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Theoretical investigation of light absorption and scattering by nanoparticles

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Abstract. In the framework of the dipole approximation we predict theoretically high magnitudes of: (1) oscillator strength transitions, and (2) the resonance light scattering ($\sigma_{ss}(\omega, a)$) and absorption ($\sigma_{abs}(\omega, a)$) cross-sections. We suggest that these peculiarities can be observed experimentally. We discuss the dependence of $\sigma_{abs}(\omega, a)$ and $\sigma_{ss}(\omega, a)$ on light frequency ω and radius a of the nanoparticle at different physical conditions.

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1. Introduction

Optical properties of nano-dimensional structures (NDS) which form systems consisting of small semiconducting or dielectric spherical particles (SP) with diameter, being of the order of 1-10 nm, distributed in the different material matrices [1-5] are a subject of intensive studies at the present time.

Large nonlinearity of the optical characteristics and short lifetimes of the photoexcited charged carriers in NDS allow us to treat such heterophase systems as prospective materials for the production of new optics and optoelectronic devices controlling, in particular, light signals.

The main task of this paper is to develop the theory of light absorption and scattering on the one-particle local states in the NDS. These are very interesting results for the studying the behaviour of NDS, which have a cluster structure [5-8].

Linear dimension of the semiconducting particles will be comparable with the linear dimension of the characteristic quasi-particles in semiconductors. Under this condition the effect of the spheric boundary between the two materials, having different dielectric constants (DC), can cause the size quantization of carriers energy spectra due to both the pure boundedness in space of the quantization region [4] and to the polarization interaction of charge carriers (CC) with the semiconducting SP surface [9-11].

It was proved in [9-10] that under some definite conditions the localized one – particle states (LOPS) of CC of different kinds might appear. Such states are, for example, the localized states of charge carriers in semiconductors near semiconductor or dielectric particle, called *the localized surface states* [11-12], or the localization of CC inside of the small spherical volume, called *the volume localized states*. The surface localization may appear near an outer or inner spherical interface between the two dielectric media and are called *the outer surface states* [11,12] or *the inner surface states* [13], respectively.

At present there exists a sufficient amount of experimental data confirming the existence of LOPS's under consideration. For example, the localization of CC and excitons have been studied in the copper chloride [4], cadmium sulphate [4,14] as well as SP in isoelectronic solid state solutions A_2B_6 [15,16]. In addition, the localization of the charge carriers on the antimony particle in *n*-type germanium and on the mercury particle placed in the dense mercury steam have been observed in [17] and [18], respectively.

2. One-particle local states of charge carriers

The problem of the CC localization near the spheric interface between two dielectric media has been analyzed in [9] using the following simple model of NDS: a SP with diameter a and dielectric permittivity (DPr) ϵ_2 is surrounded by matrix with DPr ϵ_1 and a quasiparticle with charge e

moving either in the medium with DPr ϵ_1 and effective mass m_1 near the interface surface (the outer problem (OP)) or in the medium with DPr ϵ_2 and effective mass m_2 inside spherical volume (the inner problem (IP)). In addition, the polarization interaction $U(r, a)$ (r denotes the distance between a charge carrier and the SP center) with the surface charges induced on the spherical interface has been taken into account. We point out that the magnitude of $U(r, a)$ depends on the relative dielectric permittivity $\epsilon = \epsilon_1/\epsilon_2$.

Let us write down the expression $U(r, a)$ for the interaction of charge carrier with the surface of SP [9] in the following forms: for the OP ($r > a$)

$$U(r, a) = \frac{e^2 \beta}{2\epsilon_1 a} \left[\frac{a^2}{r^2 - a^2} - \left(\frac{a}{r}\right)^2 F\left(1, \alpha, \alpha + 1; \left(\frac{a}{r}\right)^2\right) \right] \quad (1)$$

for the IP ($r < a$)

$$U(r, a) = \frac{e^2 \beta}{2\epsilon_2 a} \left[\frac{a^2}{a^2 - r^2} - \frac{1 - \alpha}{\alpha} F\left(1, \alpha, \alpha + 1; \left(\frac{r}{a}\right)^2\right) \right] \quad (2)$$

where $F(1, \alpha, \alpha + 1; z^2)$ is the Gauss hypergeometric function, $\alpha = \epsilon_1/(\epsilon_1 + \epsilon_2)$ and $\beta = (\epsilon_1 - \epsilon_2)/(\epsilon_1 + \epsilon_2)$

There exist two possibilities for the charge carriers moving near the SP:

1. The polarization interaction $U(r, a)$ (cf. Eqs (1),(2)) leads to the attraction of CC by the SP surface and formation of the outer (for $\epsilon < 1$ [9,10]) and inner (for $\epsilon > 1$ [11]) surface states;
2. For $e < 1$ the polarization interaction (2) causes repulsion of CC from the inner surface of SP and formation of the volume localized states [12,13] near the center of SP. This effect is called the *volume localization*.

We notice, that in [9-13] the potential $U(r, a)$ for IP has been obtained in the form of infinite series with respect to variable $(r/a) < 1$. It has been shown in [9-13] that with decreasing of the SP diameter a there appears the size-quantization effect reverting from CC localization because of the relative diminishing of the potential energy (1) or (2) in comparison with the kinetic energy. The smallest critical SP diameter a_c , for which the local state is emerging, is approximately equal to b_i denoting the average distance of charge carrier localized in the ground state over the flat interface surface

$$a_c \sim b_i = \frac{6}{|\beta|} a_{B_i}, \quad (3)$$

where $a_{B_i} = \epsilon_i \left(\frac{m_0}{m_i} \right) \left(\frac{\hbar^2}{m_0 e^2} \right)$ is the Bohr diameter of CC in the medium with ϵ_i ($i = 1, 2$); m_i and m_0 are the

effective masses in the medium with ϵ_i and the free carrier mass, respectively.

The energy spectrum of inner and outer surface states determined in [5,9,11] employed the variational approach with the variational function in the form: for inner surface states [5,11]

$$\chi_l(x, S_2) = A(S_2 - x)^{l+1} x(2S_2 - x) \exp(-\gamma_l(S_2)x) \quad (4)$$

and for outer surface states [5,9]

$$\chi_l(x, S_1) = Bx(S_1 + x) \exp(-\mu_l(S_1)x) \quad (5)$$

where l denotes the orbital quantum number of CC, $S_i = ab_i$, x is the CC distance from the SP surface in b_i units (cf. Eq. (3)). The dependence of the variational parameters $\gamma_l(S_2)$ and $\mu_l(S_1)$ on the radius S_i is depicted in Figs 1 and 2, respectively, for $l = 0, 1$.

It has been shown in [12,13] that the lowest-energy spectrum $E_{l,i}(S_2)$ of the CC volume local states in potential (2), for $\epsilon < 1$, is given by

$$E_{l,i}(S_2) = \omega_l(S_2) \left(t + \frac{3}{2} \right)$$

$$\omega_l(S_2) = \frac{2\sqrt{3}}{S_2^{3/2}} \times \sqrt{\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha}} \sqrt{\frac{\beta L^2}{2S_2(1+\alpha)} + \frac{(8+11\alpha+5\alpha^2)L^2}{2S_2(2+\alpha)}} \quad (6)$$

where $t = 2n_r + l = 0, 1, 2, \dots$ is the principal quantum number, $n_r = 0, 1, 2, \dots$ is the radial quantum number, $\omega_l(S_2)$ denotes the CC vibration frequency measured in

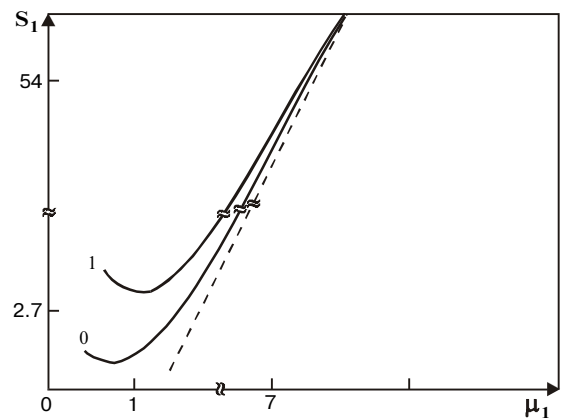


Fig. 1. Dependence of variational parameter $\mu_l(S_1)$ on the SP radius S_1 . The curves labeled by 0 and 1 correspond to quantum number values $l = 0$ and $l = 1$, respectively. The dotted line corresponds to $S_1 = (2/3)\mu_1$.

the $R_y = \frac{\hbar^2}{2m_2 a_{B_2}^2}$ units and $L_y^2 = l(l+1)$.

In [17] the energy spectrum of CC localized near the interface of two different dielectric materials has been calculated theoretically under assumption that dielectric permittivities and potential $U(r, a)$ are continuous at the interface. It can be shown that inclusion of noncontinuities of DP's and potential $U(r, a)$ leads to the corrections of order of $\leq 7\%$ in spectra $E_{i,l}(S_2)$ (for $l=0,1$) for all CC localized states [5,9–13] mentioned above.

3. Dipole moments of the charge carrier transitions

In the range of frequencies corresponding to the energies of the considered states of CC localized near spheric interface between two media, the electromagnetic wave length is much longer than the linear dimension of all local states which are of order of b_i (cf. Eq.(3)). Therefore, their behaviour under electromagnetic field action can be well described in the framework of the dipole approximation. Moreover, operators of the dipole moments for the outer and inner problem have the following form [20]:

$$\bar{D}(\mathbf{r}) = \left[1 - \frac{\beta}{1+\alpha} \left(\frac{a}{r} \right)^3 \right] e\mathbf{r} \quad (7)$$

and

$$\bar{D}(\mathbf{r}) = \frac{3\varepsilon_1}{2\varepsilon_1 + \varepsilon_2} e\mathbf{r}, \quad (8)$$

respectively.

In order to estimate the magnitudes of the dipole moments for CC transitions induced by the external electromagnetic field we will study the transitions between low-lying local states of all considered types taking into account the parity of the corresponding states. For example, transitions between s - and p -states are permitted for the dipole transitions, and changes $l \rightarrow l \pm 1$ of the orbital quantum number are accompanied with these transitions.

In order to calculate the matrix element of the dipole moment $D_{1,0}(S_i)$ for the charge carrier transition from s - to p -state, we assume that the monochromatic homogeneous electromagnetic wave $\mathcal{E}(\omega, t)$ propagates parallelly to the OZ-axis, where ω denotes the light frequency. As the external perturbation, causing the studied dipole transitions, we take for the OP and IP the dipole moments defined by Eqs (8),(9) which are induced by the external electromagnetic field $\mathcal{E}(\omega, t)$. Finally, the matrix elements of the transition dipole moments from s - to p -state of CC can be written in the form:

$$\bar{D}_{1,0}(S_i) = \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \int_{r_1}^{r_2} r^2 \psi_1(r, S_i) \bar{D}(r) \psi_0(r, S_i) dr, \quad (9)$$

where $\psi_1(r, S_i)$ and $\psi_0(r, S_i)$ are the wave functions of charge carrier in s - and p -state, respectively; θ and ϕ denote the polar and azimuthal angles, respectively; $r_1 = a, r_2 = \infty$ for outer local states and $r_1 = 0, r_2 = a$ for inner local states.

In the case of volume states such dipole transitions corresponds to the transition between the quantum states with $t = 0$ ($l = 0$) and $t = 1$ ($l = 1$). Using the formulae for wave functions of quantum oscillator and taking into account Eqs (6),(8) and (9) we can obtain the magnitude of the matrix elements of dipole transition moments $D_{1,0}$ between states with $t = 0$ and $t = 1$ localized in the SP center [21].

$$D_{1,0} = \frac{S_2^{3/4} e b_2 (3\varepsilon_1 / (2\varepsilon_1 + \varepsilon_2))}{3^{7/6} 2^{3/2} \left[\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha} \sqrt{\frac{\beta}{S_2(1+\alpha)}} + \frac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)} \right]^{1/4}} \quad (10)$$

In order to estimate magnitudes of the matrix elements of dipole transitions corresponding to the inner $D_{0,1}(S_2)$ and outer $D_{0,1}(S_1)$ surface states we will use the variational wavefunctions (4) and (5), respectively. Taking into account the formulae (4), (8) and (5), (7) we can get the matrix elements of dipole transitions (9) between the ground state with $l = 0$ and the first excited states with $l = 1$:

for inner surface states [23]

$$D_{1,0}(S_2) = \frac{L_2 S_2 e b_2}{3\sqrt{2}}$$

$$L_2 = 2^5 \frac{\sqrt{3\varepsilon_1} \mu_0^{7/2} \mu_1^{9/2}}{(2\varepsilon_1 + \varepsilon_2) (\mu_0 + \mu_1)^9} L_2^{(1)} L_2^{(2)} L_2^{(3)} \quad (11)$$

where

$$L_2^{(1)} = (\bar{\mu}_0 + \bar{\mu}_1)^5 - 16(\bar{\mu}_0 + \bar{\mu}_1)^4 + 125(\bar{\mu}_0 + \bar{\mu}_1)^3 - 570(\bar{\mu}_0 + \bar{\mu}_1)^2 + 1470(\bar{\mu}_0 + \bar{\mu}_1) - 1680,$$

$$L_2^{(2)} = \frac{1}{\sqrt{8\bar{\mu}_0^4 - 36\bar{\mu}_0^3 + 78\bar{\mu}_0^2 - 90\bar{\mu}_0 + 45}},$$

$$L_2^{(3)} = \frac{1}{\sqrt{4\bar{\mu}_1^6 - 30\bar{\mu}_1^5 + 123\bar{\mu}_1^4 - 330\bar{\mu}_1^3 + 585\bar{\mu}_1^2 - 630\bar{\mu}_1 + 315}},$$

$$\bar{\mu}_0 = \gamma_0(S_2)S_2,$$

$$\bar{\mu}_1 = \gamma_1(S_2)S_2$$

and for outer surface states

$$D_{1,0}(S_1) = 2^{-1/2} L_1 S_1 e b_1,$$

$$L_1 = 2^{5/2} \frac{(\mu_0 \mu_1)^{5/2}}{(\mu_0 + \mu_1)^6} \frac{\varepsilon_2 - \varepsilon_1}{(2\varepsilon_1 + \varepsilon_2)} \frac{3(\mu_0 + \mu_1)^2 + 12(\mu_0 + \mu_1) + 20}{\sqrt{3(\mu_0^2 + 3\mu_0 + 3)(\mu_1^2 + 3\mu_1 + 3)}} \quad (12)$$

where

$$\mu_0 = \gamma_0(S_1)S_1,$$

$$\mu_1 = \gamma_1(S_1)S_1.$$

and $j_0(x), j_1(x)$ are the Bessel spherical functions.

The variational parameters $\mu_0(S_1), \mu_1(S_1), \gamma_0(S_2), \gamma_1(S_2)$ occurring in the trial wave functions of the inner and outer surface localized states are determined [7,9].

We point out that numerical figures occurring in Eqs (10)-(12) result in calculations of dipole moments transitions [11] using variational or oscillator-type wave [22] functions for inner [4] and outer [5] surface states or volume [21] local states, respectively.

Let us note that Eq. (11), defining the dipole transition moment $D_{1,0}(S_2)$ for the inner surface states, is valid provided both inequalities $\exp(-\gamma_1(S_2)) \ll 1$ and $\exp(-\gamma_0(S_2)) \ll 1$ are satisfied. These conditions are fulfilled for arbitrary values of the variational parameters $\gamma_0(S_2)$ and $\gamma_1(S_2)$ (cf. Fig. 2) and for all magnitudes of the SP radii $S_2 > S_c$.

As the example, let us write down the magnitudes of the dipole transition moments $D_{1,0}(S_1)$ (cf. Eq. (12)) for the outer problem and $D_{1,0}(S_2)$ (cf. Eq. (11)) for the inner problem in SP with radius $S_1 = S_2 = S = 4$ [21]:

$$D_{1,0}(S_1) = 7.6 \frac{\varepsilon_2 - \varepsilon_1}{2\varepsilon_1 + \varepsilon_2} e b_1, \quad (13)$$

$$D_{1,0}(S_2) = 4.2 \frac{\varepsilon_1}{2\varepsilon_1 + \varepsilon_2} e b_2. \quad (14)$$

The magnitudes of $D_{1,0}(S_1)$ and $D_{1,0}(S_2)$ given by Eqs (13) and (14) are practically constant with increasing SP radius and increase slightly at $S \rightarrow \infty$. This property follows from the fact that radius a of the dipole has the linear dimension of the order of b_i radius state, which practically does not vary.

We should note that due to this property the variational functions (4) and (5) used for the estimation of the matrix element magnitudes of the transitions moments (cf. Eqs (10)-(15)) do not lead to a sufficient error and have an effect on the numeric prefactor being of the order of ~ 1 .

4. Light absorption and scattering

The results obtained so far and concerning the dipole moments of transitions for the volume states $D_{1,0}$ (cf. Eq. (10)) for the inner and outer states given by Eqs (11) and (12),

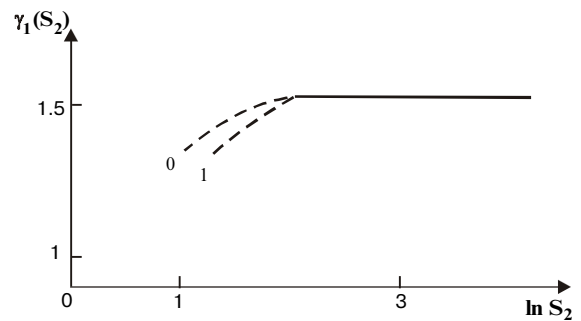


Fig. 2. Variational parameter $\gamma_1(S_2)$ versus $\ln(S_2)$, where S_2 denotes the SP radius S_2 ; the depicted numbers specify magnitudes of the quantum number l .

respectively, allow us to derive the light absorption and scattering in the considered NDS in the electromagnetic waves frequency range corresponding to the energies of LOPS's discussed in the previous section.

The absorption cross-section $\sigma_{abs}(\omega, a)$ of the SP with radius a is related to its polarization $A''(\omega, a)$ [22] in the form:

$$\sigma_{abs}(\omega, a) = \frac{4\pi}{c} \omega A''(\omega, a), \quad (15)$$

where c is light velocity in vacuum.

If the states of the charge carrier in SP are quantized, then their polarization $A''(\omega, a)$ can be easily found if we assume that the SP forms a single giant ion. In this case the polarization $A(\omega, a)$ of the charged SP can be expressed by the matrix elements of dipole transition moments between quantum states [22,23]. For sufficiently low temperatures fulfilling the inequality $T < E_B = (\frac{\hbar^2}{2m_i a_B^2})$, which are of the order of 1-10 K at $a_B \sim 10 \cdot 10^2 \text{ \AA}$, i.e. for temperatures less than the binding energy E_B of the considered states, we can write down the explicit form of the polarization

$$A''(\omega, a) = \frac{e^2}{m_i} \sum_j \frac{f_{0j}(a)}{\omega_j^2(a) - \omega^2 - i\omega\Gamma_j(a)}, \quad (16)$$

where

$$f_{0j}(a) = \frac{2m_i}{\hbar e^2} (\omega_j(a) - \omega_0(a)) |D_{j0}(a)|^2 \quad (17)$$

is the oscillator strength of the charge carrier with effective mass m_i transition from the ground to the j -th quantum state, ω_0 is the energy of the ground state of the charge carrier, $D_{j0}(a)$ denotes the matrix element of the dipole transition moment, $\hbar\omega_j(a)$ is the energy of the j -th quantum state and $\Gamma_j(a)$ means the width of the j -th excited state [10,12].

If the bound states are absent, then $D_{j0}(a)/e$ will be proportional to the linear dimension a of the delocalization range of the charge carrier (for large j we have $D_{j0}(a) = a \cdot e/j$ [22]). Moreover, the oscillator strength $f_{0j}(a) \sim |D_{j0}/e|^2 \sim a^2$. In this case polarization $A''(\omega, a)$ (cf. Eq. (16)) and the absorption cross-section (15) for SP with radius a have the following forms:

$$A''(\omega, a) = \omega \cdot f(\omega) a^2 \quad (18)$$

$$\sigma_{abs}(\omega, a) = \frac{4\pi}{c} \omega^2 \cdot f(\omega) a^2$$

since $f(\omega)$ does not depend on a , and $f(\omega)$ is weakly varied with ω [21]:

$$\sigma_{abs}(\omega, a) \sim \omega^2 a^2 \quad (19)$$

If the bound states of CC were formed near the interface between two different media, then the main contribution to $A''(\omega, a)$, for $T < E_B$, would come from the transitions between quantum states of a discrete spectrum. The contribution of a single resonance state into (16) can be written in the form

$$A''(\omega, a) = f_{01}(a) \cdot F(\omega_1, \omega), \quad (20)$$

where

$$F(\omega_1, \omega) = \frac{e^2}{m_i} \cdot \frac{1}{\omega_1^2 - \omega^2 - i\omega\Gamma_1(a)} \quad (21)$$

has the usual resonance form and in the vicinity of the resonance it is independent of a [23]. Let us note that we have isolated single resonance contribution corresponding to the transition between the ground s - and p -states discussed above for which $f_{01}(a)$ is defined by Eqs (17) and (10)-(12).

It is evident that a restriction of our studies to the consideration of the s - and p -states does not influence on the obtained behaviour of $A''(\omega, a)$ and $\sigma_{abs}(\omega, a)$ as the function of the SP radius a since the dependence of the oscillator strength $f_{01}(a)$ has the same character for other states (cf. Eq. (17)).

From Eqs (15), (20) and (21) it follows that the resonance light

$$\sigma_{abs}(\omega, S_i) = 4\pi \frac{\omega}{c} f_{01}(S_i) F(\omega_1, \omega), \quad (22)$$

$$f_{01}(S_2) = \frac{\frac{m_2 b_2^2}{\hbar} \left(\frac{\epsilon_1}{2\epsilon_1 + \epsilon_2} \right) S_2^{3/2} \omega_1}{4 \cdot 3^{1/3} \sqrt{\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha}} \cdot \sqrt{\frac{\beta}{S_2(1+\alpha)}} + \frac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)}} \quad (23)$$

where the oscillator strength $f_{0i}(S_i)$, in accordance with Eqs (10) and (17), for the volume states localized in the SP center with radius S_i takes the form Eq. (23).

The transition oscillator strength $f_{0i}(S_i)$ exhibits, for the surface and the inner states, the same dependence on SP radius S_i and is given by

$$f_{0i}(S_i) = \frac{m_i b_i^2}{\hbar} \cdot S_i^2 L_i^2 \omega_i, \quad (24)$$

where the parameters L_i for inner ($i = 2$) and outer ($i = 1$) surface states are defined by Eqs (11) and (12), respectively.

We point out that the comparison of Eqs (22)÷(24) exhibits the different dependence character for the light absorption cross-section $\sigma_{abs}(\omega, a)$ on frequency ω and radius a of the dielectric particle, under different physical conditions.

If the bound states near the spheric interface between two media do not exist, then, in accordance with Eq. (19), we have

$$\sigma_{abs}(\omega, a) \sim \omega^2 a^2.$$

In the case of the volume states of CC the absorption cross-section $\sigma_{abs}(\omega, a)$ is defined by Eqs (22), (23) and

$$\sigma_{abs}(\omega, a) \sim F(\omega_1, \omega) \cdot \omega \cdot a^{3/2}. \quad (25)$$

For the surface inner and outer states the absorption cross-section $\sigma_{abs}(\omega, a)$, in accordance with (22) and (24), has the same dependence on SP radius a :

$$\sigma_{abs}(\omega, a) \sim F(\omega_1, \omega) \cdot \omega a^2. \quad (26)$$

Thus, the localization of CC on the spheric interface and inside the NDS leads to qualitatively different dependence of the absorption of electromagnetic field on radius SP a and light frequency ω . This fact gives an additional experimental possibility to spectroscopic findings and studies of the considered localized states of charge carriers.

A similar possibility also gives the elastic scattering of electromagnetic waves with frequency ω on the SP. The scattering cross-section $\sigma_{sc}(\omega, a)$ is given by [22]

$$\sigma_{sc}(\omega, a) = 2^7 \cdot 3^{-3} \cdot \pi^3 |A''(\omega, a)|^2 \cdot \left(\frac{\omega}{c}\right)^4 \quad (27)$$

and, according to Eqs (20)÷(24) and (18), $\sigma_{sc}(\omega, a)$ exhibits different dependences on the radius a and frequency ω for various types of considered localized states.

In fact, for inner and outer surface states, according to Eqs (20), (21), (24), the cross-section $\sigma_{sc}(\omega, a)$ of the elastic scattering of electromagnetic waves is given by the formula

$$\sigma_{sc}(\omega, a) = 2^7 \cdot 3^{-1} \cdot \pi^3 \cdot \left(\frac{\omega}{c}\right)^4 \times F^2(\omega_1, \omega) \cdot L_i^2 \cdot \left(\frac{m_i b_i^2 \omega_1}{\hbar}\right)^2 \cdot S_i^4. \quad (28)$$

The scattering cross-section $\sigma_{sc}(\omega, S_2)$, for the volume local states, according to Eqs (20), (21) and (23), has the following form:

$$\sigma_{sc}(\omega, S_2) = \frac{2^3 \cdot 3^{-5/3} \cdot \pi^3 \cdot \left(\frac{\omega}{c}\right)^4 \cdot F^2(\omega_1, \omega) \cdot \left(\frac{\epsilon_1}{2\epsilon_1 + \epsilon_2}\right)^4 \cdot S_2^3 \cdot \left(\frac{m_i b_i^2 \omega_1}{\hbar}\right)^2}{\frac{8\beta}{3(1+\alpha)} + \frac{2(7+5\alpha)}{2+\alpha} \cdot \sqrt{\frac{\beta}{S_2(1+\alpha)} + \frac{8+11\alpha+5\alpha^2}{S_2(2+\alpha)}}}. \quad (29)$$

If the bound states near the SP do not occur, then, according to Eq. (18), the dependence of the cross-section $\sigma_{sc}(\omega, S_i)$ (cf. Eq. (27)) on ω and S_i is given by

$$\sigma_{sc}(\omega, S_i) = 2^7 \cdot 3^{-1} \cdot \pi^3 \cdot \left(\frac{b_i}{c}\right)^4 \omega^6 \cdot f^2(\omega) \cdot S_i^4. \quad (30)$$

The optical coefficient of attenuation $\eta(\omega, a)$, which takes into account the light absorption and scattering effects on the considered LOPS's near the small dielectric particles with radius a and concentration N , may be written in the form

$$\eta(\omega, a) = N[\sigma_{abs}(\omega, a) + \sigma_{sc}(\omega, a)] \quad (31)$$

The last expression may be used for a group of noninteracting NDS. This condition is fulfilled if the inequality

$$a_c \ll \frac{1}{N^{1/3}} \quad (32)$$

holds true, where $a_c \sim b_i$ (cf. also Eq. (3)) denotes the SP's radius. The condition (32) is fulfilled for concentration not exceeding $N \leq 10^{14} \text{ cm}^{-3}$ of small cadmium sulphate or cadmium selenite particles with $a_c \sim b_i \sim 10^2 \text{ \AA}$ (cf. [9]-[13] as documented in experimental studies [1, 2, 14].

5. Comparison of the theory with experimental data

Finally, we briefly discuss physical situations for which the results obtained in this paper are of the particular importance. An interesting feature of the studied states of charge carriers localized on the spheric interface is removing of degeneracy with respect to the orbital quantum number l . As a result the distance between the energy sublevels will be sufficiently small [7-11].

If consider that: (A) The dipole moments of transitions for all the considered local states, according to Eqs (10)-(12), have the large magnitudes of $D_{1,0}(a) \sim e \cdot b_i \gg 1D$ (i.e. $D_{1,0}(a)$ exceeds typical magnitudes for the transition dipole moments of the nonorganic semiconducting materials for which $D_{1,0}(a) \sim 10^{-1} D$ [24]; (B) The allowed dipole transitions induced by the electromagnetic field between the nearest energy sublevels correspond to a change in the quantum number l by ± 1 , then it is evident that the discussed NDS are strong non-linear media for the electromagnetic radiation. The last circumstance represents a special interest for construction of novel nonlinear devices in a wide wavelength range with its magnitude depending on the nature of the contacting materials.

As an example, let us consider a nano-dimensional system which consists of the spherical semiconducting parti-

cle of CdS with radius $a \cong 54 \text{ \AA}$ placed in the boric-silicate glass. In this system it was found experimentally the oscillator-like energy spectrum for the hole states localized near the particle centre with the energies of $\sim 10^2 \text{ meV}$ [25]. In [26],[27] it was shown that the spectrum of the hole volume local states studied experimentally in [25] consists of a series of equidistant energy levels, and the distance between these energy sublevels is proportional to $a^{-3/2}$. The dipole moments of hole transitions between such equidistant sublevels, according to Eq. (10), take high values, i.e. $D_{1,0} \sim 10 \div 20 D$.

Let us carry out a qualitative estimation of the cross-section $\sigma_{abs}(\omega, a)$ and $\sigma_{sc}(\omega, a)$ of light on the above mentioned local states of CC for the isolated transition $|0\rangle \rightarrow |1\rangle$ under the experimental conditions of [1]-[4],[23]. Let us assume that the light frequency ω is not comparable with the resonance frequency ω_1 of the local state in the SP and let us suppose also that the broadening Γ_1 of the energy level $E_1 = \hbar\omega_1$ is sufficiently small, i.e. $(\Gamma_1/\omega_1) \ll 1$. Then we can estimate the magnitudes of the cross-sections $\sigma_{abs}(\omega, a)$ and $\sigma_{sc}(\omega, a)$ using Eqs (15), (23), (24) and (27) for which the SP polarization takes the form

$$A''(a) = \frac{e^2}{m_i} \cdot \frac{f_{01}(a)}{\omega_1^2}. \quad (33)$$

In Table I the estimated magnitudes of: the oscillator strength $f_{01}(a)$ (cf. Eqs (23) and (24)), the polarization $A''(\omega, a)$ (cf. Eq. (33)), the light absorption cross-sections $\sigma_{abs}(\omega, a)$ (cf. Eq.(15)) and $\sigma_{sc}(\omega, a)$ (cf. Eq. (27)) corresponding to the specified above one-particle localized states are listed for NDS containing the semiconducting (dielectric) matrix and dispersed in it the semiconducting (or dielectric) SP with radius $a \geq a_c$ [7-11].

From the presented estimations it follows that the cross-section $\sigma_{abs}(\omega, a)$ takes super high magnitudes of $\sigma_{abs}(\omega, a) \approx 1.3 \cdot 10^{-16} \text{ cm}^2$ for light absorption on the hole volume local states in the semiconducting GaAs particle embedded in boric-silicate glass matrix (or in another medium with

Table I. Parameters of electron and hole bound states localized on nano-dimensional structures, made from materials specified in the second column, with dielectric permittivity ϵ_2 (cf. the third column) placed in dielectric materials with dielectric permittivity ϵ_1 ; a - the radius of SP, m_h - the effective hole mass in SP, E_1 - the electron or hole bound energy, f_{01} - the oscillator strength transition, A'' - the polarization of SP, σ_{abs} and σ_{sc} - the cross-section of light absorption and scattering, respectively.

ϵ_1	SP	$a(\text{\AA})$ (S)	ϵ_2	m_h/m_0	E_1 (meV)	f_{01} (10^{-1})	A'' (10^{-24} cm^3)	σ_{abs} (10^{-24} cm^2)	σ_{sc} (10^{-24} cm^2)
1.5	GaAs	340 (4)	13.2	0.62	50	3.56	$2.6 \cdot 10^2$	$1.3 \cdot 10^8$	$2.4 \cdot 10^{-3}$
1.5	CdS	54 (12.65)	9.3	5	50	0.18	$1.6 \cdot 10^2$	$4 \cdot 10^4$	$6 \cdot 10^{-13}$
1	He	420 (4)	1.06		0.65	3.81	$4.8 \cdot 10^3$	$1.6 \cdot 10^4$	$1.6 \cdot 10^{-17}$

$\epsilon_1 \ll \epsilon_2$). We point out that the magnitude of $\sigma_{abs}(\omega, a) \approx 1.3 \cdot 10^{-16} \text{ cm}^2$ is 10^8 times larger than the typical atomic values of $\sigma_{abs}(\omega, a) \sim 10^{-24} \text{ cm}^2$ [24].

The observed values of the cross-section $\sigma_{abs}(\omega, a)$ take $\sim 10^{-20} \text{ cm}^2$ for the light absorption by the inner surface states of the electron in the bublon (a macroscopic spherical cavity in a liquid superfluid) embedded in superfluid helium [30] as well as by the hole volume local states in the cadmium sulphate SP placed in boric-silicate glass matrix [25,28]. In addition, the elastic light scattering cross-section σ_{sc} by the mentioned one-particle local states will be sufficiently small in comparison with σ_{abs} , i.e. $(\sigma_{sc}/\sigma_{abs}) \sim 10^{-11}$.

This leads to the conclusion that the light attenuation coefficient $\eta(\omega, a)$ (cf. Eq. (31)) is determined mainly by processes of light absorption by one-particle local states of charge carriers. The magnitude of $\eta(\omega, a)$, corresponding to light absorption by local states of NDS described in Table I, takes the values of $\eta(\omega, a) \sim (10^{-2} \div 10^{-6}) \text{ cm}^{-1}$ for the SP with radius $a > a_c$ (cf. Eq. (3)) fulfilling condition (32) for concentration of $N = 10^{14} \text{ cm}^{-3}$.

In this way the large absorption cross-section σ_{abs} of the electromagnetic field by one-particle local states of charge carriers occurring in the vicinity of the semiconducting (dielectric) particle in NDS offers an interesting possibility of applying such heterophase structures to new materials which strong electromagnetic waves absorption (in particular: the light) in a wide wavelength range with its magnitude depending on the nature of the materials in contact.

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