n, k take only integer non-negative values. They are the number of energy and the number of the orbit center distance from the origin of coordinates.

$$\varepsilon_{n,k} = 2\beta H[n + 1/2 + \gamma(n,k)], \qquad \rho_{n,k}^2 = \lambda^2[n + 1/2 + \gamma(n,k)],$$
$$R_{n,k}^2 = \lambda^2[k + 1/2 + \gamma(n,k)]. \quad (9)$$

Here $\rho_{n,k}$ is the radius of the orbit of a classical particle that has the same energy. The radial coordinate of the center of this orbit is $R_{n,k}$. The function $\gamma(n,k)$ is obtained by transformation $\gamma(N, |l|) \to \gamma(n,k)$ with the formulae (8). On the infinite plane the number n entirely determines the eigenvalue of the energy, and the number k does the same for the eigenvalue of the operator $\widehat{R^2}$ (see (5)). Functions (3) are eigenfunctions for each of these operators. Consequently each of these quantum numbers is degenerated. The function $\gamma(n,k)$ describes the removal of the degeneracy by the insertion of the boundary barrier. The other pairs of the quantum numbers will be also used in what follows.

The operator of the orbital magnetic moment is:

$$\widehat{m_z} = -\frac{e}{2c}(x\widehat{\dot{y}} - y\widehat{\dot{x}}) = -\beta\left(\widehat{L_z} + \frac{r^2}{\lambda^2}\right). \quad (10)$$

Using (6) we can see that the formula (10) is in accord with (7). It is necessary to use the formula (see [9]):

$$z\Phi(a,c;z) = a\Phi(a+1,c;z) - (2a-c)\Phi(a,c;z) + (a-c)\Phi(a-1,c;z) \quad (11)$$

for computing the averaged value of the magnetic moment (10) in the state $\psi_{n,l}(r,\varphi)$. Then we obtain:

$$\langle m_z \rangle_{n,l} = -\frac{\varepsilon_{n,l}}{H}. \quad (12)$$

This formula is in accord with the time average of the magnetic moment of a classical charged particle in a magnetic field.

Its result accidentally agrees with the result of the commonly used formula

$$\langle m_z \rangle_{n,l} = -\partial\varepsilon_{n,l}/\partial H \quad (13)$$

only in the absence of the boundary barrier. The formula (10) can be obtained as the result of the symbolical differentiation

$$\widehat{m_z} = -\left(\frac{\partial\widehat{\mathcal{H}}}{\partial H}\right)_{\text{symb}}, \quad (14)$$

where the operator (1) is differentiated as a customary function. This differentiation is symbolical because the real derivation of an operator with respect to a parameter is the derivation of its matrix. It is generally believed that the average value of the operator $(\partial\widehat{\mathcal{H}}/\partial H)_{\text{symb}}$ obtained by a symbolical differentiation is identically equal to the derivation of the operator $\widehat{\mathcal{H}}$ eigenvalue. This inference is not always true. If there is a potential energy $u(r)$, the formula (10) does not change. A potential energy can be such that it alters only the energy eigenvalue. This alteration can non-linearly depend on the magnetic field as it is in the case of the boundary barrier. In doing so the view of the eigenfunctions does not alter. As this takes place the formulae (12) and (13) should lead to the distinct results. The consequence of this mistake for statistical thermodynamics will be considered in section three.

The DHF has real positive zeros only when its first parameter a is negative $a < 0$. The number of these zeros is equal to the smallest integer that is greater than, or equal to $(-a)$. Let us take $a = -N - \gamma$. The DHF zeros are denoted as $\xi_j(N + \gamma, c)$, where j is the zero's number when the

zeros are arranged in the increasing order. If $j \leqslant N$, the zero $\xi_j(N + \gamma, c)$ decreases with increasing γ in the interval $[\zeta_{N+1,j}^{(c-1)}, \zeta_{N,j}^{(c-1)}]$, where $\zeta_{N,j}^{(c-1)}$ is the j numbered zero of the associated Laguerre polynomial $L_N^{(c-1)}(z)$. It is shown in section five that the largest zero of the function $\Phi(-N - \gamma, c; z)$ also depends on γ monotonously: $\xi_{N+1}(N+\gamma, c) \to \infty$ when $\gamma \to 0$, and $\xi_{N+1}(N+\gamma, c) \to \zeta_{N+1,N+1}^{(c-1)}$ when $\gamma \to 1$.

We consider the distribution of the eigenvalues in the interval of the energy values $[\beta H(2n + 1), \beta H(2n + 3)]$. The number of states in this interval is equal to the number of such k values at which the wave functions with fixed n can satisfy the boundary condition. The eigenvalues are classified by the zero of DHF $\Phi(-N - \gamma, |l| + 1; z)$ which coincides with the boundary value $z_0 = R_0^2 / \lambda^2$. From the beginning we investigate the spectrum when $n \ll z_0$. The maximum distance of the orbit center from the origin of coordinates in this case can be close to R_0. Therefore, the maximum k is much greater than n. It is evident from (8) that then $N = n$, $|l| = k - n \gg N$. In section five it is shown that when $\sqrt{k} \gg n$, the minimum zero and the maximum zero of the Laguerre polynomial $L_n^{(k-n)}(z)$ are described in the first order by the formulae:

$$\zeta_{n,1}^{(k-n)} \cong k - \sqrt{\frac{kn(n-1)}{2}}, \qquad \zeta_{n,n}^{(k-n)} \cong k + \sqrt{\frac{kn(n-1)}{2}}. \qquad (15)$$

All other zeros in this approximation are equal in the order of polynomial: $\zeta_{n,j}^{(k-n)} \cong k$. The Laguerre polynomial zeros increase when k increases and n is fixed. If at some k value we have $z_0 > \zeta_{n+1,n+1}^{(k-n)}$, the boundary condition can be satisfied by the greater zero of the DHF $\Phi(-n - \gamma, k - n + 1; z)$ with the definite value $\gamma(n, k)$. It is possible, if k is less than or equal to quantity D_n, to obtain from the equation:

$$\zeta_{n+1,n+1}^{(D_n - n)} = \frac{R_0^2}{\lambda^2}. \qquad (16)$$

Using (15) we obtain:

$$D_n = z_0 - \sqrt{\frac{n(n+1)}{2} z_0}. \qquad (17)$$

When $R_0 \backsim 1$ cm and $H \backsim 10^4$ E then $z_0 \backsim 8 \cdot 10^{10}$. These numbers determine the values n for which we can make such an approximation.

There are two other intervals of the values $k > D_n$ where the boundary condition can be satisfied by other zeros of the DHF $\Phi(-n - \gamma, k - n + 1; z)$. In this approximation all Laguerre polynomial intermediate zeros coincide, i. e. they differ in the quantities that are congruent with unity. Therefore, we can find the value $\gamma(n, k)$ for any k from the interval $[z_0 - \sqrt{n(n-1)z_0/2}, \ z_0]$ so that intermediate zero of the DHF is equal to z_0. If the minimum zero of the DHF satisfies the boundary condition, the values k should be within the interval $[z_0 + \sqrt{n(n-1)z_0/2}, \ z_0 + \sqrt{n(n+1)z_0/2}]$. The number of the states in the energy interval under consideration is equal in sum of the numbers of the k values over all the intervals of the possible values. This is equal to $D_0 = z_0$, i. e. degeneration of the Landau levels that was obtained in work [1]. Hence the density of the states obtained by averaging over the interval $2\beta H$ in work [1] is the same as in the present work.

The same consideration can be done when $n \backsim z_0$. The greater zero of the Laguerre polynomial in this case is determined by the formula (see [9]):

$$\zeta_{N+1,N+1}^{(c-1)} \cong \frac{\pi^2}{4} \frac{(N + 1 + c/2 - 3/4)^2}{N + 1 + c/2} \cong \frac{\pi^2}{8}(n + k). \qquad (18)$$

Then we got from the formula (16): $D_n = (8z_0/\pi^2) - n$. The number of the states for which the greater zero satisfies the boundary condition is much smaller than $D_0 = z_0$. When $n \geqslant (8z_0/\pi^2)$ these states are absent.

It is shown in section five that

$$\gamma \backsimeq \frac{z_0^{2N+c}}{(c + N - 1)! N!} \exp(-z_0) = \frac{z_0^{k+n+1}}{k! n!} \exp(-z_0), \qquad (19)$$

if the boundary condition is satisfied by the greater zero of DHF. Therefore, the shift of the energy value $2\beta H\gamma$ and its variation with a variation of k cannot be observed for the greater part of the k values that are less than D_n. This means that the density of the states has very narrow peaks at the energy values equal in the position of the Landau level. The number of the states in this peak is D_n instead of $D_0 = z_0$ as it was asserted in work [1]. When the k value is close to D_n the $\gamma(n, k)$ value is close to unity. But the number of these states is small compared to D_n when $n \ll D_n$. It is also shown in section five that the values of $\gamma(n, k)$ can be supposed to be distributed over the interval $[0, 1)$ uniformly if the boundary condition is satisfied by any of the other zeros of DHF. These states do not make a contribution to the peak of the density of states. All zeros of the DHF except the maximum zero can be approximately obtained by the formula (see [9]):

$$\xi_i = \frac{1}{2c - 4a} j_{c-1,i}^2 \,, \tag{20}$$

if $-a + c/2 \gg 1$ and $|a| > c/2$. Here $j_{c-1,i}$ is the zero with number i of the first type Bessel function with the index $c - 1$. Then from the boundary condition and from the formulae (3) and (20) we obtain:

$$\varepsilon = \frac{\hbar^2}{2m} \frac{j_{|l|,i}^2}{R_0^2} + l\beta H. \tag{21}$$

It is interesting to note that the first term is the energy of a free electron with the angular momentum $\hbar l$ in a circular potential box, and the second term is the energy of its magnetic moment in a magnetic field. Hence the energy spectrum for these states is quasi-continuous and its density of the states is like the one for a free electron.

The main result of this research is the density of the state being broken up into the narrow peaks and the background that is obtained within the framework of the unified classification of the states. This successfully allows us to take into account a background contribution in the computation of the thermodynamical functions. Hereafter we shall see this contribution to be substantial.

It is widely believed that we can use classical mechanics and quasi-classical quantization in considering the electron gas in a metal in a magnetic field (see, for example, [2–4,10,11]). The periodical motion of a classical particle can be described as distributions of probability to find a particle in a given point or as a current density. From the above it is seen that distributions of probability density and the density of probability current for an electron are very different from the classical ones. Among other things, a particle in a quasi-classical state also alternates its energy due to the collision with a boundary. Then the number of states on the energy interval of length $2\beta H$ is not equal to $\pi R_0^2 m / 2\pi\hbar^2 = D_0 / 2\beta H$ as it should be.

3. Contradictions of common statistical thermodynamics of electron gas in magnetic field

First of all we compute the magnetic moment by means of a common method using some results of the second section. A change to the thermodynamical limit and densities of the extensive quantities is correct only for systems that can be named spatial-additive. The determinative property of these systems is that they can be mentally divided into parts, where each of them should be similar to a complete system. It was shown above that the electron gas in a magnetic field is not a spatial-additive system as long as the probability current lines envelope the origin of coordinates. Then we should compute a thermodynamical potential of the limited system taking into account the effect of the boundary.

The computation of the thermodynamical potential of a three-dimensional electron gas in a magnetic field (see [2]) leads to a formula:

$$\Omega = -\frac{2h}{\pi\hbar}\sqrt{2m}\sum_n \left(\sum_{k=0}^{D_{12}}\phi_{nk} + \sum_{k=D_{11}}^{D_0}\phi_{nk} + \sum_{k=D_{21}}^{D_{22}}\phi_{nk}\right), \qquad \phi_{nk} = \phi(x_n + 2\beta H\gamma(n,k)),$$

$$\phi(x) = \int_x^\infty (\varepsilon - x)^{1/2} f^0(\varepsilon)\mathrm{d}\varepsilon, \qquad x_n = 2\beta H\left(n + \frac{1}{2}\right),$$

$$D_{qr} = D_0 + (-1)^q\sqrt{\frac{D_0}{2}n[n + (-1)^r]}. \tag{22}$$

Here h is the height of the circle cylinder that contains the gas, and $f^0(\varepsilon)$ is the Fermi function. The integration is done over energy. We made the change of variables with the formula $p_z(\varepsilon, n) = \pm\sqrt{2m[\varepsilon - \beta H(2n+1)]}$. The summation over k is made using the results of the second section. Let us compute the formula (22) by expanding ϕ_{nk} in terms of $\gamma(n,k)$ in the first order. We suppose that $\gamma(n,k) = \bar{\gamma} \simeq 1/2$ on the both intervals of the k values where the boundary states are located. In the first sum in (22) we suppose $\gamma(n,k) = 0$. Then we obtain:

$$\Omega = -\frac{2h}{\pi\hbar}\sqrt{2m}\sum_n \left[\phi(x_n)D_0 + 2\beta H\bar{\gamma}\left(\frac{\partial\phi}{\partial x}\right)_{x=x_n}\sqrt{\frac{D_0}{2}n(n+1)}\right]. \tag{23}$$

The first term leads to the formula of thermodynamical potential that is obtained in work [1]. The second term describes the contribution of boundary states. We can take the sum in this term changing over integration since the dependence upon the magnetic field does not disappear in so doing. Then we obtain the ratio between the magnetic moment of work [1] and the magnetic moment created by the conversion of the part of Landau level states to the background. It is in the region of $-(\beta H/\zeta_{\mathrm{F}})^{5/2}(R_0 p_{\mathrm{F}}/\hbar)$ where ζ_{F} is the Fermi energy and p_{F} is the Fermi momentum. We can see that the magnetic moment created by the background can be larger than the one in the conventional theory by a factor of two or three hundred. Therefore the agreement of the results in the conventional theory with experimental data is caused by the theoretical mistakes.

Let us note some other contradictions that demonstrate the incorrectness of the conventional theory. In the second section it was shown that the averaged magnetic moment is negative in any eigenstate (see formula (12)). The averaged magnetic moment of ideal gas should be equal to

$$\overline{M_z} = \sum_{n,l}\mathcal{N}_{n,l}\langle m_z\rangle_{n,l}, \tag{24}$$

where

$$\mathcal{N}_{n,l} = [1 + \exp\left(\varepsilon_{n,l} - \mu\right)/k_{\mathrm{B}}T)]^{-1}. \tag{25}$$

Evidently $\overline{M_z}$ cannot be zero when it is computed by change of summation into integration. The same paradox in classical thermodynamics was described in work [5]. This is more obvious in quantum thermodynamics because there is the formula (24). The mistake of the conventional theory is in the formula (see [12])

$$\overline{\left\langle\frac{\partial\widehat{\mathcal{H}_g}}{\partial H}\right\rangle} = \frac{\partial\Omega_g}{\partial H}, \tag{26}$$

where $\widehat{\mathcal{H}_g}$ is the gas Hamiltonian that is the sum of the one-particle Hamiltonians. Firstly $\langle(\partial\widehat{\mathcal{H}}/\partial H)_{\mathrm{symb}}\rangle_{n,l} \neq \partial\varepsilon_{n,l}/\partial H$ generally. Secondly $\sum_{n,l}\mathcal{N}_{n,l}(\partial\varepsilon_{n,l}/\partial H) \neq \partial\Omega_g/\partial H$. Let us prove the last assertion.

$$\Omega_g(\mu, T) = \sum_{n,l}\Omega(\mu, T; \varepsilon_{n,l}) = \sum_n D_n(H)\Omega(\mu, T; n), \tag{27}$$

where $D_n(H)$ is the degeneracy of the Landau energy levels in the conventional theory. The assertion is proved by taking the derivative with respect to H from (27). $\partial\Omega(\mu, T; \varepsilon_{n,l})/\partial H =$

$\mathcal{N}_{n,l}(\partial \varepsilon_{n,l}/\partial H)$ but the degeneracy of energy levels also depend on H. Through this dependence the gas thermodynamical potential does not depend on the magnetic field when it is computed by change of summation into integration.

Let us compute the averaged magnetic moment by means of a formulae (24) using (25) and (12).

$$\overline{M_z} = -\frac{1}{H} \sum_{n,l} \mathcal{N}_{n,l} \varepsilon_{n,l} = -\frac{\overline{E_g}}{H} = \frac{T^2}{H} \left(\frac{\partial}{\partial T} \frac{\Omega_g}{T} \right). \tag{28}$$

This formula can't be correct because Ω_g does not depend on the magnetic field. We should obtain the magnetic moment that is inversely proportional to the magnetic field. It pinpoints that the Fermi function of distribution (25) is not applied in the case of an electron gas in a magnetic field. It is in accord with the conclusion of the work [5].

4. Principles of quantum statistical mechanics of electron gas in magnetic field

It was shown in work [5] that the density of distribution of the classical statistical ensemble for a charged particle gas in a magnetic field depends on the component of the system's angular momentum that is parallel to the magnetic field. The angular momentum also plays an important part in the quantum statistical mechanics in a magnetic field. In the second section we showed that the equilibrium state wave functions should be also the eigenfunctions of the angular momentum. All one-particle eigenstate wave functions are also the angular momentum eigenfunctions. They exponentially decrease at great distance from the origin of coordinate. The remoteness of this asymptotic domain for the one-particle state depends on $|l|$. It is conditioned by the quadratic potential term of the operator $\widehat{\mathcal{H}_e}$ and cannot be abandoned by any potential $u(r)$. Formula (4) demonstrates that if the one-particle energy eigenvalue $\varepsilon_{N,l}$ is fixed, the possible positive values of the angular momentum number can have only values $l \leqslant \varepsilon/2\beta H$. Therefore, if the energy and the angular momentum of the electron gas are fixed, the negative values of the one-particle angular momentum number should be also limited. Consequently both the degeneracy of the gas energy value and the phase space domain volume in classical statistical thermodynamics should be finite. We can make the same conclusions that the magnetic field should be the mandatory parameter instead of the volume and the electron gas density in the magnetic field is non-homogeneous. It is common knowledge that a magnetic field confines a two-dimensional plasma. That is another reason why the electron gas in a magnetic field cannot be considered a spatial-additive system.

In the case under investigation we should derive the function of distribution from the ground principles. To compute the degeneracy of a state of a two-dimensional electron gas in a magnetic field without a potential field, the generating function can be taken as follows:

$$F = \prod_{n=0}^{\infty} \prod_{l=n}^{-\infty} \left(1 + xy^{2n+1}z^{-l} \right). \tag{29}$$

Here the unity of energy is taken as βH. This formula is the extension of the corresponding formula from [13]. The degeneracy of a gas macroscopic state $C(N; E; L)$ is equal to the coefficient of the term $x^N y^{E/\beta H} z^{-L}$ when the function $F(x, y, z)$ is represented as expansion in the powers of arguments. Here N is a number of electrons, E is a fixed gas energy and L is a fixed angular momentum regarding the mass center measured by a unit $\hbar$. This degeneracy can be obtained by computing the integrals regarding the closed paths using the steepest descent method (see [13]).

Let us consider a composite system made up of an electron gas in a magnetic field and other subsystem (thermostat), a particle number of which is considerably greater than the one of the electron gas. Quantities of the electron gas are marked by index 1, quantities of the thermostat are marked by index 2, and quantities of the all system are not marked by any index. The energy of the full system is conserved. The electron gas angular momentum is equal to zero. We suppose

that the magnetic field does not effect the thermostat. Then the probability that the electron gas has the energy E_1 is:

$$w(E_1) = \frac{C_2(N_2; E - E_1)}{C(N_1, N_2; E; L_1 = 0)} \,. \tag{30}$$

In this equation we suppose that the energies are expressed by integers in the unity $\Delta\epsilon$ that is independent of the magnetic field. We need to compute four integrals regarding closed pathes in the planes of complex variables x_1, x_2, y and z for $C(N_1, N_2; E; L_1 = 0)$ evaluation. The corresponding saddle points ξ_1, ξ_2, θ and ζ should satisfy the equations:

$$\sum_{n_1=0}^{\infty} \sum_{l=n_1}^{-\infty} \frac{(-l)\xi_1\theta_1^{2n_1+1}\zeta^{-l}}{1 + \xi_1\theta_1^{2n_1+1}\zeta^{-l}} = 1, \qquad \sum_{n_1=0}^{\infty} \sum_{l=n_1}^{-\infty} \frac{\xi_1\theta_1^{2n_1+1}\zeta^{-l}}{1 + \xi_1\theta_1^{2n_1+1}\zeta^{-l}} = N_1; \tag{31}$$

$$\frac{\beta H}{\Delta\epsilon} \sum_{n_1=0}^{\infty} \sum_{l=n_1}^{-\infty} \frac{(2n_1 + 1)\xi_1\theta_1^{2n_1+1}\zeta^{-l}}{1 + \xi_1\theta_1^{2n_1+1}\zeta^{-l}} + \sum_{n_2=0}^{\infty} \frac{n_2\xi_2\theta^{n_2}}{1 + \xi_2\theta^{n_2}} = (E - E_1) + E_1 \,,$$

$$\sum_{n_2=0}^{\infty} \frac{\xi_2\theta^{n_2}}{1 + \xi_2\theta^{n_2}} = N_2 \,, \qquad \theta_1 = \theta^{\beta H/\Delta\epsilon}. \tag{32}$$

Let us use $N_1 \ll N_2$ as it is commonly performed. We neglect the first term in the left member of the first equation from (32) in the zeroth order. We also neglect the last term in the right member of this equation. Following those approximations, the equations (32) relate only to the thermostat. They coincide with the equations for the saddle points of the numerator of (30). We determine θ from these equations. As in [13] $\theta = \exp(-\Delta\epsilon/k_{\mathrm{B}}T)$. In the first approximation the first equation (32) is the equation for the electron gas:

$$\sum_{n_1=0}^{\infty} \sum_{l=n_1}^{-\infty} \frac{(2n_1 + 1)\xi_1\theta_1^{2n_1+1}\zeta^{-l}}{1 + \xi_1\theta_1^{2n_1+1}\zeta^{-l}} = \frac{\Delta\epsilon E_1}{\beta H} \,. \tag{33}$$

Let us add this equation to the first equation (31) and take into account that $E_1/\beta H \gg 1$. (Hereafter we shall denote the electron gas energy $\Delta\epsilon E_1$ as E_1). If we assume $\zeta = \theta_1$ in the obtained equation, then summation indexes n_1 and l will get only as the linear combination:

$$2n_1 + 1 - l = n + k + 1 = t. \tag{34}$$

Therefore, we can change to the summation over t. The quantum numbers n and l or k (see (8)) completely determine the electron state. Then the quantum number t is t-fold degenerated. We obtain the equations for finding the saddle points of the electron gas:

$$\sum_{t=1}^{\infty} t\frac{t\xi_1\theta_1^t}{1 + \xi_1\theta_1^t} = \frac{E_1}{\beta H} \,, \qquad \sum_{t=1}^{\infty} t\frac{\xi_1\theta_1^t}{1 + \xi_1\theta_1^t} = N_1 \,. \tag{35}$$

These equations coincide with the equation set that defines the function of distribution of the effective gas of the N_1 particles which are described by Hamiltonian $\widehat{\mathcal{H}}_e$ (see (2)). The saddle point ξ_1 determines the chemical potential μ:

$$\xi_1 = \exp\frac{\mu}{k_{\mathrm{B}}T}, \qquad \beta Ht = \tau, \qquad \mathcal{N}_t = \frac{t}{1 + \exp\left(\tau - \mu\right)/k_{\mathrm{B}}T} \,. \tag{36}$$

The chemical potential $\mu(T, N_1)$ is determined by the second equation (35). When $T \to 0$ then $\mu_0 = \beta H\sqrt{2N_1}$. If the spin degeneracy is taken into account, then $\mu_0 = \beta H\sqrt{N_1}$.

We can clarify these results in the following manner. The Hamiltonian of electron gas can be presented as the sum of the Hamiltonian of effective particle gas and the operator that is proportional to a full angular momentum of this gas like this is done in formula (1) for one

electron. These operators are commutative. Therefore, the energy eigenvalues of the electron gas can be presented as the sum of the corresponding eigenvalues. We consider only the electron gas states in which the full angular momentum is equal to zero. Therefore, in this case the statistical thermodynamics should be determined by the Hamiltonian of effective gas. This consequence of the angular momentum conservation law was not taken into account in the conventional theory.

The average magnetic momentum of the electron gas in the magnetic field is obtained from the formulae (12), (24) and (35):

$$M_z = -\beta \sum_{t=1}^{\infty} t \frac{t \xi_1 \theta_1^t}{1 + \xi_1 \theta_1^t} = -\frac{E_1}{H} \,. \tag{37}$$

This formula is applicable when the wave functions have the view like in (3). Derivation of the function of distribution in the formulae (29)–(36) is founded on the supposition that the Hamiltonian of the system can be represented as a sum of the one-particle Hamiltonians. To do this it is necessary to describe the particle interaction by the self-consistent field. The sum of this field with the neutralizing background field should not be zero because the electron gas density is non-homogeneous. This potential field should modify the view of the eigenfunctions. It differs from those in the formula (3).

For a qualitative analysis we assume that the potential energy is as follows:

$$u(r) = \begin{cases} u_1 > 0, & \text{when } 0 < r < R_1 \,, \\ 0, & \text{when } R_1 < r < R_0 \,, \\ u_0 \gg u_1, & \text{when } R_0 < r. \end{cases} \tag{38}$$

The investigation of operator $\widehat{\mathcal{H}_e}$ spectrum can be carried out in the same way as in section two. Density of states also has got very narrow peaks at the effective energy values $\tau = \beta H t$ and the uniform background. Quantum numbers (t, l) define the assignment of state to the peak or to the background. This can be illustrated by quantization of classical orbits that are obtained by Hamiltonian $\widehat{\mathcal{H}_e}$. If $u(r) \equiv 0$, these orbits are ellipses with a center in the origin of coordinates. The orbit with the energy τ and the angular momentum χ has the axes, the squares of which are

$$\rho_{\pm}^2 = \frac{2}{m\omega^2} \left(2\tau \pm \sqrt{4\tau^2 - \omega^2 \chi^2} \right). \tag{39}$$

By quantization of the adiabatic invariants we obtain $\chi = \hbar l$, $\tau = \beta H t$. The axes of the corresponding orbit are obtained by formula:

$$\rho_{\pm}^2 = \lambda^2 \left(t \pm \sqrt{t^2 - l^2} \right). \tag{40}$$

Let us determine the potential by the formula (38). If $R_1 < \rho_-(t, |l|) \leqslant \rho_+(t, |l|) < R_0$, the orbit and its energy remain unchanged. These states fall into the peaks of the density of states. If one or both of these inequalities do not hold, the quantum condition for the radial adiabatic invariant should be a transcendental equation for $\tau(t, |l|; H)$. It can be shown that in this case $\tau(t, |l|; H) = \beta H f(t, |l|) > \beta H t$. These states fall into the background of the density of states.

The states can be depicted by points with integer coordinates at the (t, l)-plane. All states fall into the sector bounded by the bisectors of the first and fourth quadrants. In figure 1 the upper half of the symmetric diagram is shown with $R_0^2/\lambda^2 = 50$, $R_1^2/\lambda^2 = 5$. The states with $\rho_+(t, |l|) < R_0$ and $R_1 < \rho_-(t, |l|)$ are positioned on the triangle ABC that is constructed both by the segment BC of the parabola $R_0^2/\lambda^2 = t + \sqrt{t^2 - l^2}$ and the segment AB of the parabola $R_1^2/\lambda^2 = t - \sqrt{t^2 - l^2}$. The states positioned in this domain have the effective energy values $\beta H t$. Effective energy values of other states are above $\beta H t$. The states with $\tau < \beta H \sqrt{N_1}$ are completed at zero temperature. When a magnetic field increases, the parabolas move to the right. Therefore, the energy of electron gas should nonlinearly vary with the magnetic field. In this model a magnetic moment is proportional to the energy of electron gas.
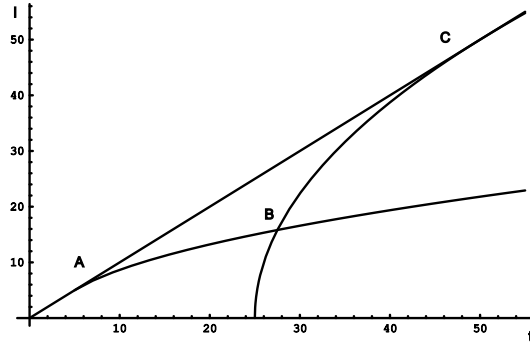
Figure 1. The domains of quantum numbers t and l in which the effective energy is determined by different formulae.

5. Supplement. Some details about features of zeroes of Laguerre polynomial and degenerate hypergeometric function

Let us consider the Laguerre polynomial $L_\nu^b(z) = \Phi(\nu, b+1; z)$ where $\nu \geqslant 2$, $b > 0$ are integers. Thus we have established that its maximum and minimum zeroes are described by the formulae

$$\zeta_{\nu,\nu}^b \cong b + \sqrt{\frac{b\nu(\nu-1)}{2}}, \qquad \zeta_{\nu,1}^b \cong b - \sqrt{\frac{b\nu(\nu-1)}{2}}, \tag{41}$$

in a first approximation when $\nu^2 \ll b$. Other zeroes are approximately equal in b, that is order of polynomial.

According to the definition of Laguerre polynomial

$$L_\nu^b(z) = \sum_{k=0}^{\nu} \frac{(-z)^k \nu! b!}{(\nu-k)! k!} \frac{1}{(b+k)!}. \tag{42}$$

Let us introduce $[(b+k)!]^{(-1)} \cong [b!]^{-1} b^{-k} (1 - b^{-1} \sum_{j=0}^{k} j)$. In (42) we denote $zb^{-1} = y$ and make some identical transforms:

$$
\begin{aligned}
L_\nu^b(z) &\cong \sum_{k=0}^{\nu} \frac{(-y)^k \nu!}{(\nu-k)! k!} \left(1 - b^{-1} \sum_{j=0}^{k} j\right) = \left[(1-y)^\nu - \frac{1}{2b} \sum_{k=0}^{\nu} \frac{(-y)^k \nu! k(k+1)}{(\nu-k)! k!}\right] \\
&= \left[(1-y)^\nu - \frac{1}{2b} \frac{\mathrm{d}}{\mathrm{d}y} y^2 \frac{\mathrm{d}}{\mathrm{d}y} \sum_{k=0}^{\nu} \frac{(-y)^k \nu!}{(\nu-k)! k!}\right] = \left[(1-y)^\nu - \frac{1}{2b} \frac{\mathrm{d}}{\mathrm{d}y} y^2 \frac{\mathrm{d}}{\mathrm{d}y} (1-y)^\nu\right] \\
&= (1-y)^{\nu-2} \left[y^2 \left(1 - \frac{\nu}{2b} - \frac{\nu^2}{2b}\right) - 2y \left(1 - \frac{\nu}{2b}\right) + 1\right].
\end{aligned}
\tag{43}
$$

The obtained approximate expression for the Laguerre polynomial has zeroes that coincide with (41) in the first order after multiplying by b. Thus, this proves the statement.

The distribution of states in the energy value interval $2\beta H$ is governed by the distributions of the values $\gamma(n, k)$ between zero and unity in each interval of the possible values k. Let us introduce the DHF at the point z_0 as:

$$\Phi(-N-\gamma, c; z_0) = P_N(\gamma, z_0) - \gamma A_N(\gamma, z_0) - \gamma(1-\gamma) T_N(\gamma, z_0), \tag{44}$$

$$P_N(\gamma, z_0) = \sum_{i=0}^{N} \frac{(-N-\gamma)_i z_0^i}{(c)_i i!}, \qquad A_N(\gamma, z_0) = \frac{(-1)^N (1+\gamma)_N z_0^{N+1}}{(c)_{N+1}(N+1)!},$$

$$T_N(\gamma, z_0) = \frac{(-1)^N (1+\gamma)_N z_0^{N+2}}{(c)_{N+2}(N+2)!} \sum_{i=0}^{\infty} \frac{(2-\gamma)_i z_0^i}{(c+N+2)_i (N+3)_i}, \tag{45}$$

where

$$(b)_i = b(b+1)\cdots(b+i-1) = \Gamma(b+i)/\Gamma(b), \ (b)_0 = 1 \tag{46}$$

is the Pochhammer symbol. It is evident from the DHF expansion that A_N and T_N have the same sign. Then DHF can have zero at z_0 only if P_N has the same sign. The values of P_N, A_N and T_N are slightly varied when γ takes on the values between zero and unity. Let us neglect this dependence. Then we put (44) equal to zero and solve this like equation for γ:

$$\gamma = \frac{1}{2} + \frac{A_N}{2T_N} - \frac{1}{2}\sqrt{1 + \frac{2A_N}{T_N} - \frac{4P_N}{T_N} + \left(\frac{A_N}{T_N}\right)^2}. \tag{47}$$

Here the sign of the radical is chosen in order to get $0 < \gamma < 1$.

We consider the problem of the maximum zero of DHF $\Phi(-N-\gamma, c; z)$. This zero cannot be described by the formula (20) since it should vanish continuously when $\gamma \to 0$. This is possible only when $\xi_{N+1} \to \infty$ in so doing. When $\gamma = 1$ then $\xi_{N+1} = \zeta_{N+1,N+1}^{(c-1)}$. Therefore, when each $z_0 > \zeta_{N+1,N+1}^{(c-1)}$ it is possible to take $\gamma < 1$ such that z_0 is the greatest zero of the function $\Phi(-N-\gamma, c; z)$. When $z_0 \gg 1$ we can change the expression of $T_N(z_0)$ as a power series (45) into the first term of its asymptotic expansion. To derive this expansion we consider the identity:

$$\Phi(-N-\gamma, c; z) \equiv \sum_{k=0}^{N} \frac{(-N-\gamma)_k}{(c)_k} \frac{z^k}{k!} + \frac{(-N-\gamma)_{N+1}}{(c)_{N+1}} \frac{z^{N+1}}{(N+1)!} + \frac{(-N-\gamma)_{N+2}}{(c)_{N+2}}$$

$$\times \int_0^z \mathrm{d}z_1 \cdots \int_0^{z_{i-1}} \mathrm{d}z_i \cdots \int_0^{z_{N+1}} \mathrm{d}z_{N+2} \Phi(2-\gamma, c+N+2; z_{N+2}). \tag{48}$$

This identity is easily proved by integrating the power series of function $\Phi(2-\gamma, c+N+2; z_{N+2})$. We correlate (48) with (44) and see that

$$T_N(z_0) = \frac{(-1)^N N!}{(c)_{N+2}} \int_0^z \mathrm{d}z_1 \cdots \int_0^{z_{i-1}} \mathrm{d}z_i \cdots \int_0^{z_{N+1}} \mathrm{d}z_{N+2} \Phi(2, c+N+2; z_{N+2}). \tag{49}$$

We assume $\gamma = 0$. This can be done in computating the $T_N(z_0)$. We express $\Phi(2, c+N+2; z)$ by integral representation (see ([9])):

$$\Phi(2, c+N+2; z) = \frac{\Gamma(c+N+2)}{\Gamma(2)\Gamma(c+N)} \int_0^1 \exp(zu) u(1-u)^{c+N-1} \mathrm{d}u, \tag{50}$$

and obtain:

$$T_N(z_0) = \frac{(-1)^N N! \Gamma(c)}{\Gamma(c+N)} \int_0^1 \left[\exp(zu) - \sum_{k=o}^{N+1} \frac{(zu)^k}{k!}\right] u^{-N-1}(1-u)^{c+N-1} \mathrm{d}u. \tag{51}$$

This expression is convenient for T_N numerical computation. In formula (49) the antiderivative of each integration at inferior limit is equal to zero. Therefore the function $\Phi(2-\gamma, c+N+2; z)$ can be changed by its asymptotic approximation in this formula. This approximation is

$$\Phi(2, c+N+2; z) \simeq \Gamma(c+N+2) \frac{\exp(z)}{z^{c+N}} \left(1 - \frac{c+N}{z}\right). \tag{52}$$

For the integration in (49) we use the formula:

$$\int^x \frac{\exp(t)}{t^n}\mathrm{d}t = \frac{\exp(x)}{x^n}\left[\sum_{l=0}^{L}\frac{(n)_l}{x^l} + O\left(\frac{1}{x^{L+1}}\right)\right]. \tag{53}$$

Let us find the coefficient of the term $\exp(z_0)/z_0^{c+N+s}$ in the asymptotic expansion. To obtain this factor we can pick the terms of expansion (53) that have $l = m_j$ in the integration over each z_j in the formula (49) so that $\sum_{j=1}^{N+2} m_j = \sigma$. Here $\sigma = s$ for the first item of (52) and $\sigma = s - 1$ for the second one. The consequence of (46) is $(b)_n(b+n)_m = (b)_{n+m}$. Therefore, the result of integration does not depend on the set of numbers m_j. It is proportional to $(c+N)_s/(c+N)_{s-\sigma}$. The number of these items is equal to the number of the disposition of σ indistinguishable objects at $N + 2$ boxes. The net result is

$$T_N(z_0) = (-1)^N N!(c-1)!\frac{\exp(z_0)}{z_0^{c+N}}\left[\sum_{s=0}^{S}\frac{(c+N)_s}{z_0^s}\binom{s+N}{s} + O\left(\frac{1}{x^{S+1}}\right)\right]. \tag{54}$$

If we restrict ourselves to the first term of this sum we obtain:

$$T_N \simeq (-1)^N N!(c-1)!\frac{\exp(z_0)}{z_0^{c+N}} . \tag{55}$$

The greatest zero of the polynomial P_N is smaller than z_0. That is why this polynomial can be changed by its last term. The formula (19) has been obtained. If the value of k is in other intervals of the possible values, the equation (47) does not have a great parameter. It is evident from the fact that zero of $\Phi(-N - \gamma, c; z)$ in this case is located between zeroes of the Laguerre polynomials that are the function $\Phi(-N - \gamma, c; z)$ with $\gamma = 0$ and $\gamma = 1$. The quantity of T_N does not take part in the computation of these zeroes.

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Квантова статистична мехніка електронного газу в магнітному полі

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Розглянуто власні стани електрону в магнітному полі. Одержано густину електричного струму і середній магнітний момент. Досліджено густину станів для двовимірного електрону в колі обмеженому нескінченним потенціальним бар'єром. З використанням цих результатів показано, що загальноприйнята квантова статистична механіка електронного газу в магнітному полі приводить до помилкових результатів. Магнітний момент електронного газу може бути обчислений як сумма середніх моментів заповнених станів. Таке обчислення приводить до результату, що відрізняється від того, який одержується як похідна термодинамічного потенціалу по магнітному полю. Наведено й інші протиріччя у загальноприйнятій статистичній термодинаміці електронного газу в магнітному полі. Зроблено висновок, що ці протиріччя виникають внаслідок використання неправильного статистичного оператора. Нова квантова функція розподілу виведена з основних принципів, беручи до уваги закон збереження кутового моменту. Ці результати узгоджуються з теорією, яка була виведена у рамках класичної статистичної термодинаміки у попередній роботі.

Ключові слова: *електронні стани, магнітне поле, кутовий момент, середній магнітний момент, квантова функція розподілу, квантова статистична термодинаміка*

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