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Nonequilibrium action principles for quasi-fields*

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Quasiparticle descriptions of nonequilibrium physics hold the promise of tracking the relevant degrees of freedom in calculations of statistical systems, as well as providing an important formal perspective on quantum field theories away from equilibrium. A formulation of nonequilibrium, quasi-particle field theory is presented here. The Schwinger closed time path generating functional and generalized sources are used to develop schematic field theories which can be adapted to real problems. The importance of the source method for effective nonequilibrium theories is argued and the physical interpretation of the formalism is discussed, including its possible applicability to biological dynamics.

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1. Introduction

Scientific studies are usually performed for one of two reasons: either to directly address and explain a phenomenon or to indirectly explore the meaning of a methodology. In the latter, one hopes to better understand how dynamical formulations of physics fit into the greater scheme of things – about universal qualities and structure. It is from both these perspectives that it is interesting to approach the problem of quasi-particles.

Quasi-particles are "dressed excitations" of a quantum field. They are dynamical entities which drag with them a cloud of correlating interactions from their surrounding environment. In large statistical systems all particles are surrounded by a bath or reservoir of average environmental behaviour, whether it is a heat bath, a phenomenological *source* or an explicitly interacting quantum field. Particles' surroundings modify their behaviour in complex ways and often it is possible to find a simpler *effective* description in terms of a new set of dynamical objects or *quasi-particles* which absorb part of that complexity into redefinitions of the basic variables.

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The fact that quasi-particle descriptions track the important changes in dynamical objects and hide the less obvious details of the interaction clouds is an attractive idea: it suggests that conceptual and perhaps calculational simplifications can be made essentially by a change of variable. In practice the use of quasi-particle descriptions may prove too difficult to use as a calculational scheme, since they involve hierarchies of non-local correlations and these are notoriously difficult to work with, as we know from gauge theories. On the other hand a deeper understanding of the way in which quasiparticle formalism works can help us to build effective theories whose local remnants yield tractable problems at sufficiently long wavelengths and might also be important in revealing universal aspects. Some progress has been made here using rather different approaches [1–3].

Nonequilibrium field theory presents its own special problems to effective theory building. By its very nature, the effective dynamical variables of a nonequilibrium field theory change with time and often also from place to place, so the identification of appropriate variables which track these changes is a difficult problem. In fact the problem of an effective description of spacetime varying quasi-particles is in many respects identical to the study of gauge theories. In a gauge theory one deals with phases which vary in space and time; the effects of those phases can also be represented through a transformation in terms of a vector potential which provides an 'environmental background' for 'particles' or excitations of a field. In a nonequilibrium field theory one is interested in statistical environmental changes which vary in space and time. It is perhaps not surprising then that these can be represented in terms of a 'statistical vector potential'.

The primary purpose of this review is to demonstrate that the formal structure of nonequilibrium field theory has much in common with covariant field theories, but in a wider context, i.e. to reassert the importance of covariant formulation and hidden symmetry.

2. Inhomogeneous field theory

Nonequilibrium systems remember their history: they are not translationally invariant in time and often not translationally invariant in space. A formalism which captures the long-term changes in the system is required. This will typically involve two scales: one scale at which long term average changes take place and another over which fluctuations occur. To measure a scale we need two points so we expect two point nonequilibrium Green functions to have special properties, so we look for a formalism in which these scales can be separated from one another.

Between any two space-time points x and x' it is useful to parameterize functions in terms of 'rotated' variables [1]:

$$\tilde{x} = (x - x'), \qquad \overline{x} = \frac{1}{2}(x + x'). \tag{1}$$

The odd variables $\tilde{x}$ (the variable conjugate to the momentum) characterize 'translational invariance' while the even variables $\overline{x}$ represent the opposite of this: inhomogeneity.

Many problems in nonequilibrium field theory can be addressed with the help of the closed time path (CTP) generating functional [4–6]. The CTP is a field theoretical prescription for deriving expectation values of physical quantities, given a description of the state of the field at some time in the past. The generating functional requires an artificial duplicity in the field, so the closed-time path action is described as a two-component field ϕ_A, where $A = +, -$.

$$S_{\rm CTP} = \int {\rm d}V_x {\rm d}V_{x'} \frac{1}{2} \phi^A S_{AB} \phi^B. \tag{2}$$

The CTP field equations in the presence of sources may be found by varying this action with respect to the $+$ and $-$ fields. The fact that we can present a quasi-particle prescription directly in terms of an action principle implies that it is a well-defined canonical system, as one would expect from general renormalization arguments. Following a similar line of argument to Lawrie [7], a general Gaussian, quadratic form for a closed time path action may be expressed in terms of general sources. Calzetta and Hu approached the problem in essentially the same fashion using a more general formalism than Lawrie in [6]. These sources can be thought of as external quantities or as the renormalized shadows of higher loop contributions due to self-interactions. They must be non-local quantities in general. This is testified to by the importance of hard-thermal loop effective actions which contain non-local regulatory terms. The non-locality leads to power-law type dissipation [8], rather than exponential dissipation, amongst other things so it is important to retain it [1,8]. This seems to defy the usual wisdom of effective field theory, in which only local terms are retained [9]. In fact there is no contradiction when care is taken in choosing the order of the approximation to which one is calculating. One reserves the ability to make an approximate local expansion later once the dynamical effects of the nonequilibrium development have been better understood. We write

$$S_{AB}(x, x') = \begin{pmatrix} \hat{\alpha} & \hat{\beta} \\ -\hat{\beta}^* & -\hat{\alpha}^* \end{pmatrix}, \tag{3}$$

where the indices A, B run over the $\pm$ labels of the CTP fields,

$$\hat{\alpha} = (-\Box + m^2)\delta(x, x') + I(x, x'), \qquad \hat{\beta} = J(x, x') + K^\mu(x, x') \overset{\leftrightarrow}{D^K_\mu} \tag{4}$$

and a Hermitian derivative has been defined to commute with the function $K^\mu(x, x')$:

$$\overset{x}{D^K_\mu} \equiv \overset{x}{\partial_\mu} + \frac{1}{2} \frac{\overset{x}{\partial_\mu} K_\nu(x, x').}{K_\nu(x, x')} \tag{5}$$

for time reversibility. Notice the general form of the ‘connection’ term in this derivative. This inhomogeneous, conformal structure crops up repeatedly in non-equilibrium development. The currents associated with sources I, J, K^μ are not necessarily conserved since their behaviour is not completely specified by the action, but the action is differentially reversible. The sum of rows and columns in this operator is zero, as required for unitarity and subsequent causality. The significance of the off-diagonal terms involving K^μ can be seen by writing out the coupling fully:

$$K^\mu(x,x') \cdot \left(\phi_+ D^K_\mu \phi_- - \phi_- D^K_\mu \phi_+\right). \tag{6}$$

The term in parentheses has the form of a current between components ϕ_+ (the forward moving field) and ϕ_- (the backward moving field). When these two are in equilibrium there will be no dissipation to the external reservoir and these off-diagonal terms will vanish. This indicates that these off-diagonal components (which are related to off-diagonal density matrix elements, as noted earlier) can be understood as the mediators of a detailed balance condition for the field. When the term is non-vanishing, it represents a current flowing in one particular direction, pointing out the arrow of time for either positive or negative frequencies. The current is a 'canonical current' and is related to the fundamental commutator for the scalar field in the limit $+ \to -$.

The system can be analyzed, and in principle solved, by looking for the Green functions associated with this system. These can all be expressed in terms of the Wightman functions $G^{(\pm)}(x,x')$ using the relations

$$G^{(+)}(x,x') = \left[G^{(-)}(x,x')\right]^* = -G^{(-)}(x',x). \tag{7}$$

The Wightman functions are the sum of all positive or negative energy solutions, satisfying the closed time path field equations, found by varying the action above. Thus they are the embodiment of the dispersion relation between $\mathbf{k}$ and $\omega = k^0$.

$$\tilde{G}(x,x') = G^{(+)}(x,x') + G^{(-)}(x,x'), \qquad \overline{G}(x,x') = G^{(+)}(x,x') - G^{(-)}(x,x'). \tag{8}$$

$\overline{G}(x,x')$ is the sum of all solutions to the free field equations and, in quantum field theory, becomes the so-called *anti-commutator* function. The symmetric and anti-symmetric combinations satisfy the identities

$$\overset{x'}{\partial_t}\, \overline{G}(x,x')\Big|_{t=t'} = 0, \tag{9}$$

and

$$\overset{x'}{\partial_t}\, \tilde{G}(x,x')\Big|_{t=t'} = \delta(\mathbf{x},\mathbf{x}'). \tag{10}$$

The latter is the classical dynamical equivalent of the fundamental commutation relations in the quantum theory of fields. Other Green functions may be constructed from these to model the processes of emission, absorption and fluctuation respectively:

$$\begin{aligned}
G_r(x,x') &= -\theta(t,t')\tilde{G}(x,x'),\\
G_a(x,x') &= \theta(t',t)\tilde{G}(x,x'),\\
G_F(x,x') &= -\theta(t,t')G^{(+)}(x,x') + \theta(t',t)G^{(-)}(x,x').
\end{aligned} \tag{11}$$

It may be verified that, since $G^{(+)}(k,\overline{x})$ depends only on the average coordinate, the commutation relations are preserved (see equation (10)) even with a time-dependent action. The general solution for the positive frequency Wightman function may be written

$$G^{(+)}(x,x') = -2\pi \mathrm{i} \int \frac{\mathrm{d}^{n-1}k}{(2\pi)^{n-1}} \mathrm{e}^{\mathrm{i}k_\mu \tilde{x}^\mu} \frac{(1+f(k_0,\overline{x}))}{2|\omega|}, \tag{12}$$

where $f(k_0, \overline{x})$ is an unspecified function of its arguments and it is understood that $k_0 = |\omega|$ (this describes the dispersion of the plane wave basis). The ratio of Wightman functions describes the ratio of emission and absorption of a coupled reservoir (the sources). As observed by Schwinger [10,11], all fluctuations may be thought of as arising from generalized sources via the Green functions of the system. Thus a source theory is an effective description of an arbitrary statistical system.

In an isolated system in thermal equilibrium, we expect the number of fluctuations excited from the heat bath to be distributed according to a Boltzmann probability factor [12].

$$\frac{\text{Emission}}{\text{Absorption}} = \frac{-G^{(+)}(\omega)}{G^{(-)}(\omega)} = \mathrm{e}^{\hbar\beta|\omega|}. \tag{13}$$

$\hbar\omega$ is the energy of the mode with frequency ω. This tells us the rate flow from the 'heat-bath' in equation (6) is in balance. It is a classical understanding of the well-known Kubo-Martin-Schwinger relation [13,10] from quantum field theory. In the usual derivation, one makes use of the quantum mechanical time-evolution operator and the cyclic property of the trace to derive this relation for a thermal equilibrium. The argument given here is identical to Einstein's argument for stimulated and spontaneous emission in a statistical two state system, and the derivation of the well-known A and B coefficients. It can be interpreted as the relative occupation numbers of particles with energy $\hbar\omega$. This is a first hint that there might be a connection between heat-bath physics and the two level system.

Finally, it is useful to define quantities of the form

$$F_\mu = \frac{\partial_\mu f}{f} = \frac{1}{2}\overline{\partial}_\mu \ln(f), \tag{14}$$

$$\Omega_\mu = \frac{\partial_\mu \omega}{\omega} = \frac{1}{2}\overline{\partial}_\mu \ln|\omega(\overline{x})|. \tag{15}$$

which occur repeatedly in the field equations and dispersion relations for the system and characterize the average rate of development of the system. Note the similarity in form to the connection term in the derivative of equation (5).

3. Inhomogeneous scaling and gauge formulation

Inhomogeneous field theory can be presented in a natural form by introducing a 'covariant derivative' D_μ which commutes with the average development of the field. This derivative is thus physical, in the sense of being a Hermitian operator. This description parallels the structure of a gauge theory (in momentum space) with a complex charge, generalized chemical potentials and quantum field theories in curved spacetime (see the local momentum space expansion approach of [6] and the curved spacetime formulation of Nicola [14]). Consider the derivative

$$D_\mu = \partial_\mu - a_\mu \tag{16}$$

and its square

$$D^2 = \Box - \partial^\mu a_\mu - 2a^\mu \partial_\mu + a^\mu a_\mu. \tag{17}$$

Derivatives occur in the field equations and in the dispersion relation for the field and they are thus central to the dynamics of the field and the response (Green) functions. As with a gauge theory, the effect of derivatives on spacetime dependent factors may be accounted for in a number of equivalent ways, by redefinitions of the field. In a gauge theory, we call this a gauge transformation and we usually demand that the theory be covariant, if not invariant under such transformations. In a nonequilibrium field theory, we require only covariance, since it is normal to deal with partial systems in which conserved currents are not completely visible and thus invariance need not be manifest.

In order to solve the closed time path field equations, it is useful to solve the dispersion relation, giving the physical spectrum of quasi-particles in the system. In the Keldysh diagrammatic expansion of Schwinger's closed time path generating functional, one expands around free particle solutions. By starting with a quasi-particle basis here we can immediately take advantage of resummations and renormalizations which follow from the unitary structure (specifically two-particle irreducible or daisy diagrams). It also allows one to track changes in the statistical distribution through the complex dispersion relation, instead of using real Vlasov equations coupled to real equations of motion.

Let us briefly review the formalism introduced in [1]. In order to make its meaning clearer we can simplify the notation and gloss over some of the details which are not essential to the conceptual makeup. For a technical discussion readers are referred to [1], with the change of notation

$$(I, J, K_\mu) \to (A, B, \gamma_\mu). \tag{18}$$

Substituting the form of the Green function in equation (12) into the equation of motion, one obtains the dispersion relation

$$\begin{aligned} & k^2 + m^2 + \overline{I}(k,\overline{x}) + \frac{\mathrm{i}}{2}(\overline{\partial}_\mu \overline{I})(T^\mu - v_g^\mu/\omega) - \mathrm{i}\tilde{J} - \overline{\partial^\mu K_\mu} \\ & - (F-\Omega)^2 - 2\mathrm{i}k^\mu (F-\Omega)_\mu - 2\mathrm{i}k^\mu \overline{K}_\mu - \overline{K}^\mu (F-\Omega)_\mu \quad = \quad 0, \end{aligned} \tag{19}$$

where the quantities within are defined by

$$T_\mu = \frac{\partial f/\partial_{k^\mu}}{(1+f)}, \qquad v_g^\mu = \frac{\partial \omega}{\partial k_\mu} = k_i/\omega(k), \qquad T^\mu - v_g^\mu/\omega = \frac{1}{G^{(+)}(k)}\frac{\partial G^{(+)}}{\partial k_\mu}. \tag{20}$$

Notice that the gradient of the plasma distribution or spectral envelope T^μ is responsible for classical Landau damping of the field modes due to scattering from the wave modes of $I(x,x')$. Also, the imaginary part of this dispersion relation is just the generalization of the Vlasov transport equation

$$\begin{aligned} k^\mu \overline{\partial}_\mu f(k,\overline{x}) = (1+f)k^\mu F_\mu &= \omega C, &(21)\\ [\partial_t + v_g^i \partial_i] f &= C, &(22)\end{aligned}$$

where C is a 'collision term' (see, for instance, [15]). The 'collision terms' are non-zero here because the sources play the role of mediating interactions, just as Landau damping is essentially collision by scattering off a third-party field. One could separate out the imaginary part and solve it as a separate equation but, rather than do this, it is useful to retain a complex dispersion relation and complex $\omega(k)$. This will make it possible to reveal which of the key players of this formulation have equivalent roles in their effect of the dynamics below.

Without any approximation, it is straightforward to show that, in the general inhomogeneous case,

$$\begin{aligned} (-\Box + m^2)G^{(+)}(x,x') = -2\pi\mathrm{i}\int \frac{\mathrm{d}^{n-1}k}{(2\pi)^{n-1}} \frac{(1+f)}{2|\omega|}\mathrm{e}^{\mathrm{i}k(x-x')},\\ \Big[-(\mathrm{i}k_\mu + F_\mu - \Omega_\mu)^2 - \partial^\mu(\mathrm{i}k_\mu + F_\mu - \Omega_\mu)\Big] = 0. \qquad (23)\end{aligned}$$

It is then natural to rewrite this by making the identification

$$a_\mu = F_\mu - \Omega_\mu + \overline{K}_\mu = -\partial_\mu S_E(k) + \overline{K}_\mu\,. \qquad (24)$$

Indeed, it does not require a great leap of insight to see that all of these effects come from the $\overline{x}$-dependence of $G^{(+)}$ in equation (12). We can even extend the generalization to include the case of a system of finite size which is expanding or contracting, by letting

$$a_\mu \;=\; F_\mu - N_\mu - \Omega_\mu + \overline{K}_\mu\,. \qquad (25)$$

The N_μ term then arises from the $\overline{x}$ dependence of the momentum space measure:

$$\int \frac{\mathrm{d}^n k}{(2\pi)^n} \to \prod_\mu \left(\frac{1}{L_\mu}\sum_{l_\mu}\right) \qquad (26)$$

giving a contribution

$$N_\mu = \frac{\partial_\mu(L_0 \ldots L_{n-1})}{(L_0 \ldots L_{n-1})}. \qquad (27)$$

This contribution makes a more intimate contact with quantum cosmological models of expanding universes and serves to emphasize the fact that scaling is a general phenomenon, regardless of whether it is a homogeneous (global) rescaling or an inhomogeneous (local) rescaling. The connection now embodies the effect of changing statistical distributions and quasi-particle energies (ω is solved in terms of the sources through the dispersion relation) and the rarefaction of the field by expansion through N_μ. It also shows that the source K^μ plays essentially the same role as these effects and it is thus capable of absorbing or "resumming" them. Furthermore, the field a_μ is related to a measure of the rate of increase of the entropy S_E of the mode. This in turn means that all of these are related to the behaviour of a density matrix, since off-diagonal elements labelled by K^μ represent a density matrix in this formulation.

In terms of the covariant derivative, one now has:

$$\begin{aligned}(-D^2+m^2)G^{(+)}(x,x') &= \\ &-2\pi\mathrm{i}\int\frac{\mathrm{d}^{n-1}k}{(2\pi)^{n-1}}\frac{(1+f)}{2|\omega|}\{-(\mathrm{i}k_\mu-\overline{K}_\mu)^2)-\partial^\mu(\mathrm{i}k_\mu-\overline{K}_\mu)\},\end{aligned} \tag{28}$$

$$\begin{aligned}&\left[-\overset{x}{\Box}+m^2\right]G^{(+)}(x,x') \\ &+\int \mathrm{d}V_{x''}\left[\overline{I}(x,x'')-\tilde{J}(x,x'')-\overline{K}^\mu(x,x'')\overset{x''}{\partial_\mu}-\overline{\partial^\mu K_\mu}\right]G^{(+)}(x'',x') = 0.\end{aligned} \tag{29}$$

In the inhomogeneous case there is no dispersion relation consisting of continuous frequencies in general, so the dispersion relation will only exist for a discrete set. It is convenient to divide the discussion into two parts: determining the dispersion relation and the nature of the restricted set of values which satisfy the dispersion relation.

The problem to be addressed is contained in the following form in momentum space:

$$\begin{aligned}(-\Box+m^2)G^{(+)}(x,x')+\int \mathrm{d}V_{x''}\frac{\mathrm{d}^nk}{(2\pi)^n}\frac{\mathrm{d}^np}{(2\pi)^n}\mathrm{e}^{\mathrm{i}k(x-x'')+\mathrm{i}p(x''-x')} \\ \times S(p,x''+x')G^{(+)}(k,x+x') = \lambda G^{(+)}(x,x').\end{aligned} \tag{30}$$

The integral over x'' is no longer a known quantity in general, but it is possible to extract an overall Fourier transform by shifting the momentum $p\to p+k$ and defining the average variable of interest $\overline{x}=\frac{1}{2}(x+x')$:

$$\begin{aligned}(k^2+\mathrm{i}k^\mu\overline{\partial}_\mu-\frac{1}{4}\overline{\Box}+m^2)G^{(+)}(k,\overline{x}) \\ +\int \mathrm{d}V_{x''}\frac{\mathrm{d}^np}{(2\pi)^n}\mathrm{e}^{\mathrm{i}(x''-x')}S(k,x+x'')G^{(+)}(k+p,x''+x') = \lambda G^{(+)}(k,\overline{x}).\end{aligned} \tag{31}$$

In order to find eigenvalues, required for a stable expansion in Fourier space, it is necessary to extract the factor of $G^{(+)}(k,\overline{x})$ from this expression. This is not necessarily possible for arbitrary values of k. It is possible, however, if the momenta are restricted to a set expressed by the property

$$G^{(+)}(k+p,x''+x')=G^{(+)}(k,x''+x'). \tag{32}$$

The differential equation satisfied by $G^{(+)}(x,x')$ is thus, in terms of the new derivatives,

$$\left[-D^2+m^2+\overline{K}^2(k,\overline{x})+\overline{I}(k,\overline{x})-\tilde{J}(k)+\frac{\mathrm{i}}{2}(\overline{\partial}_\mu\overline{I})(T^\mu-v_g^\mu/\omega)\right]_k G^{(+)}(x,x')=0, \tag{33}$$

where the appearance of the subscript k to the bracket serves to remind that the equation exists under the momentum integral. This relation relates the time and space dependence (lack of translational invariance) to the spectral content of the field, i.e. it forges the link between $\overline{x}$ dependence and k_μ dependence.

The presence of the D^2 implies the existence of quasi-particles: an effective symbol which replaces a conventional one. The positive frequency field may be 'gauge transformed' using the integrating factor (Wilson line)

$$\phi(k) \quad \rightarrow \quad \phi(k)\mathrm{e}^{\int a_\mu \mathrm{d}\overline{x}^\mu}. \tag{34}$$

We might also refer to this as a quasi-particle transformation. It is also related to renormalizations [16]. This shows the explicit decay (amplification) of the k-th field mode. This transformation also has a nice physical interpretation in terms of the entropy of the modes, defined above. The Wilson line is the negative exponential of an entropy, showing how the field decays as the energy of a mode becomes unavailable for doing work, i.e. as its entropy rises. Note however that this does not describe the total entropy of the system, only a measure of the mode in question.

The above transformation should not be confused with similarity transformations on the closed time path action. Because of unitarity, the plus and minus components of closed time path fields have to satisfy a global $O(1,1)$ symmetry, which allows a certain freedom in the way one chooses to set up the solution of the system. One can choose, for instance, to work with Feynman Green functions and Wightman functions, or with advanced and retarded functions, or with general mixtures of these. The only constraint imposed by unitarity is that the sum of rows and columns in the action (i.e. in the argument of the exponential in the generating functional) remains zero when plus and minus labels are removed. The transformations considered here are field redefinitions. This need not even be a symmetry of the system, since a nonequilibrium system is often an incomplete (open) system.

The reason for the similarity in form between a gauge theory and a theory of field rescalings is that both are linked through the conformal group. This also explains the connection to the curved spacetime approach already referred to: general spacetime metrics are not of interest, but conformal rescalings are. Gauge theories bear the structure of the conformal group, not the Lorentz group and time dependent perturbations and changes of variable are also connected with inhomogeneous rescalings of the conformal group. One will not normally see a conformal symmetry in the original action because the effective field theories we are discussing are incomplete: they describe partial systems, in which we ignore heat baths and external influences etc. It can be argued that there one should study generalized descriptions of nonequilibrium field theory such that they are fully invariant with respect to conformal transformations. This should be explored further.

4. Real gauge theories

In order to further understand this source formulation and the meaning of the effective quantities we have introduced, it is useful to look at a real gauge theory.

The familiar example of scalar electrodynamics is chosen to preserve the scalar field structures which have already been introduced, as well as to extend them to a simple gauge symmetry. The action for scalar electrodynamics, in terms of a complex scalar field Φ is,

$$S = \int \mathrm{d}V_t \left\{ \hbar^2 c^2 (D^\mu \Phi)^\dagger (D_\mu \Phi) + m^2 \Phi^\dagger \Phi + \frac{\lambda}{3!} (\Phi^\dagger \Phi)^2 + \ldots + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right\}, \qquad (35)$$

where $D_\mu = \partial_\mu + \mathrm{i} e A_\mu$. This can also be written in terms of a two component field ϕ_A:

$$\begin{aligned} S \;=\; \int \mathrm{d}V_t \Bigg\{ & \frac{1}{2} \hbar^2 c^2 (\partial^\mu \phi_A)(\partial_\mu \phi_A) - e\hbar c (\partial^\mu \phi_A) \epsilon_{AB} A_\mu \phi_B \\ & + e^2 \phi_A \phi_A A^\mu A_\mu + \frac{\lambda}{4!} (\phi_A \phi_A)^2 + \ldots + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \Bigg\}. \end{aligned} \qquad (36)$$

The field equations are easily worked out and supplemented with generalized sources, just as in the scalar case. One thing which makes a true gauge theory different from the scalar example is that it cannot be purely quadratic. The interaction between scalar and vector field introduces third and fourth order terms. To simply drop these would turn the example into a number of decoupled scalar theories, at least at the pure state level. However, we may focus on the quadratic part of the model in an open statistical ensemble, in which non-zero mean-fields are generated by virtue of an external influence such as a heat bath, or ambient electromagnetic field. This preserves some of the gauge structure even at the level of quadratic dynamical fields, by introducing background mean-fields $\overline{\phi}_a(x)$ and $\overline{A}_\mu(x)$. These fields are not necessarily equilibrium states.

An immediate comparision which one can make between electrodynamics and the quasi-particle formalism, is between the current expressed in equation (6) and the conserved current of electrodynamics. The $J \cdot A$ coupling is reflected in the forms

$$\mathcal{J}^\mu_{11} A_\mu = e\epsilon_{ab} (\phi_a \partial_\mu \phi_b) A^\mu(x), \qquad \mathcal{J}^\mu_{12} K_\mu = \epsilon_{AB} (\phi_A D^K_\mu \phi_B) K^\mu(x, x'). \qquad (37)$$

From this one sees that K_μ plays the part of a non-local current which operates in the A, B space (emission/absorption). It trades off non-conservation in one part of the system against non-conservation in another part and therefore provides a mechanism for redistributing stuff (energy, charge etc.) around the system. This is also closely analogous to the phenomenon of Landau damping. The electrodynamical current can also be interpreted in the same way, but there it is a redistribution of the phase of the field with respect to the $U(1)$ symmetry which takes place. Owing to the relationship between gauge symmetry and unitarity, this amounts to saying that A_μ and K_μ are like each others' imaginary complements.

To investigate the formal similarities further, it is useful to introduce a notation to distinguish the indices. There are now two sets, the $O(1,1)$ indices $A, B = +, -$ which represent the closed time path unitarity space, and the $U(1)$ electromagnetic indices $a, b = 1, 2$ which represent the gauge symmetry. They can be combined:

$$\begin{aligned}\phi_A &= (\phi_+, \phi_-), &(38)\\ \phi_a &= (\phi_1, \phi_2), &(39)\\ \phi_A^a &= (\phi_+^1, \phi_+^2; \phi_-^1, \phi_-^2). &(40)\end{aligned}$$

We also have to deal with two flavours of field, the scalar and the vector field. These can be grouped into a generic vector by defining:

$$\Psi_\alpha = (\phi_a, A_\mu). \tag{41}$$

The action then takes the much simplified form

$$S_{\rm CTP} = \int \Psi_\alpha\, S_{\alpha\beta}\, \Psi_\beta = \int \Psi_\alpha \left[\begin{array}{cc} S_{ab} & S_{a\nu} \\ S_{\mu b} & S_{\mu\nu} \end{array}\right] \Psi_\beta. \tag{42}$$

Each term is a two-by-two matrix with closed time path components. The two-by-two structure is labelled by capital roman indices, i.e. the whole thing has indices S_{AB}. Introducing corresponding sources,

$$\text{scalar} \to \text{vector}, \qquad I \to I_{\mu\nu}\,, \qquad K^\lambda \to K^\lambda_{\ \ \mu\nu}\,, \tag{43}$$

it is possible to cast all of the fields in the same mould. The meaning of such sources can be then examined in the context of scalar electrodynamics. If we consider the terms which would appear in the different blocks for scalar electrodynamics expanded around a background field $\overline{\phi}_a$ and $\overline{A}_\mu$, the terms then look like equation (3) with a generalized form:

$$S_{ab} = \left(\begin{array}{cc} \hat{\alpha}_{ab} & \hat{\beta}_{ab} \\ -\hat{\beta}^*_{ab} & -\hat{\alpha}^*_{ab} \end{array}\right), \quad S_{\mu\nu} = \left(\begin{array}{cc} \hat{\alpha}_{\mu\nu} & \hat{\beta}_{\mu\nu} \\ -\hat{\beta}^*_{\mu\nu} & -\hat{\alpha}^*_{\mu\nu} \end{array}\right), \quad S_{a\nu} = \left(\begin{array}{cc} \hat{\alpha}_{a\nu} & \hat{\beta}_{a\nu} \\ -\hat{\beta}^*_{a\nu} & -\hat{\alpha}^*_{a\nu} \end{array}\right), \tag{44}$$

where the terms appearing are schematically of the form,

$$\begin{aligned}\hat{\alpha}_{ab} &= (-\Box + m^2)\delta(x,x')\delta_{ab} + I_{ab}(x,x') + e\epsilon_{ab}\delta(x,x')\,\overline{A}^\mu(x)\ \overset{\leftrightarrow}{\partial}_\mu\,,\\ \hat{\beta}_{ab} &= J_{ab}(x,x') + K^\mu_{ab}(x,x')\ \overset{\leftrightarrow}{D}{}^K_\mu + e\epsilon_{ab}\delta(x,x')\overline{A}^\mu\partial_\mu\,,\\ \hat{\alpha}_{\mu\nu} &= -\Box\,\delta(x,x')g_{\mu\nu} + I_{\mu\nu}(x,x') + e^2\overline{\phi}^2_{(\pm)}\,g_{\mu\nu}\,,\\ \hat{\beta}_{\mu\nu} &= e^2\overline{\phi}^2_{(\pm)}\,g_{\mu\nu} + J_{\mu\nu}(x,x') + K^\lambda_{\mu\nu}(x,x')\ \overset{\leftrightarrow}{D}{}^K_\lambda\,,\\ \hat{\alpha}_{a\nu} &= \epsilon_{ab}\overline{\phi}^{(\pm)}_b\partial_\mu\,,\\ \hat{\beta}_{a\nu} &= \epsilon_{ab}\overline{\phi}^{(\pm)}_b\partial_\mu\,. \end{aligned} \tag{45}$$

Generic sources I, J, K have, once again, been introduced. This is not to imply that all of these sources will be required or relevant for every calculation. We include them all here for generality, to show how they relate to more familiar terms, and how the parts of linear combinations provide essentially equivalent contributions to the dynamics. To write out every permutation of A, B indices would only be nebulous, so these permutations have been written $(\pm)$ etc. It is difficult not to confuse the various indices in these expressions, but the fact that this is so, also indicates just how analogous their roles are. The point of this 'gauge like' formulation is to acknowledge the covariance of underlying group transformations at work.

The derivative for the gauge field could have been written more symmetrically using a notation like $\overleftrightarrow{D}^A_\mu$, but for gauge theories there is always a gauge condition of the form

$$\partial_\mu A^\mu = \chi(\phi_a), \tag{46}$$

which is required in order to give the vector A_μ representation of the Lorentz group the same properties as the $F_{\mu\nu}$ representation. This condition allows us to rewrite connection terms in the form of other generic terms, already included, so there is no harm in writing partial derivatives here. Comparing the terms in equation (45) it is easy to see how the gauge field appear in much the same way as the sources K_μ. ϵ_{ab} is equivalent to a factor of $\mathrm{i} = \sqrt{-1}$ in a complex formulation of the scalar field.

Since some interactions can be replaced by generic sources at the quadratic level, a complex problem is abstracted into a simpler one, which can be discussed schematically, unencumbered by irrelevant detail. In particular we see how the background fields act as sources themselves and how the dynamical fields 'communicate' or interact via channels represented by these sources. Schwinger's source theory was clearly influenced by Shannon's information theory, in which data are transmitted from source to sink. Schwinger's insight was to realize that such generic communication is at the heart of all interaction between field fluctuations. Sources turn closed systems into open systems by allowing them to communicate either by overlapping, or by contact at a boundary. This is the heat-reservoir model of thermodynamics, thinly disguised and generalized.

5. Conformal covariance

In the preceding section we have identified a 'covariant derivative' with a pseudo-gauge field a_μ, for inhomogeneous (i.e. nonequilibrium) systems. This connection is, in fact, a familiar object in field theories of all kinds. It typically occurs together with transformations of the conformal group, or transformations which are position dependent. The conformal group contains the Poincaré group as a subgroup, and its importance is already known in connection with covariance of gauge theories, though it is perhaps not widely appreciated, as exemplified by the long-standing confusion over definitions of the energy-momentum tensor [17,18]. (Such confusion disappears when one acknowledges the fact that the gauge theory really belongs to the conformal group, not the Poincaré group, because of the freedom to perform spacetime dependent gauge transformations at arbitrary points.) Conformal transformations are usually expressed in terms of a position dependent scaling of the metric tensor:

$$g_{\mu\nu}(x) \to \Omega^2 \; g_{\mu\nu}(x). \tag{47}$$

Most discussions of conformal symmetry are restricted to two dimensional Euclidean space where the conformal symmetry is automatic for analytic functions by virtue of the Cauchy-Riemann relations. This restriction is not relevant here since we do not specifically need to use the analytic properties of the fields. The inhomogeneous (Abelian) part of a gauge transformation U has the form

$$\begin{aligned} \Phi &\rightarrow U\Phi, \\ A_\mu &\rightarrow A_\mu + U^{-1}(\partial_\mu U); \end{aligned} \tag{48}$$

in general relativity the trace of connection is

$$\Gamma^\sigma_{\lambda\sigma} = \frac{1}{2} g^{-1}_{\mu\nu}\, \partial_\lambda\, g_{\mu\nu}, \tag{49}$$

giving a transformation rule of

$$\Gamma^\sigma_{\lambda\sigma} \rightarrow \Gamma^\sigma_{\lambda\sigma} + \frac{\partial_\lambda \Omega}{\Omega} g^\mu{}_\mu. \tag{50}$$

Note also the form of equation (20) which relates to Landau damping, which is similar in the conjugate space. Clearly the form,

$$\Gamma_\mu = \partial_\mu \ln X = \frac{\partial_\mu X}{X}. \tag{51}$$

has a general significance. Indeed, logarithms which add scaling corrections are familiar in a different context: the renormalization group. There they are picked up by analytic continuation away from spacetime dimensionalities where correlation ‘loops’ are scale invariant. This can be seen by power counting. In a nonequilibrium theory, what is perhaps surprising is that these renormalization group logarithms, and the spatial inhomogeneity logarithms seen earlier, are connected. While k and $\tilde{x}$ are conjugate variables, k and $\overline{x}$ are a complementary pair, related by the dispersion relation.

The main reason for pointing out this connection is to motivate a deeper study of conformally covariant theories, but practical insights might also be made possible. In [19] it was shown, in an explicit example, how an inhomogeneous scaling transformation can be used to provide an exact quasi-field solution to the two-level atom, without having to adopt the standard rotating wave-approximation.

6. Beyond quasi-particles

Can a quasi-particle field theory be shown to have a significance beyond elementary and composite excitations of fields? Could we model other dynamical systems with more complex interactions, understanding how the nature of the dynamical entities changes with the nonequilibrium development?

One current area of interest is in biology. In biological development, very small interactions within cells induce changes at the molecular level (genes/proteins) whose final consequences lead to large and very dramatic changes in the way a total system develops macroscopically (phenotype). What starts as a few proteins bumping into one another, ends up as plant and animal life. The transition is a dramatic one and clearly cannot be described by any simple field theory because it involves complex interactions with time varying boundary conditions coupling a whole hierarchy of scales. In equilibrium one could coarse grain cells into a 'flesh-field' and things would then be well-described by a continuum hypothesis, but during development (far from equilibrium) that cannot be true. No simple, local field theory gives rise to such complicated, reproducible self-organization from a microscopic code.

What occurs in the evolution and development of biological organisms must therefore be due to the complicated time-dependent interactions between neighbouring composite objects, whose boundaries communicate modifications to dynamics for the duration of their contact. While each arbitrary part of a system follows microscopic laws in every detail, the totality of a complex system exceeds the sum of its parts because it involves structural information about how to put those parts together, i.e. the boundary conditions between neighbouring elements in a system. Cooperation and competition between neighbouring cells introduces huge complications. Could any recognizable features of this process be reproduced in a field theoretical model with time-dependent boundary conditions? This is clearly a tall order, but with small progressive steps it might be possible to fill in some of the 'magic' behind biological development, which is currently beyond field theoretical models.

The wisdom of effective field theory is that dynamical behaviour is, for many purposes, independent of the underlying nature of its key players. Precise details of interactions can be ignored to a determinable level of approximation. Effective field theories are particularly applicable to statistical systems where coarse graining is a central feature. Sources provide us with a way of modelling external influences. Could they be further used to model cells which interact through permeable cell boundaries? Cells, after all, have the generic structure of quasi-particles: a core surrounded by a cloud of screening material. Could a dynamical model, with appropriate interactions, provide a schematic description of biological systems by modelling them internally and externally, i.e. from the viewpoints of a cell and of a cluster of cells:

- A super-system of time-dependent cells which interact on contact.
- A single cell which experiences time-dependent interactions on its boundary due to other cells.

Such constructions might be useful in modelling the immune system, for instance. Whether or not this approach can lead to usable biological models is one thing, but in any event it would be a useful testing ground for nonequilibrium analyses at more microscopic levels, where field theory is prevalent. It is also important, in principle, to understand how the microscopic to macroscopic transition occurs in detail in genetic development.

7. Conclusions

A proper study of field theories which are covariant with respect to a conformal symmetry is needed, in order to understand nonequilibrium field theory. To date no specific detailed studies have been carried out in more than two dimensions.

A natural place to begin would be to take the simplest case of the quasi-gauge formalism presented here. This would be a quadratic scalar field theory with some specified time-dependent, external influence which can be thought of as a “boundary condition”. This problem has already been discussed in [19–22] in the context of quantum optics, but there has not been sufficient occasion to take these studies to an extensive conclusion. The simplest cases are those where one has a predetermined model of what the sources should look like, based on prior knowledge of the external environment to which a field theory is coupled. Hopefully more instructive examples can be found so that one might explore the statistical behaviour of a system in response to time-dependent interactions with external agent, such as in a biological cell interaction. Again it is expected that a covariant formulation would lead to insights about which effects are relevant and which are merely detail.

It is, of course, natural to suppose that changes of variable, i.e. changes of perspective will lead to transformations analogous to gauge transformations. After all, we are perturbing a system with sources which vary in space and time. This is precisely the nature of a gauge theory. Methods of solution which rely on diagonalization of the action will also involve transformations which depend on space and time. All such transformations demand covariant derivatives and transforming auxiliary fields. The amplification of modes makes this a recipe for a kind of space-time dependent renormalization group. Some authors have suggested making coupling constants run with time, but there are canonical restrictions associated with making coupling constants depend on space-time coordinates [16]. The idea of completion by general covariance is also reminiscent of the Vilkovisky-DeWitt effective action [23]; probably this also has an interpretation in nonequilibrium physics.

All we have presented here is formalism, but formalism with an explanatory value is not to be sniffed at. Whether practical benefits might emerge from quasi-particle descriptions in nonequilibrium field theory is an open question. More work is needed to overcome both the technical and conceptual difficulties involved.

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Нерівноважний принцип дії для квазіполів

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Квазічастинкові описи нерівноважної фізики дозволяють простежувати відповідні ступені вільности в обчисленнях статистичних систем і забезпечують важливий формальний підхід до квантової теорії поля поза рівновагою. Тут запропоновано формулювання нерівноважної квазічастинкової теорії поля. Для розвитку схематичних теорій поля, здатних описувати реальні системи, використовуються швінґерівський твірний функціонал за шляхами, замкненими в часі, і узагальнені джерела. Обґрунтовується важливість методу джерел для ефективних нерівноважних теорій, а також його застосовність до біологічної динаміки.

Ключові слова: *нерівноважна теорія поля*

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