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Parametric modeling of global TEC fields

A. S. Prokhorenkov[1*], *V. Ya. Choliy*[1,2]

[1]Taras Shevchenko National University of Kyiv, Glushkova ave., 4, 03127, Kyiv, Ukraine
[2]Main Astronomical Observatory of NAS of Ukraine, Akademika Zabolotnoho St., 27, 03680, Kyiv, Ukraine

We applied the autoregressive moving average statistical modelling to analysis and forecasting of the global Total Electron Content maps. Autoregressive moving average is a parametric time series and fields modelling method, widely used in applied statistics. The methodology is applied to Total Electron Content modelling first-time. We present the new software and first results.

Key words: modeling and forecasting; instruments and techniques

INTRODUCTION

The Total Electron Content (TEC) is one of the most important descriptive characteristics of the Earth ionosphere. It should be taken into account in determination of scintillation and group delay of a radio wave. TEC can be determined by measuring the phase delays of received radio signals from satellites located above the ionosphere.

Important aspect in ionosphere study is a good representation of TEC. So, in 1996 The International GPS Service for Geodynamics (IGS), which provides precise GPS orbits, Earth orientation parameters (EOPs), station coordinates, satellite clock information and other related stuff, proposed a new product. It is the Global Ionosphere TEC maps in RINEX IONEX [4] format. These are epoch specific 2D and 3D TEC maps. They are permanently built by IGS for easy exchange, compare and combine the TEC maps. Every TEC file contains a sequence of TEC maps for the given day. IONEX files can be found on NASA web-site[1]. The main problem with IONEX maps is that they are published with a delay, which may be critical for some ionosphere-related applications.

THE METHOD

Different methods can be applied to process and forecast ionosphere state at time point. In the present work autoregressive moving average `ARMA[p,q]` [1] statistical algorithm was introduced to process and forecast TEC field.

We consider a TEC value above some Earth's point with known coordinates λ and φ as a time series. Let assume that the value z_t at the moment t depends on p previous values $(z_{t-1}, z_{t-2}, \ldots, z_{t-p})$ as a linear combination. From another point of view z_t depends upon linear combination of q previous values of white noise, with zero mean and unknown dispersion $(\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-q}, \varepsilon_t = \mathcal{N}(\mu = 0, \sigma^2))$. See Fig. 1 for the flowchart of the ARMA process.

General equation for the `ARMA[p, q]` model is:

$$z_t - \sum_{i=1}^{p} \varphi_i \cdot z_{t-i} = \varepsilon_t - \sum_{i=1}^{q} \psi_i \cdot \varepsilon_{t-i}, \qquad (1)$$

where, φ_i are parameters of the autoregressive part, ψ_i are parameters of the moving average part.

To use the model (1) one need to identify the process by selecting appropriate p and q and then determine $p + q + 1$ parameters: φ_i, ψ_i and σ^2.

By introducing the autocorrelation function (ACF):

$$r_k = \frac{\mathrm{E}[(z_t - \bar{z})(z_{t+k} - \bar{z})]}{\sqrt{\mathrm{E}[(z_t - \bar{z})^2]\mathrm{E}[(z_{t+k} - \bar{z})^2]}}, \qquad (2)$$

where $\mathrm{E}[z_t]$ is an expected value of a random variable, autoregressive part of the general equation (1) may be transformed into Yule-Walker [5, 6] linear equation, solution of which is preliminary values of

*andrej.prokhorenkov@gmail.com
[1]`ftp://cddis.gsfc.nasa.gov/pub/gps/products/ionex/`

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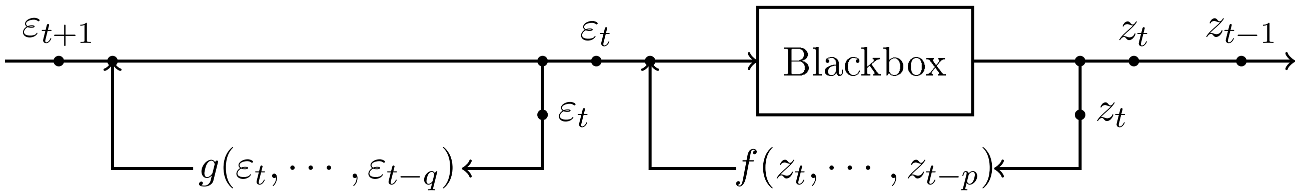

Fig. 1: General representation of ARMA process.

the autoregressive AR[p] coefficients:

$$\underbrace{\begin{pmatrix} r_q & r_{q+1} & r_{q+2} & \cdots & r_{q+p-1} \\ r_{q+1} & r_q & r_{q+1} & \cdots & r_{q+p-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{q+p-1} & r_{q+p-2} & r_{q+p-3} & \cdots & r_q \end{pmatrix}}_{\mathbf{R}} \times$$

$$\times \underbrace{\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_p \end{pmatrix}}_{\mathbf{\Phi}} = \underbrace{\begin{pmatrix} r_{q+1} \\ r_{q+2} \\ \vdots \\ r_{q+p} \end{pmatrix}}_{\mathbf{r}} \qquad (3)$$

for given p and q.

Initial value for the moving average MA[q] parameters may be determined from the following system of non-linear equations:

$$r_k = \begin{cases} \dfrac{-\psi_k + \psi_1\psi_{k+1} + \cdots + \psi_{q-k}\psi_q}{1 + \psi_1^2 + \cdots + \psi_q^2}, & k \leq q; \\ 0, & k > q, \end{cases} \qquad (4)$$

using, for example, the Newton-Raphson algorithm.

Precise values of AR and MA parameters may be determined only using some optimization procedure, for example, Levenberg-Marquardt algorithm [2, 3]. As the equation are quite non-linear it causes the main computational difficulties.

RESULTS AND CONCLUSIONS

- We developed the software to identify the `ARMA` model and to calculate values of AR and MA parameters.
- For testing purposes we selected IONEX maps for days around solstices and equinoxes of 2010.
- Plots for autocorrelation and partial-autocorrelation functions (PACF) for several Earth's points along the prime meridian are presented ($\varphi = -87.5°$, $\varphi = \pm 0.0°$ and $\varphi = +87.5°$) in Fig. 2.
- From the graphs we can roughly determine the degrees of AR and MA parts of the model. Oscillation pattern of the autocorrelation function shows that $p \geq 2$. Exponential decay of the partial-autocorrelation function hints on $q \sim 1$. In total we have got `ARMA`[≥ 2, ~ 1].

  During equinoxes we have less representative oscillations near poles, which can be interpreted as smaller values of AR coefficients. On equator we have mostly the same behaviour both for autocorrelation and partial-autocorrelation functions regardless of the day. This confirms the hypothesis that on equator the seasonal position of the Earth plays a minor role in ionosphere state. Differences between ACF on poles during solstice are obvious and caused by ionizing Solar cosmic rays. Partial autocorrelation functions look similar for the first few hours (except for one pole during solstice, where we have some reduction of PACF which determines the same phenomena.
- We have a hope that our software will be quite useful in analysis and forecasts of ionospheric TEC fields.

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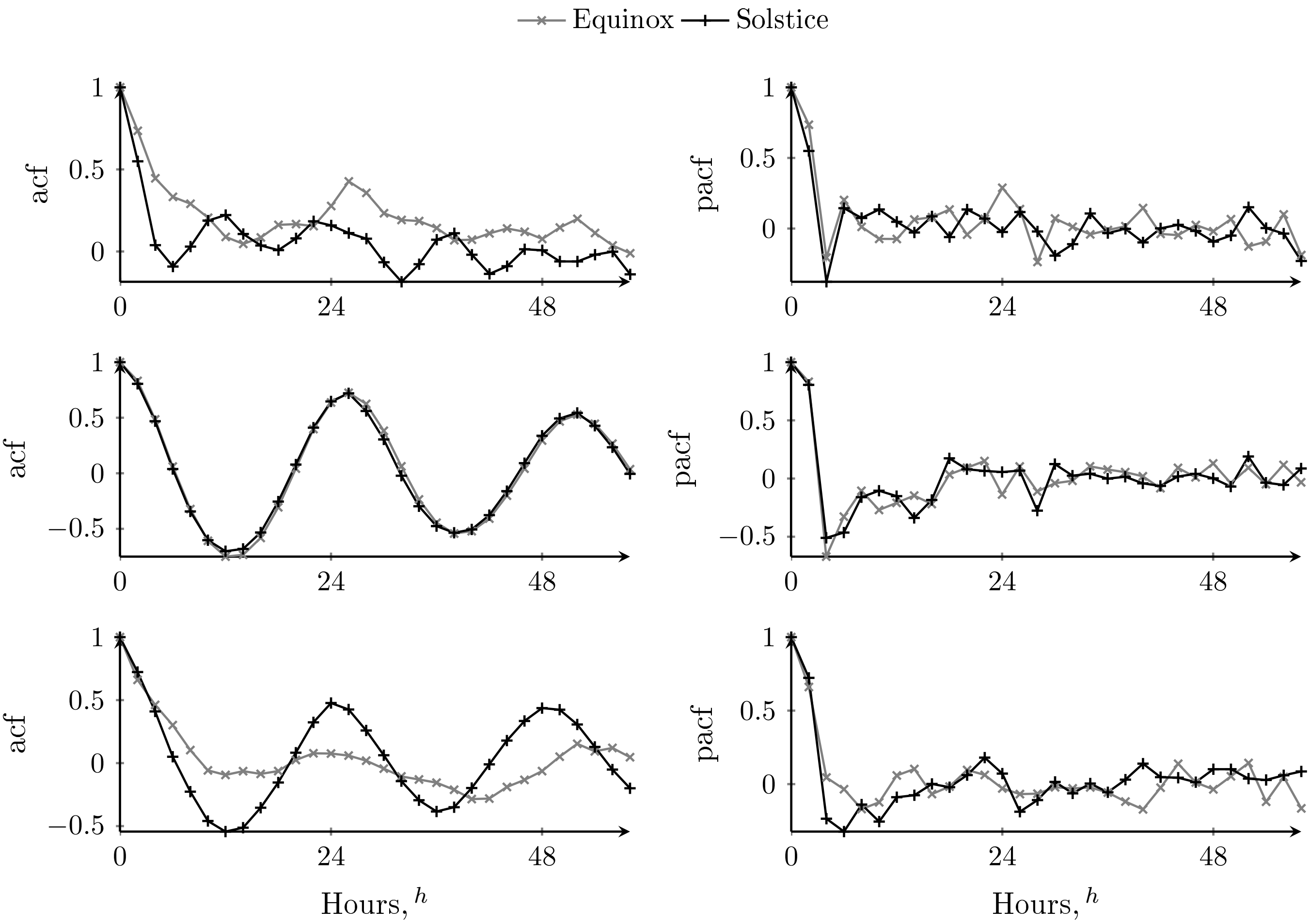

Fig. 2: Autocorrelation and partial autocorrelation functions for different latitudes.