The modernity of the research of Romanian astronomer Constantin Popovici

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We present the model of the photogravitational field proposed by the romanian mathematician Constantin Popovici, basic equations. We give the variation of orbital energy in the photogravitational two bodies problem and energy integral. In this case the orbital energy is not conserved.

Introduction

The Constantin Popovici model was first proposed in 1923 (see e.g. [12]). In the following we try to synthesize the remarkable results of Professor Constantin Popovici, as well as other contemporary researchers in the photogravitational two bodies problem.

During the first part of the twentieth century, Romanian astronomer Constantin Popovici (1876-1956) proposed an amendment to the law resulting from combining the Newtonian gravitation attraction and radiation repulsion. He added a term which varies with the radial velocity [12], that leads to the law being applied to the attractive and repulsive centrifugal forces.

It was also used by the Italian scientist Giuseppe Armellini in his work on the generalization of Newton’s law of universal gravitation (see e.g. [1]).

Born in 1878 in Iasi, Constantin Popovici achieved his elementary school, high school and higher education in his hometown. He graduated from the Faculty of Science of the “Al. I. Cuza” University in 1900, having a degree in mathematics. In 1905 he went to Paris, where he obtained an another license in mathematics, then prepared and submitted a doctorate thesis in mathematics at the Sorbonne, in March 1908.

Back in his home country, he was appointed a professor of astronomy, geodesy and celestial mechanics at the University of Iasi. In 1938 he moved to Bucharest, where he taught at the Department of Astronomy of the Bucharest University until 1940 and led the Astronomical Observatory since 1937 till 1943.

In astronomy he studied the influence of light pressure on the movement of bodies within the solar system. Constantin Popovici was an honorary member of the Romanian Academy (1948) and member (1949). He died on November 26, 1956 and was buried in the “Belu” cemetery in Bucharest.

Constantin Popovici model has been reconsidered in the last decade by several Romanian scientists. E. g., M.-C. Anisiu [2, 3, 4] formulated the photogravitational problem of two body in Cartesian coordinates and investigated the existence of equilibrium points; V. Mioc and C. Blaga [10, 11] using McGehee type variables provided a qualitative study of the system of differential equations for the new variables; M. Barbosu and T. Oproiu [5] determined the nature of equilibrium points for the two bodies photogravitational problem.

The photogravitational problem (for two, three or more bodies) has attracted more attention in recent years [6, 8, 9, 15].

Constantin Popovici model. Basic equations

In Popovici C. model for the projection of the force onto the direction of the radius-vector one considers the following relation [7]:

$$F = -\frac{A}{r^2} + \frac{R}{r^2} - R\frac{\dot{r}}{c \cdot r^2}, \tag{1}$$

where the first term corresponds to Newtonian attraction force (with A being the attraction of the luminous body at unit distance $r = 1$), the second term represents the force caused by the light pressure of the central body (with R being the repulsion caused by light pressure at unit distance), and the last term introduced by Popovici represents an increase of attraction force due to the finite speed of light.

An expression similar to (1) was used by Giuseppe Armellini ($1887-1958$) to generalize Newton's law of universal gravitation. Armellini used the expression:

$$F = -\frac{Gmm'}{r^2}(1+\epsilon\dot{r}), \tag{2}$$

where m and m' are the masses of interacting bodies, G is the gravitational constant, r is the distance between bodies and ϵ is the Armellini constant. The value of the constant was determined by comparison between theory and observations. The law proposed by Armellini was first published in [1].

Starting from the relation (1) and using the notations $k = A - R$ and $\epsilon = \frac{R}{ck}$ get (here ϵ is not the same as in the relation introduced by Armellini):

$$F = -\frac{A-R}{r^2} - \frac{R\dot{r}}{cr^2} = -\frac{k}{r^2}(1+\epsilon\dot{r}). \tag{3}$$

The following form was used by romanian researchers using the notation $l = R/c$ [3] :

$$F = -\frac{k}{r^2} - \frac{R\dot{r}}{cr^2} = -\frac{k}{r^2}(1+l\dot{r}). \tag{4}$$

The variation of orbital energy in the photogravitational two bodies problem

The orbital energy in the Popovici model is not conserved. Starting from the relation (3):

$$m\ddot{\vec{r}} = -\frac{k}{r^2}(1+\epsilon\dot{r})\cdot\frac{\vec{r}}{r}, \tag{5}$$

one can derive:

$$\frac{d}{dt}(m\dot{\vec{r}}^2) = -2\frac{d}{dt}\left(\frac{k}{r}\right) - 2\epsilon k\left(\frac{\dot{r}}{r}\right)^2 \tag{6}$$

In the manuscript [12] one can find the following Popovici's theorem:

"L'énergie ne se conserve plus. Cette quantité

$$E = \frac{v^2}{2} - \frac{k}{r} \tag{7}$$

que l'on appelle dans la Mécanique newtonienne l'énergie, varie dans le même sens avec le temps. La relation suivante, qui est en même temps l'équation du mouvement:

$$\frac{dE}{dt} = \alpha k\left(\frac{r'}{r}\right)^2, \quad r' = \frac{dr}{dt}, \tag{8}$$

$\frac{dE}{dt} > 0$ *attraction;* $\frac{dE}{dt} < 0$ *répulsion, nous fait voir comment l'énergie est dépensée par le mécanisme de la propagation".*

Theorem 1. The variation of the energy of an infinitesimal particle (during relative motion) has the following property:

$$\dot{E} = -\epsilon k\left(\frac{r'}{r}\right)^2, \tag{9}$$

where $\epsilon = \frac{R}{ck}$. In this case if the energy does not conserve, then there is no prime integral.

Remark: If $\dot{E} = 0$ one can find a first integral of energy to generalize the classical one, where energy varies with time [7, 13] .

Theorem 2. In the photogravitational two bodies problem, the C. Popovici model, the energy integral takes its generalization form:

$$\sqrt{I - 2\rho\alpha\sqrt{I-\rho^2}} \cdot e^S = H, \quad \text{where } S = \frac{\alpha}{\sqrt{1-\alpha^2}} \arctan\frac{\sqrt{I-\rho^2} - \rho\alpha}{\rho\sqrt{1-\alpha^2}}, \tag{10}$$

where $I = 2E + \frac{k^2}{C^2}$, $\rho = \frac{C}{r} - \frac{k}{C}$, $\alpha = \frac{\epsilon k}{2C}$ and H is a constant of integration.

Proof:

Starting from the relation which can be obtained from theorem 1, $\frac{dE}{dt} = \frac{dE}{dr} \cdot \frac{dr}{dt} = -\epsilon k \left(\frac{\dot{r}}{r}\right)^2$, and knowing that the theorem of the areas remains valid in this case, and the force is central, one can obtain:

$$\left(\frac{dr}{dt}\right)^2 + \frac{C^2}{r^2} - \frac{2k}{r} = 2E,$$

and get the equality:

$$\frac{dE}{dr} = -\frac{\epsilon k}{r^2}\sqrt{2E + \frac{k^2}{C^2} - \left(\frac{C}{r} - \frac{k}{C}\right)^2}. \tag{11}$$

Using the notations $I = 2E + \frac{k^2}{C^2}$, $\rho = \frac{C}{r} - \frac{k}{C}$ one obtains:

$$\frac{dE}{dr} = -\frac{\epsilon k}{r^2}\sqrt{I-\rho^2}. \tag{12}$$

Then differentiating relations one gets:

$$\frac{dI}{d\rho} = 2\frac{dE}{d\rho} = \frac{2\epsilon k}{C}\sqrt{I-\rho^2}. \tag{13}$$

Use the notation $\alpha = \frac{\epsilon k}{2C}$ a simplified relation can be obtained:

$$\frac{dI}{d\rho} = 4\alpha\sqrt{I-\rho^2}. \tag{14}$$

Changing the variable $I \to u$ as $I = \rho^2(1+u^2)$ one derives:

$$\frac{dI}{d\rho} = 2\rho(1+u^2) + 2\rho^2 u\frac{du}{d\rho}, \tag{15}$$

or:

$$2\rho^2 u\frac{du}{d\rho} = 4\alpha\rho u - 2\rho(1+u^2), \tag{16}$$

from which one can obtain the following differential equation:

$$-\frac{udu}{u^2 - 2\alpha u + 1} = \frac{d\rho}{\rho}, \tag{17}$$

that can be reorganized as:

$$\frac{d\rho}{\rho} = -\frac{(u-\alpha)du}{(u-\alpha)^2 + 1 - \alpha^2} - \frac{\alpha du}{(u-\alpha)^2 + 1 - \alpha^2}. \tag{18}$$

Integrating this relation one can obtain:

$$\ln\rho = -\frac{1}{2}\ln(u^2 - 2\alpha u + 1) - \frac{\alpha}{\sqrt{1-\alpha^2}}\arctan\frac{u-\alpha}{\sqrt{1-\alpha^2}} + H_1, \tag{19}$$

or

$$H=\rho\sqrt{1-2\alpha u+u^2}\cdot e^{S_1},\ S_1=\frac{\alpha}{\sqrt{1-\alpha^2}}\arctan\frac{u-\alpha}{\sqrt{1-\alpha^2}}. \tag{20}$$

And coming back to the first notation one can obtain:

$$\sqrt{I-2\rho\alpha\sqrt{I-\rho^2}}\cdot e^S=H,\quad \text{where } S=\frac{\alpha}{\sqrt{1-\alpha^2}}\arctan\frac{\sqrt{I-\rho^2}-\rho\alpha}{\rho\sqrt{1-\alpha^2}},\quad \text{q.e.d.} \tag{21}$$

Remark: $\epsilon=0$ means that the following equalities take place: $S=0$, $e^S=1$ and $I=2E+\frac{k^2}{C^2}=H^2$, thus $E=\frac{1}{2}\left(H^2-\frac{k^2}{C^2}\right)=h$ is constant. It means that the energy integral from the keplerian motion exists.

Conclusions

We discuss the photogravitational model proposed by the Romanian astronomer Constantin Popovici. The term introduced by Constantin Popovici in order to consider the fact that the light pressure propagates with a finite velocity unlike the attraction force which propagates instantaneously is also important. This has a breaking effect, similar to the motion in a resisting medium that had been marked out by Radziewsky since 1950 [14].

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