Determination of the reference frames deflections from optical observations of GNSS satellites

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Optical observations of navigation satellites were carried out at Terskol observatory during 2007-2010 with the aim to determine the deflection angles between GPS and GLONASS dynamical reference frames. There were three different observations strategies: celestial equator crossing, intersection of visible satellite paths, occultation of astrometric stars with the satellite. We present methodology of observation, data processing and the first results.

Introduction

During seasons of 2007-2010 optical observation of navigation (GPS and GLONASS) satellites were carried out at Terskol observatory (3128 m above sea level, Northern Caucasus, Russia). In total, more than 3000 raw satellite images were caught. The first testing images have 5 sec exposure while the most of others — 1 sec. Raw images served as a sources of combined ones. Examples are presented in Fig. 1.

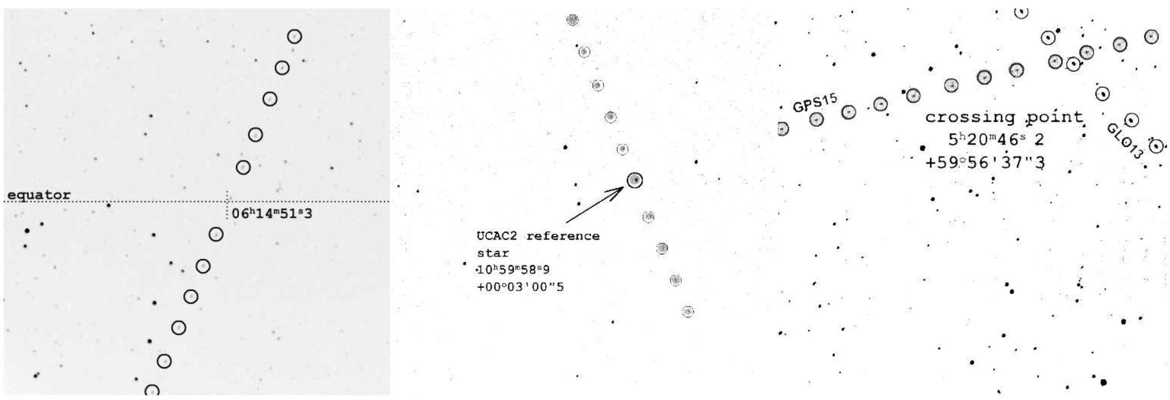

Figure 1: Three kinds of observations (*left to right*): equator crossing, reference star occultation, mutual paths crossing.

Three different types of events were interesting for us. First of all if there was a visible satellite path crossing with celestial equator (left part of Fig. 1). Such events may be easily forecasted. Processing of the pictures consists in fixing the place on the equator where the satellite crossed it. Sometimes, the satellite occults reference astrometric star (central part of Fig. 1). As the star is a point-like source it is very difficult to forecast such events. That is why they were observed mostly occasionally. The third type of events was visible intersection of satellite paths (right part of Fig. 1). It is quite possible, as inclinations of GPS and GLONASS orbits are different. The mutual intersections are quite rare: $1-2$ observable events per night.

The last type of events needs very precise ephemeris. Good example is presented on the right part of Fig. 1. According to ephemeris we planed to catch the intersection near the center of telescope field of view. Despite of that, the real intersection point lies definitely not in the center of the image, which can be clearly seen on right part of Fig. 1.

Satellite positions

Navigation messages in RINEX format and final orbits in SP3 files were used to determine satellite positions according to GPS [3] and GLONASS [4] interface control documents.

For GPS the keplerian elements $(a,\ e,\ i_0, \Omega_0,\ \omega_0,\ M_0,\ \Delta n)$ their time derivatives $\left(\frac{d\Omega}{dt},\ \frac{d\omega}{dt},\ \frac{di}{dt}\right)$ and perturbation model $(C_{ic},\ C_{is},\ C_{rc},\ C_{rs},\ C_{uc},\ C_{us})$ were extracted for the moments, closest to the observations. Coordinates in orbital reference frame were then deduced according to the following algorithm:

$$
\begin{array}{rcl}
n & = & \sqrt{\frac{\gamma M_\oplus}{a^3}} + \Delta n, \\
M & = & M_0 + n(t - t_e), \\
E - e\sin E & = & M \quad \to E, \\
\tan v & = & \frac{\sqrt{1-e^2}\sin E}{\cos E - e} \quad \to v, \\
u & = & \omega + v, \\
\Omega & = & \Omega_0 + \frac{d\Omega}{dt}(t - t_e) + \Omega_\oplus(t - t_0), \\
i & = & i_0 + C_{ic}\cos(2u) + C_{is}\sin(2u) + \frac{di}{dt}(t - t_e), \\
\omega & = & \omega_0 + C_{uc}\cos(2u) + C_{us}\sin(2u), \\
r & = & a(1 - e\cos E) + C_{rc}\cos(2u) + C_{rs}\sin(2u),
\end{array}
\tag{1}
$$

and the coordinates itself:

$$
\mathbf{r} = \left(\frac{a(1-e^2)}{1+e\cos v}\cos u, \quad \frac{a(1-e^2)}{1+e\cos v}\sin u, \quad 0 \right), \tag{2}
$$

where $\gamma M_\oplus$ is the geocentric gravitational constant, $\Omega_\oplus$ is the Earth's rotation velocity, t_e is the moment of ephemeris data.

The difference $(t - t_e)$ never exceeds 2^{h} for GPS and 30^{m} for GLONASS. Then coordinates (2) are transformed to equatorial reference frame (in our case it is ICRF – International Celestial Reference Frame):

$$
\mathbf{r}_{ICRF} = \mathbb{R}(-\Omega) \cdot \mathbb{P}(-i) \cdot \mathbb{R}(-\omega)\mathbf{r}, \tag{3}
$$

where $\mathbb{P}$ and $\mathbb{R}$ are rotation matrices:

$$
\mathbb{P}(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}, \qquad \mathbb{R}(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}
$$

SP3 final orbits [5] contain satellite positions for every 15 min. They should be interpolated to find position on desired moment.

In contrary to GPS, determination of GLONASS position implies numerical integration of satellites equations of motion:

$$
\left\{
\begin{array}{rcl}
\frac{d\mathbf{r}}{dt} & = & \mathbf{V}, \\
\frac{d\mathbf{V}}{dt} & = & \gamma M_\oplus \cdot \frac{\mathbf{r}}{r^3} \cdot \left(-1 + \frac{3C_{20}a_\oplus^2}{r^2}\left(k - 5\frac{z}{r^2}\right)\right) + \mathbf{j}_{\leftmoon} + \mathbf{j}_\odot,
\end{array}
\right.
\tag{5}
$$

where $k = 3$ for z and $k = 1$ for x and y. These equations account for perturbation from second zonal geopotential garmonics C_{20} and direct influence of the Sun $(j_\odot)$ and the Moon $(j_{\leftmoon})$. One should select the data from RINEX file for the closest possible moment and integrate to the moment of interest. There are no SP3 files for GLONASS.

Deflection angles

Coordinates $\mathbf{r}_s$ and $\mathbf{r}'$ in two different reference frames satisfy the Helmert transform [1]:

$$
\mathbf{r}_s = \begin{pmatrix} \mu_1 & R & -Q \\ -R & \mu_2 & P \\ Q & -P & \mu_3 \end{pmatrix} \mathbf{r}' + \mathbf{a}, \tag{6}
$$

where P, Q and R are rotation angles around x, y and z correspondingly, μ_i are the scale factors, $\mathbf{a}$ is the center shift vector. Having coordinates of the same point in two reference frames one can solve (6) for rotation angles, scales and shift vector.

Unfortunately, we cannot account GLONASS satellite coordinates in GPS dynamical reference frame and vice versa. That is why the third reference frame — optical one — was used.

If we have transformation angles from optical frame to the first and the second ones: P_1, Q_1, R_1 and P_2, Q_2, R_2, they can be presented as ephemeris values B_e with small corrections ΔB as $B_e + \Delta B$. Thus they satisfy the following relation [6]:

$$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} = \begin{pmatrix} 0 & \cos P_{e1} & \sin P_{e1} \cos Q_{e1} \\ 1 & 0 & \sin Q_{e1} \\ 0 & -\sin P_{e1} & \cos P_{e1} \cos Q_{e1} \end{pmatrix} \begin{pmatrix} \Delta P_1 - \Delta P_2 \\ \Delta Q_1 - \Delta Q_2 \\ \Delta R_1 - \Delta R_2 \end{pmatrix}. \quad (7)$$

Ephemeris values for rotation angles may by calculated from:

$$\tan P_1 = \tan i \sin \Omega; \quad \sin Q_1 = \sin i \cos \Omega; \quad \tan(R_1 + \omega) = \tan \Omega / \cos i. \quad (8)$$

Equations (7) can be modified to trivial ones:

$$P = \Delta P_2 - \Delta P_2, \quad Q = \Delta Q_1 - \Delta Q_1, \quad R = \Delta R_3 - \Delta R_3 \quad (9)$$

in the case of geostationary satellites or when a satellite crosses the equator.

Results and conclusions

We used Astrometrica software [7] with UCAC2 astrometric catalogue [2] for processing the pictures. Three kinds of observations were used to deduce P, Q, R between GPS and GLONASS dynamical Reference frames. Typical result is present below:

$$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} = \begin{pmatrix} 2.38 \pm 1.88 \\ 1.56 \pm 1.32 \\ -0.90 \pm 1.06 \end{pmatrix}; \qquad \mathbf{a} = \begin{pmatrix} 0.070 \pm 1.420 \\ -0.703 \pm 1.420 \\ 1.413 \pm 1.128 \end{pmatrix}, \quad (10)$$

angles in arcsec, shifts in km.

Differences between two reference frames are above their errors, especially P, Q and z components of $\mathbf{a}$. They are quite large and need further analysis.

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