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Theory of free-carrier absorption in the presence of a quantizing magnetic field in quasi-one-dimensional quantum well structures

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Abstract. The theory of free-carrier absorption is given for a quasi one-dimensional semiconducting structures in a quantizing magnetic field for the case when carriers are scattered by polar optical phonons and acoustic phonons and the radiation field is polarized perpendicular to the magnetic field direction. The usual resonance condition $P\omega_c = \Omega + \omega_0$, where P is an integer and w_0 and w_c are the optical-phonon frequency and cyclotron frequency, respectively, becomes $P\omega = \Omega + \omega_0$' with $\tilde{\omega}$ equal to $\sqrt{\omega_c^2 + \omega^2}$. The magnetic field dependence of the absorption for the transverse configuration can be explained in terms of phonon-assisted transitions between various Landau levels of carriers.

Keywords: quantizing magnetic field, quantum well.

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1. Introduction

The application of magnetic field to crystal changes the dimensionality of electronic levels and leads to a redistribution of the density of states. Quantum well wires (QWW) in a magnetic field have been the subject of a few investigations [1-5]. In [1] rectangular QWWs were treated in the decoupled approximation. Concerning the theoretical work on magnetotransport in QWWs, we are aware of the Hall resistivity treatments of Refs 2 and 3 and of magnetophonon oscillations in Ref. 4. In [5] the field-induced change in optical anisotropy was studied for a quasi - two- dimensional (Q2D) system subject to a periodic modulation. The effect of a magnetic field on conductance quantization in quasi-one –dimensional (Q1D) systems is reviewed in [6]. An essential progress in techniques of growth on patterned substrates and of cleaved-edge overgrowth has led to QWWs with very good optical properties [7–10], thus renewing the interest for the basic properties of Q1D systems.

In this work, we are interested in the effect of magnetic field on the free carrier absorption (FCA) in semiconductor QWWs. Over the past two decades the investigation of FCA in low-dimensional systems has been very intense. FCA is one of the powerful means to understand the scattering mechanisms of carriers. In bulk semiconductors, it accounts for the absorption of electromagnetic radiation of frequencies Ω such that $\hbar\Omega < E_g$, where E_g is the band gap [11]. In quantum well (QW) structures, apart from the direct interband and intersubband optical transitions, the optical absorption can also take plase via indirect intrassubband optical transitions in which carriers absorb or emit photons with a simultaneous scattering by phonons or other imperfections. The quantum theory for FCA in Q2D structures is well developed both in the absence [12–22] and in the presence of quantizing magnetic fields [23]. In the works [23], we have extended the theory of FCA in Q2D systems in the presence of a quantizing magnetic field when phonon scattering is important, and it was found that FCA coefficient oscillates as a function of the magnetic field and photon frequency with resonances occuring when $P\omega_c = \Omega \pm \omega_0$, where ω_c, Ω and ω_0 are the cyclotron, photon and phonon frequencies, respectively, and where P is an integer. The theory FCA has been studied theoretically in quasi-one dimensional (Q1D) structures only in the absence of quantizing magnetic field [24–27].

In this paper, we extend the quantum theory of FCA developed previously to take into account the presence of quantizing magnetic fields. We consider FCA for the case when carriers are scattered by the alloy disorder, acoustic phonons and boundary roughness. We will present a calculation of FCA coefficient for electromagnetic radiation polarized along the length of the wire. The magnetic field is assumed to be perpendicular to the wire axis, so that the dispersion of one-dimensional subbands is strongly modified.

2. Formalism

We consider Q1D electron gas confined in a wire of dimensions L_x, L_y, L_z. We model transverse confinement via an infinite square well approximation to a heterojunction quantum well (z axis) and a parabolic potential of frequency ω (x axis). Moreover, a magnetic field B, parallel to the z axis, is applied to the wire. The electrons are free in the direction of the wire (y axis). Correspondingly, the one-electron eigenfunctions Ψ_{Nlk_y} and energy eigenvalues E_{Nlk_y} are given by

$$\Psi_{Nlk_y} = \left(\frac{2}{L_y L_z}\right)^{1/2} \Phi_N(x - x_0) e^{ik_y y} \sin\left(\frac{l\pi z}{L_z}\right) \quad (1)$$

$$E_{Nlk_y} = \left(N + \frac{1}{2}\right)\hbar\tilde{\omega}_c + \frac{\hbar^2 k_y^2}{2\tilde{m}^*} + l^2 E_0 \quad (2)$$

where $N = 0,1,2,\ldots$, $l = 1,2,3,\ldots$, and $E_0 = \pi^2\hbar^2 / 2m^* L_z^2$, k_y is the wave vector in the y direction, m^* is the effective mass of the electron, $\omega_c = eH/m^*c$ is the cyclotron frequency, $\tilde{\omega} = \sqrt{\omega_c^2 + \omega^2}$, $\tilde{m} = m^*\tilde{\omega}^2/\omega^2$. Moreover, $\Phi_N(x - x_0)$ is the well-known harmonic-oscillator wave function centered at $x_0 = \tilde{b}\tilde{R}^2 k_y$ with $\tilde{b} = \omega_c/\tilde{\omega}$ and $\tilde{R}^2 = \hbar/m^*\tilde{\omega}$.

The FCA coefficient α, which is related to the quantum-mechanical transition probabilities in which the carriers absorb or emit a photon with the simultaneous scattering of carriers by phonons, is given by [12]

$$\alpha = \frac{\in^{1/2}}{n_0 c}\sum_i W_i f_i \quad (3)$$

Here $\in$ is the dielectric constant of material, n_0 is the number of photons in the radiation field and f_i is the free-carrier distribution function. The sum is over all the possible initial states *'i'* of the system. The transition probabilities W_i can be calculated using the standard second-order Born golden rule approximation:

$$W_i = \frac{2\pi}{\hbar}\sum_{fq}\left[\left|\langle f|M_+|i\rangle\right|^2 \delta\left(E_f - E_i - \hbar\Omega - \hbar\omega_q\right) + \left|\langle f|M_-|i\rangle\right|^2 \delta\left(E_f - E_i - \hbar\Omega + \hbar\omega_q\right)\right] \quad (4)$$

Here E_i and E_f are the initial and final state energies, respectively, of electrons, $\hbar\Omega$ is the photon energy, $\hbar\omega_q$ is the phonon energy, and $\langle f|M_\pm|i\rangle$ are the transition matrix elements from the initial state to the final state for the interaction between electrons, photons and phonons. These transition matrix elements can be represented by the following expression:

$$\langle f|M_\pm|i\rangle = \sum_\alpha\left(\frac{\langle f|H_R|\alpha\rangle\langle\alpha|V_s|i\rangle}{E_i - E_\alpha \mp \hbar\omega_q} + \frac{\langle f|V_s|\alpha\rangle\langle\alpha|H_R|i\rangle}{E_i - E_\alpha - \hbar\Omega}\right) \quad (5)$$

where H_R is the interaction Hamiltonian between the electrons and the radiation field, V_s is the scattering potential due to the electron-phonon interaction.

Using the wavefunctions given by the expression (1), the matrix elements of the electron-photon interaction Hamiltonians can be written as

$$\langle k_y' N'l' \,|H_R|k_y Nl\,\rangle = -\frac{e\hbar}{m^*}\left(\frac{2\pi\hbar n_0}{V\Omega\in}\right)^{1/2}\left(\varepsilon\kappa\right)\delta_{\kappa_y\kappa_y'}\delta_{NN'}\delta_{ll'} \quad (6)$$

where V is the volume of the crystal. Here the radiation field is polarized along the wire, ε is the polarization vector of the radiation field.

We shall use two different scattering processes: polar-optical scattering and acoustic -phonon scattering. The matrix elements $\langle k_y' n'l'|V_s|k_y nl\rangle$ of electron-phonon interaction corresponding to the above two processes are equal to

$$\langle k_y'\; n'l|V_s|k_y nl\rangle = C_j'\delta_{k_y',k_y\pm q_y} J_{nn'}\;(q_x q_y)\Lambda_{ll'}(q_z) \quad (7)$$

where $J_{n',n}(q_x, q_y)$ is the overlap integral of the harmonic wave functions:

$$J_{N',N}(q_x q_y) = \int_{-\infty}^{\infty} dx \exp(iq_x x)\Phi_{N'}\left(x - \tilde{b}\tilde{R}^2 k_y - \tilde{b}\tilde{R}^2 q_y\right)\Phi_N\left(x - \tilde{b}\tilde{R}^2 k_y\right) \quad (8)$$

$$\Lambda_{ll'}(q_z) = \frac{2}{L_z}\int_0^{L_z} dz \exp(iq_z z)\sin\left(\frac{l'\pi z}{L_z}\right)\sin\left(\frac{l\pi z}{L_z}\right) \quad (9)$$

$$C_j'^2 = C_j^2 F_j(q)$$

The function $\Lambda_{ll'}(q_z)$ given by Eq. (8) is crucial for our calculation whose suitable approximation was discussed by Ridley [28].

For the electron-polar-optic phonon interaction we have

$$F_{POL} = \frac{N_0^\pm}{q^2 V},\; C_{POL}^2 = 2\pi e^2\hbar\omega_0 \in'^{-1},$$

$$\in'^{-1} = \left\{\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right\}.$$

Here, ε_∞ and ε_0 are the high-frequency and static dielectric constants of the semiconductor, respectively. As usual, we take phonon energy $\hbar\omega_q = \hbar\omega_0 \approx$ const.

$$N_0=\left[\exp\left(\frac{\hbar\omega_0}{K_BT}\right)-1\right]^{-1},\ N_0^-=N_0,\ N_0^+=N_0+1.$$

where $N_0^-\left(N_0^+\right)$ describes the annihilation (creation) of the phonon.

When acoustic phonon scattering is dominant, one may obtain

$$C_{AC}^2=\frac{E_d^2K_BT}{2\rho\upsilon_s^2V},\ F_{AC}(q)=1$$

In the case of bulk materials and at extremely strong magnetic fields, the electronic wave functions have small absolute values of momentum components parallel to the applied magnetic field. Therefore, we can neglect the q_z dependence in the interaction potential given by $C_j^{\bar{y}}$.

The electron distribution function for quasi-one-dimensional nondegenrate electron gas in the presence of magnetic field can be shown as follows:

$$f_{Nlk_y}=\frac{2(2\pi)^{1/2}\hbar n_eL_xL_z\sinh(\hbar\tilde{\omega}/2K_BT)}{\delta(\tilde{m}K_BT)^{1/2}}\times$$
$$\times\exp\left\{-\left[\frac{(N+1/2)\hbar\tilde{\omega}+l^2\mathrm{E}_0}{K_BT}\right]\right\}\exp\left(-\frac{\hbar^2k_y^2}{2\tilde{m}^*K_BT}\right)\qquad(10)$$

where $\delta=\sum_l\exp\left(l^2\mathrm{E}_0/K_BT\right)$, n_e is the concentration of electrons.

Below, we shall use the following identities:

$$\int_0^\infty\left|J_{nn'}(q_x,q_y)\right|^2q_\perp dq_\perp=\frac{1}{R^2}$$
$$\int_0^\infty\left|J_{nn'}(q_x,q_y)\right|^2q_\perp^3dq_\perp=\frac{2}{R^4}(N'+N+1)\qquad(11)$$
$$\int_0^\infty\left|\Lambda_{ll'}(q_z)\right|^2dq_z=\frac{2\pi}{d}\left(1+\frac{1}{2}\delta_{ll'}\right)$$

Now we make the same approximation as in [4], i.e. we take $\frac{\hbar^2}{2\tilde{m}^*}\left(q_y^2-2k_yq_y\right)=0$, in δ functions. Using Eqs (4-6) and (9) in (3) and also identities (11), we obtain the following expression for the FCA coefficient for polar and acoustic phonon scattering in a Q1D semiconducting structure in the presence of a magnetic field:

$$\alpha_{POL}(H)=\frac{4\pi^2e^4\hbar\omega_0n_e\sinh(\hbar\tilde{\omega}/2K_BT)}{c\in^{1/2}\in' m^{*2}\Omega^3L_z\tilde{b}R^2\delta}\times$$
$$\times\sum_{N_fl_f}\sum_{N_il_i}\left(1+\frac{\delta_{l_fl_i}}{2}\right)\exp\left\{-\frac{1}{K_BT}\left[\left(N_i+\frac{1}{2}\right)\hbar\tilde{\omega}+l_i^2E_0\right]\right\}\times$$
$$\times\left\{N_0\delta\left(\left(N_f-N_i\right)\hbar\tilde{\omega}+\left(l_f^2-l_i^2\right)E_0-\hbar\Omega+\hbar\omega_0\right)+\right.$$
$$\left.+(N_0+1)\delta\left(\left(N_f-N_i\right)\hbar\tilde{\omega}+\left(l_f^2-l_i^2\right)E_0-\hbar\Omega-\hbar\omega_0\right)\right\}\qquad(12)$$

$$\alpha_{AC}(H)=\frac{2\pi e^2E_d^2n_e(K_BT)^{1/2}\sinh(\hbar\tilde{\omega}/2K_BT)}{c\rho\in^{1/2}\upsilon_s^2m^{*2}\Omega^3L_z\tilde{b}^2R^4\delta}$$
$$\sum_{N_fl_f}\sum_{N_il_i}\left(1+\frac{\delta_{l_fl_i}}{2}\right)\left(N_f+N_i+1\right)\times$$
$$\times\exp\left\{-\frac{1}{K_BT}\left[\left(N_i+\frac{1}{2}\right)\hbar\tilde{\omega}+l_i^2E_0\right]\right\}\times\qquad(13)$$
$$\times\left\{\delta\left(\left(N_f-N_i\right)\hbar\tilde{\omega}+\left(l_f^2-l_i^2\right)E_0-\hbar\Omega\right)\right\}.$$

It is particularly convenient to express our results in terms of the dimensionless ratio of the FCA coefficient in presence of the magnetic field to that in the absence of the field. For scattering through acoustic phonon, we adopt the results [24]

$$\alpha_{AC}(0)=\frac{2^{3/2}e^2E_d^2(K_BT)^{3/2}n_e\sinh(\hbar\omega/2K_BT)}{m^{*1/2}l_\omega(\hbar\Omega)^3\in^{1/2}\rho\upsilon_s^2cL_z\delta}\times$$
$$\times\sum_{n_fl_f}\sum_{n_il_i}\left(1+\frac{1}{2}\delta_{l_il_f}\right)\exp\left(-\frac{(n_i+1/2)\hbar\omega+l^2E_0}{K_BT}\right)\times\qquad(14)$$
$$\times Z\exp(Z)K_1(Z)$$

where

$$Z=\frac{\hbar\Omega-\left(n_f-n_i\right)\hbar\omega-\left(l_f^2-l_i^2\right)E_0}{2K_BT}$$

and $K_1(x)$ is the modified Bessel function of the second kind, $l_\omega^2=\hbar/m^*\omega$. In the quantum limit, in which only the $n_i=n_f=l_i=l_f=1$ quantum level is occupied and $\hbar\omega_c>>K_BT$, only the lowest Landau level N=0 is thermally populated, the ratio $\alpha_{AC}(H)/\alpha_{AC}(0)$ takes the particularly simple form

$$\frac{\alpha_{AC}(H)}{\alpha_{AC}(0)}=$$
$$=\frac{2\hbar^{1/2}\tilde{\omega}^4(K_BT)^{1/2}\exp(-\hbar\tilde{\omega}/2K_BT)}{\Omega\omega^{1/2}\omega_c^2K_1(\hbar\Omega/2K_BT)}\times\qquad(15)$$
$$\times\sum_{N_f}\left(N_f+1\right)\delta\left(N_f\hbar\tilde{\omega}-\hbar\Omega\right)$$

For optical phonon scattering, the ratio takes a similar form

$$\frac{\alpha_{POL}(H)}{\alpha_{POL}(0)}=F(T,\omega_c,\Omega)\qquad(16)$$

From Eqs (15)–(16), it can be seen that, the ratio depends only upon the magnetic field, absolute temperature, and photon frequency and does not depend upon such material parameters as the values of the deformation potential, sound velocity, or density of the material, although, of course, the absolute value of absorption coefficient does depend upon the numerical values of these parameters.

3. Discussion

Thus, we have obtained general expressions for FCA coefficients for QWWs in the presence of the quantizing magnetic field. From Eqs (12)-(13) it can be seen that, in the extreme quantum limit ($\hbar\widetilde{\omega} >> K_B T$, $N_i = 0$, $l_i = l_f = 1$) for polar optical phonons, the FCA coefficient oscillates as a function of the magnetic field and photon frequency with resonances occuring when $P\widetilde{\omega} = \Omega \pm \omega_0$. Since $\omega_c < \widetilde{\omega}$, for $\omega > 0$, the resonances are shifted to smaller magnetic fields. For $\omega = 0$, i.e., in the absence of confinement, $\widetilde{b}^2 = 1$, $\widetilde{\omega} = \omega_c$, and we recover the usual resonance condition $P\omega_c = \Omega \pm \omega_0$. For the elastic scattering by acoustic phonons, resonances are expected when $P\widetilde{\omega} = \Omega_0$.

The oscillatory dependence of the absorption on magnetic field can be understood in terms of the Landau subband structure of the electronic energy levels in quantizing magnetic fields. As the magnetic field, and therefore $\widetilde{\omega}$, increases there are fewer and fewer subbands to which the transition can place until finally. Every time that the ratio $(\Omega \pm \omega_0)/\widetilde{\omega}$ equals an integer value, the transition can take place with an additional subband ending as a final state.

In conclusion, we predict that FCA coefficient should increase with magnetic field with an oscillatory dependence on the field when $\Omega > \widetilde{\omega}$. The magnetic field dependence of the FCA coefficient is explained in terms of the field dependence of the scattering rates and the possibility of phonon-assisted transitions between various Landau levels when $\Omega > \widetilde{\omega}$.

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