Lucky image performance simulation on the basis of optical turbulence data obtained on Mt. Shatdzhatmaz

B. S. Safonov

Sternberg Astronomical Institute, Universitetskij pr-t, 13, 119991, Moscow, Russia
`safonov@sai.msu.ru`

Lucky imaging is a method allowing to achieve diffraction resolution on a moderate ground-based telescopes with the cost of magnitude limitation. To adequately evaluate capabilities of this technique for a given place (Mt. Shatdzhatmaz) we performed a Monte Carlo numerical simulation taking into account properties of optical turbulence measured there. Statistics of lucky imaging isoplanatic angle was evaluated on the basis of these data. Optimal atmospheric conditions for lucky imaging were obtained.

Introduction

Lucky imaging gained popularity as an alternative to adaptive optics (AO) thanks to simplicity and low cost comparing with AO. It seems that it has also much larger isoplanatic angle than in the case of simple AO. Activity in this field increased significantly in last several years thanks to appearance of EMCCD technology. Cameras manufactured using this technology have a remarkable feature — negligibly small readout noise. It allows to divide a single long exposure into a set of shorter ones, process them separately and than sum them up. In case of lucky imaging the basic algorithm of such a processing look like the following [5]:

1. Obtain a set of short-exposure images. Exposure time have to be short enough to freeze the image boiling due to the optical turbulence (OT) in atmosphere — usually it is $30 - 70$ ms.
2. Choose a reference star to work with. Most convenient option is a bright star near the frame center.
3. Choose a certain portion of the best frames. Ratio of number of these frames to the total number of frames is called frame selection ratio (FSR). Usually Strehl ratio is used as a figure-of-merit of quality of frames.
4. Sum up processed frames using well-known shift-and-add algorithm. Centering is carried out by the reference star image.

Image obtained as a result of this summation has improved angular resolution [1, 5].

Lucky imaging like other similar techniques — speckle interferometry, AO — suffers from anisoplanatism. For lucky imaging it lies in the degrading of the point spread function (PSF) in restored image with the increasing of distance to the reference star. Isoplanatic angle is an angle at which Strehl ratio is reduced in e time comparing to that of reference star (see Fig. 1).

The main purpose of this work was to determine this isoplanatic angle for a given telescope and given conditions of OT in the atmosphere. Unfortunately it is impossible to do it analytically. We performed the Monte Carlo numerical simulation of light propagation in turbulent atmosphere according to principles described in [1, 4].

In this simulation the atmosphere is represented by several infinitely thin layers with von Karman power spectrum $\Phi(k) = 0.023 r_0^{-5/3}(k^2 + k_0^2)^{-11/6}$, where r_0 is Fried radius, k is the spatial frequency and k_0 relates to outer scale $k_0 = 1/L_0$, where outer scale L_0 is the upper limit of sizes of turbulence eddies. Wavefront undergoes pure phase distortion on these turbulent layers and propagates between them within the approximation of geometrical optics. To simulate wind the turbulent layers are translated as a whole (Taylor hypothesis). PSF is calculated as a square of Fourier transform of wavefront on telescope pupil. Basic parameters of simulation are: size of phase screen 512×2048 px, scale 0.025 m/px, wavelength 806 nm[1],

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[1] I-band is widely used in lucky imaging because one should go to longer wavelength as far as possible to reduce OT effects and I-band is the most red band allowed by commercially available EMCCD cameras

outer scale 25 m, telescope diameter 2.5 m, central obscuration 0.43. Fried radiuses r_0 of turbulence profiles are set according to optical turbulence profile (OTP) data which is described in the next section.

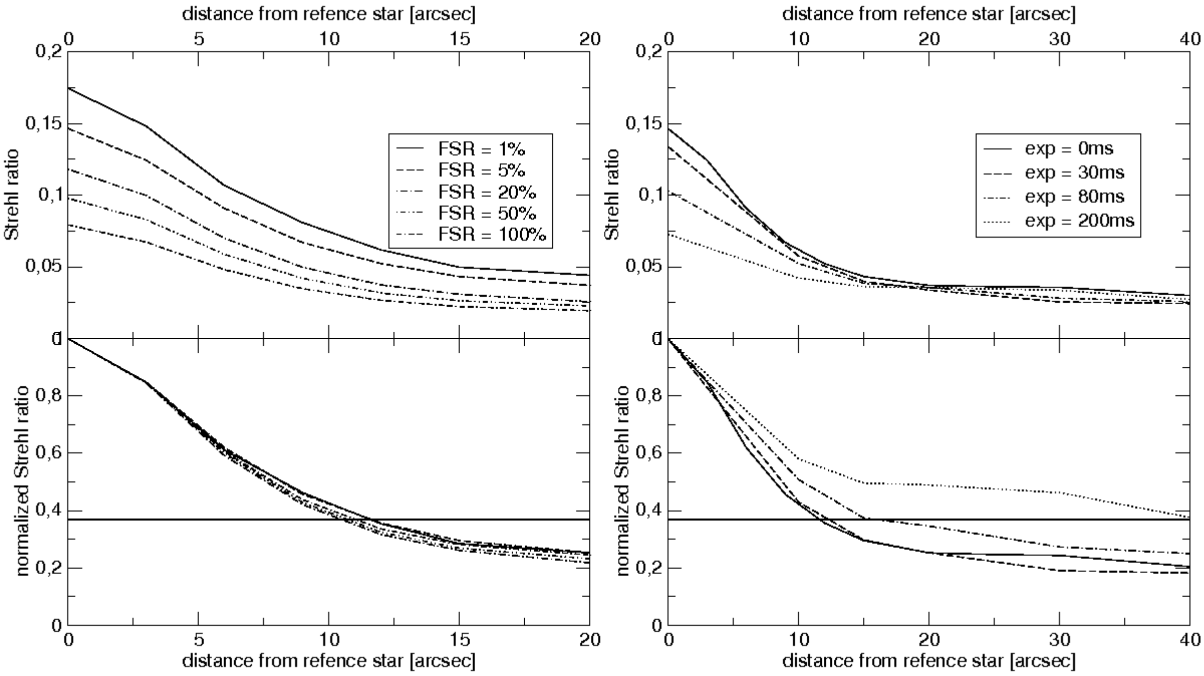

Figure 1: All graphs are computed for profile BB, thick horizontal line corresponds to $1/e$ level. *Left top:* dependence of Strehl ratio in restored image on distance to reference star for different FSRs. *Left bottom:* the same for normalized Strehl ratio. *Right top:* dependence of Strehl ratio in restored image on distance to reference star for FSR=5% and for different exposures. *Right bottom:* the same for normalized Strehl ratio.

Optical turbulence profile data

For simulation we used OT data set obtained on Mt. Shatdzhatmaz (2127 m above see level) in 2007-2009 [2] with MASS/DIMM device [3]. This summit is selected as a place of installation of 2.5-m telescope of Sternberg Astronomical Institute and located on Northen Caucasus near the Kislovodsk city.

For the period between October 2007 and November 2009 85000 OTPs were obtained. Each profile consists of 13 turbulent layers at heights 0, 0.5, 0.7, 1.0, 1.4, 2.0, 2.8, 4.0, 5.6, 8.0, 11.3, 16.0, 22.6 km. These data already can be used as input for the simulation. However simulation run for one profile takes about several hours to complete so we cannot process all profiles individually. Also it is well-known that OTP is highly variable and cannot be represented by a single arbitrary profile from data set [6]. Because of this we reduced the data set to 9 typical OTP using method described in [6].

Firstly, we selected 9 subsets from initial data set, each of them coded by two letters running over 3 values: A, B, C (this results in 9 combinations: AA, AB, AC, BA, BB, BC, CA, CB, CC). These subsets consist of profiles having similar values of OT intesity of 0-km layer (ground layer) J_{GL} and of total intensity of remaining layers — free atmosphere J_{FA}. First letter in the code indicates conditions in ground layer: A — good, B — median, C — bad, second one indicates conditions in free atmosphere in the same way. The following inequalities are conditions of falling of certain OTP in ij-subset: $p_{GL}(N_{iL}) < J_{GL} < p_{GL}(N_{iU})$ and $p_{FA}(N_{jL}) < J_{FA} < p_{FA}(N_{jU})$, where $p_{GL}(N)$ and $p_{FA}(N)$ is N-th percentiles of distribution of J_{GL} and J_{FA}, correspondingly. Also, $N_{AL} = 20\%, N_{AU} = 30\%, N_{BL} = 45\%, N_{BU} = 55\%, N_{CL} = 60\%, N_{CU} = 70\%$. Finally for each subset we computed the median profile. General parameters of resulting typical profiles are shown in Table 1. The free atmosphere part of typical profiles are shown in Fig. 2.

It can be easily seen that the shape of profiles does not depend on intensity of ground layer that can be expected due to the fact that free atmosphere and ground layer are mutually independent [6]. Meanwhile there is a noticeable difference in shapes of profiles having different free atmosphere integrals. This fact was already discovered for several sites [6].

Table 1: General parameters of typical optical turbulence profiles. Seeing β, Fried radius r_0, conventional isoplanatic patch θ_0, and lucky imaging isoplanatic angle θ_{LI} are computed for I-band

profile code	$J_{GL}10^{-13}$ [m$^{2/3}$]	$J_{FA}10^{-13}$ [m$^{2/3}$]	β [arcsec]	D/r_0	θ_0 [arcsec]	$\theta_{LI}(FSR = 5\%)$ [arcsec]
AA	1.90	0.89	0.532	8.16	4.44	13.6
AB	1.91	1.50	0.600	9.20	3.72	11.4
AC	1.89	2.70	0.716	11.0	3.08	9.0
BA	3.12	0.89	0.660	10.1	4.58	14.0
BB	3.12	1.49	0.718	11.0	3.74	10.4
BC	3.13	2.72	0.828	12.7	3.15	8.5
CA	5.19	0.89	0.847	13.0	4.56	12.6
CB	5.20	1.56	0.904	13.9	3.90	10.3
CC	5.22	2.88	1.007	15.5	3.23	7.7

Results

We used 9 typical OTPs as a model of atmosphere for the simulation. For each of them 50 realizations of turbulent atmosphere were obtained. In each realization we considered 30 moments with interval of 50 ms, which corresponds to shift of each layer of $0.2D$, where D is the telescope diameter (all layers are supposed moving with speed of 10 m/s). For each moment instantaneous PSFs were computed for reference star and for 12 stars on distances 3, 6, 9, 12, 15, 20, 25, 30, 40, 50, 65, 80 arcseconds to it. These data were then processed using lucky imaging procedure described in introduction with different FSR: 1%, 5%, 20%, 50%, 100%.

To measure the quality of restored images we used Strehl ratio as in [1, 5]. On the left top panel of Fig. 1 an example of dependence of Strehl ratio on distance to reference star for different FSR is shown. To compare rate of reducing of Strehl ratio for different FSRs it is convenient to normalize Strehl ratio by its value in reference star image (see Fig. 1, left bottom). In this representation cases of different FSRs become visually indistinguishable.

In Fig. 3 and Table 1 we summarize the values of isoplanatic angles for different turbulent profiles and FSRs. Isoplanatic angle for our mountain varies within the range of $7-16$ arcseconds and depends mainly on OT in free atmosphere. Fig. 3 also clearly demonstrates that there is no considerable dependence of isoplanatic angle on FSR.

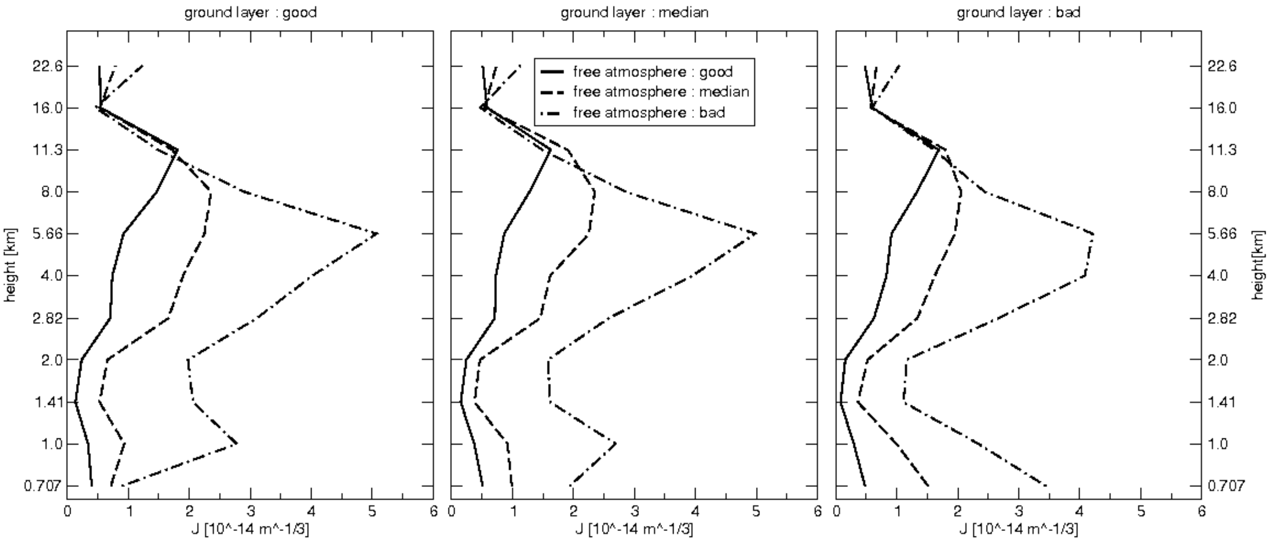

Figure 2: Typical optical turbulence profiles, free atmosphere part. Ground layer is not depicted because it has much greater value.

To make this simulation more practical we introduced a finite exposure in it. In real lucky image or speckle interferometry observation exposure is rarely short enough to neglect it. One can expect that finite exposure will blur the PSF and reduce the Strehl ratio. The simulation proves this fact as can be seen on the right top panel of Fig. 1. Strehl ratio is reduced indeed but the effect is greater for higher Strehl. It leads to the fact that formally isoplanatic angle is increased (see right bottom panel of Fig 1).

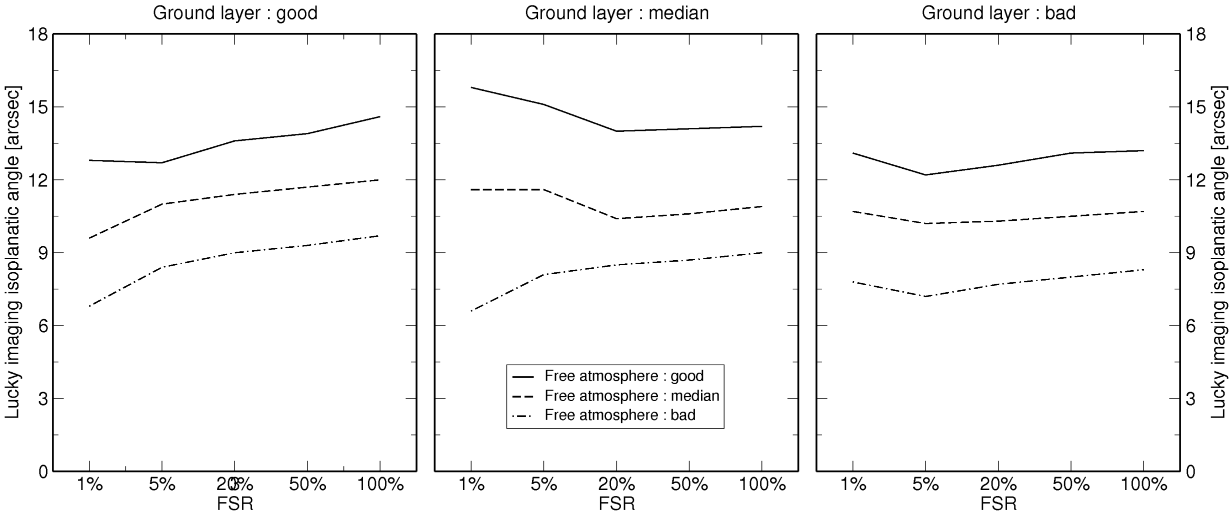

Figure 3: Isoplanatic angle for lucky imaging derived according to [5]. For profiles parameters see Table 1.

Conclusions

- Lucky imaging isoplanatic angle varies from 7 to 16 arcseconds and depends mainly on the intensity of OT in free atmosphere.
- Finiteness of exposure leads to reduction of Strehl ratio of image in all fields, especially near the reference star. It results in isoplanatic angle of $20 - 30$ arcseconds.
- This simulation also explains observed values of isoplanatic angle [5] without additional assumptions like seeing variations.
- Practical conclusions: there is no sense in construction of camera for lucky imaging with field of view more than 60 arcseconds. During selection of observational programs required large isoplanatic angle current intensity of OT in free atmosphere is pivotal.

Acknowledgement

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References

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