Response of HTS tapes to a parallel ac magnetic field in the vicinity of the superconducting transition

L.M. Fisher and I.F. Voloshin

All-Russian Electrical Engineering Institute, Moscow 111250, Russian Federation Joint Stock Company "National Technical Physics and Automation Research Institute" Moscow 115230, Russian Federation

V.A. Yampol'skii

A.Ya. Usikov Institute for Radiophysics and Electronics, National Academy of Sciences of Ukraine Kharkov 61085, Ukraine V.N. Karazin Kharkov National University, Kharkov 61077, Ukraine E-mail: yam@ire.kharkov.ua

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We have studied the temperature dependence of the ac magnetic susceptibility $\chi(T) = \chi'(T) + i\chi''(T)$ of YBCO tapes in a parallel magnetic field in both the Meissner and vortex states. For the vortex state, we have observed two maxima in the $\chi''(T)$ dependence. The position and the magnitude of one of these maxima are described well by the nonlocal critical state model. The second maximum and corresponding kink in the function $\chi'(T)$ observed close to the temperature T_c of the superconducting transition are unexpected. The origin of this maximum cannot be explained within the usual notions of the high-temperature superconductivity. We suppose that it is related to some magnetic restructuring in the superconducting layer just above T_c . Our results put a question on the correctness of the interpretation of some previous microwave experiments.

PACS: 74.25.Ha Magnetic properties including vortex structures and related phenomena; 74.25.Uv Vortex phases;

74.25.nn Surface impedance.

Keywords: high-temperature superconductivity, magnetic properties, vortex structures.

1. Introduction

After the discovery of high-temperature superconductivity (HTS), an intensive study of the main parameters of superconductors, such as the coherence length ξ and the London penetration depth λ_L , was performed (see, e.g., Refs. 1-7). These investigations are in progress up to now because the temperature dependences of ξ and λ_L reflect the character of the superconducting pairing and can be useful for the elucidation of the nature of the HTS (see review [8] and references therein). Now the μ SR spectroscopy is considered as the most effective method to measure both these parameters. However, this refined method is labor-consuming (e.g., the study in Ref. 7 was performed during several years). Moreover, the µSR method is used to measure the magnetic field distribution in superconductors in the vortex state. So, to extract the information about ξ and λ_L by this method, the vortex structure and structure of an individual vortex should be known a priori.

In this paper, we study the temperature dependence of λ_L in Y₁Ba₂Cu₃O_{7- δ} (YBCO) tapes of the second generation (2G) by measuring the ac magnetic susceptibility $\chi = \chi' + i\chi''$ with an ac magnetic field *parallel* to the tape surface. To realize the parallel geometry one needs in extreme homogeneity of the field in the sample length (~ L) and aligning its direction within an angular precision better than $d/L \sim 10^{-4}$. In spite of these difficulties and the relative smallness of the detected signal from a thin superconducting cover in the tape, we chose such a geometry of the experiment due to several reasons. First, this geometry provides the homogeneity of the exciting ac magnetic field on the sample surface, whereas, in the perpendicular geometry, the ac field distribution is essentially non-uniform. In addition, the parallel geometry allowed us to study the interplay of the Meissner and vortex components of the ac field. Specifically, the nonlocal effects [9-11] play the important role in the ac field distribution within thin samples with thickness d of a superconductor comparable with λ_L . So far, the physical

picture of the nonlocal effects had only indirect experimental ground [10], and now we decided to validate conclusions of this nonlocal theory using now available and more proper objects for study than it was in papers [9–11].

In this paper, we have observed two maxima in the $\chi''(T)$ dependence for the tapes in the vortex state. The position and the magnitude of one of them are in a good qualitative agreement with the predictions of the nonlocal theory of the critical state [9–11]. Another peak in the dependence $\chi''(T)$ and the corresponding kink in the function $\chi'(T)$ were observed close to T_c . The height and position of this maximum depend weakly on the applied ac and dc magnetic fields and do not depend on frequency in the range $f < 10^5$ Hz. For samples with different oxygen deficit, this peak is always observed very close to the corresponding T_c . The origin of the additional maximum cannot be explained within the usual notions of the type II superconductors. We suppose it may be related to some magnetic restructuring in the superconductor near T_c .

2. Experimental details

The most measurements of χ' and χ'' are performed on recent commercial 2G HTS tapes SF12050 and SF12100 (SuperPower). These tapes have epitaxial HTS covers with almost perfect microstructure and their characteristics are very uniform along the tape. This conclusion is based on the results of the x-ray diffractometry study. According to the results of this study, the superconducting layers are quasi-single-crystals. The layers have 3D texture with small (1°–2°) misorientation of the crystallographic **c** axes of the crystallites. The misorientation of the crystallographic **a** and **b** axes in the tape plane is not observed.

We used samples 10–12 mm in width and several centimeters in length. The YBCO layers were *roughly* 1 or 2 μ m in thickness. The thickness of the upper silver cover and hastelloy substrate were 1–2 μ m and 100 μ m (or 50 μ m), respectively. We could change the oxygen deficit in the samples by means of the heat treatment in the oxygen-helium gas mixture.

A sample was mounted between two silicon singlecrystal slabs each $3.7 \times 10 \times 110$ mm in size. Due to high heat capacity and heat conductivity, the silicon container provided a uniform temperature regime. Two heaters and two CuFe–Cu thermocouples were placed on the peripheral regions of the container. The last was located inside a narrow tail of the "main" glass cryostat with a heat exchanging helium gas. Two mutually-orthogonal Cu-wire coils creating ac and dc fields were mounted over the tail of the main cryostat inside the outer nonmetallic bath with the liquid nitrogen or helium. The exciting coils created a magnetic field with a homogeneity better than 10^{-4} in 1 cm³. The ac currents through both coils were in-phase. Tuning the ratio of the current amplitudes in exiting coils, we oriented the resulting ac magnetic field $\mathbf{h}(t) = \mathbf{h}_0 \cos(2\pi ft)$ precisely along the tape surface with an accuracy better than $d/L \sim 2 \cdot 10^{-4}$. This condition corresponded to the observed disappearance of the normal component of magnetization (see Ref. 12). The main single layer pick-up coil was wound close on the sample in its middle part 3 mm in width. To extract the induced voltage related purely to the superconductor response, an auxiliary coil system was connected in series with the main coil to the input of the lock-in amplifier (SR830, Stanford Research System). Unavoidable presence of a spurious signal results in a position uncertainty of measured ac susceptibility χ components. So, the obtained value of χ can be arbitrary shifted along the ordinate axis.

3. Results and discussion

3.1. Meissner state

The symbols in Fig. 1 show a fragment of the temperature dependence $\chi'(T)$ measured in the parallel geometry for a tape with a superconducting layer thickness $d \approx 1 \mu m$. The critical temperature of the tape was $T_c \approx 90$ K. The amplitude h_0 of the exciting ac field was 2 Oe that is much less than the first critical field H_{c1} for almost whole considered temperature region (except for the narrow region near T_c).

Using the measured curve $\chi'(T)$, we can obtain the temperature dependence of the London penetration depth. Its relation to χ' , measured in the parallel geometry, is described by the equation

$$\chi' = \frac{2\lambda_L}{d} \tanh\left(\frac{d}{2\lambda_L}\right) - 1.$$
 (1)



Fig. 1. (Color online) The dependence $\chi'(T)$ for the tape with $d \approx \mu m$ and $T_c \approx 90$ K (symbols) measured at the ac field with $h_0 = 2$ Oe, f = 7300 Hz. The solid line corresponds to the result of curve fitting using Eqs. (1) and (3) with $\lambda_L(T)/d = = 0.173/(1-T^2/T_c^2)^{0.47}$.

This formula follows directly from the definition of χ' [13]

$$\chi' = \frac{\omega}{\pi h_0} \int_0^{2\pi/\omega} M(t) \cos(\omega t) dt, \qquad (2)$$

where $M(t) = \overline{B(t)} - h_0 \cos(\omega t)$, $\overline{B(t)}$ is the magnetic induction averaged over the volume of the superconductor. Here we normalize χ in such a way that $\chi' = -1$ for the ideal diamagnetic. To obtain $\lambda_L(T)$ we assume the fitting function

$$\lambda_L(T) = \frac{\lambda_L(0)}{\left(1 - T^2 / T_c^2\right)^{\alpha}}.$$
(3)

We found the best fit parameters $\lambda(0) = 0.173 \,\mu\text{m}$ and $\alpha = 0.47$. The exponent $\alpha = 0.47$ is very close to 0.5 which is predicted by the BCS theory within the mean-field approximation. For high-temperature superconductors, one could expect non-mean-field behavior near T_c because of extremely small value of the coherence length. Indeed, the exponent $\alpha \sim 0.33$ over the region 8–10 K near T_c was observed in Ref. 14. However, there are experiments where the mean-field behavior of $\lambda_L(T)$ (similar to our observation) was found (see, e.g., Ref. 15).

3.2. Vortex state. Nonlocal effects

We have also measured the temperature dependence of χ for tapes in the vortex state. These measurements were carried out in the presence of the dc magnetic field **H** parallel to the ac magnetic field, $\mathbf{H} \parallel \mathbf{h}_0$. The most striking features of the dependence $\chi(t)$ were observed for the tape with the thickness of the superconducting layer $d \approx 2 \mu m$. The dc resistance of this tape vanished at T < 90.5 K.

Some results obtained for different values of the external dc magnetic field H and ac amplitudes h_0 are shown in Fig. 2. One can see two maxima in the curve $\chi''(T)$ and a sharp kink (fast fall) in the $\chi'(T)$ dependence. The right maximum and the kink are observed near T_c . The position of the right maximum does not depend on the ac amplitude and the dc magnetic field. Contrary, the position of the left maximum is sensitive to the values of h_0 and H.

Leaving the discussion of the interesting behavior of $\chi'(T)$ and $\chi''(T)$ in the vicinity of T_c for the next subsection, we consider here the origin of the left maximum in the dependence $\chi''(T)$. Similar maxima appear often for superconducting plates in the vortex state due to the size effect. The condition for this effect consists in the approximate equality of the ac penetration depth and the half-thickness of the plate. The magnitude χ''_{max} of this maximum is $3/4\pi \approx 0.24$ within the critical state model (remind that we use here the normalization of χ where $\chi' = -1$ for the ideal diamagnetic). However, the critical state model should be modified for slabs with thicknesses comparable with the London penetration depth. As was shown in Refs. 9, 10, the nonlocal effects can play an important role



Fig. 2. (Color online) The measured temperature dependence of χ'' for the tape with the superconducting layer 2 µm in thickness in the vortex state for different values of the ac amplitude and the dc magnetic field. The locations of the experimental curves $\chi''(T)$ along the vertical axis are arbitrary.

in this case. Under the condition $d \sim \lambda_L$, the local relation $B(x) = \Phi_0 n(x)$ between the magnetic induction B(x) and the vortex density n(x) is violated and should be changed by an integral relation (here Φ_0 is the magnetic flux quantum). In other words, the magnetic induction at point x is defined by the vortex distribution over the distance about λ_L near the point x.

According to the nonlocal critical state model [9,10], the value of $\chi''(T)$ is described by the formula

$$\chi'' = \frac{1}{3\pi^2} \left(\frac{l}{D}\right)^2 \frac{3D - 2l}{d},\tag{4}$$

where $D = ch_0/4\pi J_c$, J_c is the temperature dependent critical current density, $l = (d - l_0)/2$, l_0 is a solution of the transcendent equation

$$D = \frac{d - l_0}{2} + \lambda_L \coth \frac{l_0}{2\lambda_L}.$$
 (5)

According to our consideration, the real part of the ac susceptibility can be written in the integral form

$$\chi' = -\frac{1}{4\pi} + \frac{\lambda_L}{\pi^2 d} (i_1 + i_2), \tag{6}$$

$$i_{1} = \frac{1}{2} \tanh\left(\frac{d}{2\lambda_{L}}\right) \left(\arccos\Gamma - \Gamma\sqrt{1 - \Gamma^{2}}\right), \quad (7)$$

$$\Gamma = 1 - \frac{2\lambda_L}{D} \coth \frac{d}{2\lambda_L},$$

$$i_{2} = -\frac{2\lambda_{L}}{D} \int_{d/2\lambda_{L}}^{l_{0}/2\lambda_{L}} dz \coth^{2} z \left[\frac{d}{2\lambda_{L}} - z + \tanh z \right] \times \sqrt{1 - \left[1 + \frac{2\lambda_{L}}{D} \left(z - \frac{d}{2\lambda_{L}} \right) - \frac{2\lambda_{L}}{D} \coth z \right]^{2}}.$$
 (8)

The analysis of Eqs. (4) and (5) shows that the height of the maximum in the function $\chi''(T)$ is less than $3/4\pi$ and decreases when the ratio λ_L/d increases. For our tapes, the ratio d/λ_L is not very large even for T = 0 and decreases with the temperature rise. Therefore, in accordance with the nonlocal theory, the height of the observed left maximum has to be less than 0.24 and should decrease when its position shifts up to higher temperatures. In Fig. 3, we present the temperature dependences of γ' and χ'' calculated within the nonlocal theory for the parameters taken from our experiment. Namely, we used $h_0 = 40$ Oe, $d = 2 \ \mu m$, $\lambda_L(T) = \lambda_L(0)/(1 - T^2/T_c^2)^{0.47}$ with $\lambda_L(0) = 0.173 \,\mu\text{m}$, $T_c = 90$ K. We estimated the temperature dependence of the critical current density as $J_{c}(T) = 2.08 \cdot 10^{6} \cdot (1 - T/T_{c})^{2}$ A/cm². One can see a qualitative agreement of the calculation results with the experiment for the left peak of $\chi''(T)$.

3.3. Unusual behavior of $\chi'(T)$ and $\chi''(T)$ in the vicinity of T_c

As we see in Fig. 2, besides the wide maximum of $\chi''(T)$ related to the nonlocal size effect in the vortex distribution, there exists another sharp peak. The corresponding kink (fall) in $\chi'(T)$ is also observed. The position and magnitude of this peak do not practically depend on the frequency and amplitude of the ac field. Similar peak and kink are observed in the YBCO tapes with increased oxygen deficit δ . The position of the peak in the $\chi''(T)$ dependence follow the changes of the critical temperature T_c



(see curves in Fig. 4 obtained for a sample with $T_c \approx 59$ K). We made also comparative experiments in the parallel field geometry for a YBCO single crystal $2.5 \times 3.5 \times 0.01$ mm in size with $T_c = 92.8$ K and found the similar unusual fea-

Let us discuss the possible nature of this unexpected behavior of $\chi''(T)$ near T_c . The similar maxima in the ac absorption were observed in many microwave experiments

tures of the $\chi(T)$ dependence near T_c .

magnitude of about $2 \cdot 10^{-8}$. This value is much less than the peak magnitude $\approx 2 \cdot 10^{-2}$ observed in our experiment. So negligibly small influence of the conductivity change on the imaginary part of the ac susceptibility is quite natural. The point is that our experiment was performed at low frequencies and symmetric excitation with respect to the ac magnetic field when the skin depth exceeds significantly the thickness of the superconducting layer. Thus, the fluctuation mechanism cannot be responsible for the observed peak in $\chi''(T)$. The discussed maximum cannot be explained within the BCS theory as well.

The observations of curves $\chi''(T)$ with two maxima were reported in a number of papers devoted to investigation of HTC ceramic samples. One of the maxima was caused by the penetration of the magnetic flux into the whole ceramic plate (through the Josephson intergranular



Fig. 3. (Color online) The calculated temperature dependences of χ' and χ'' for the superconducting layer 2 µm in thickness in the vortex state for $h_0 = 40$ Oe.



Fig. 4. (Color online) The temperature dependence of χ for the sample with an oxygen deficit, $T_c \approx 59$ K, f = 700 Hz.

weak links) and the second was connected with the field penetration into the individual granules with relatively high critical current density. Evidently, this picture cannot be attributed to our tapes which are characterized by a good homogeneous crystallographic structure.

We consider the right maximum in the $\chi''(T)$ dependence as the response of the *magnetic subsystem* of our samples to the external ac field. The problem of antiferromagnetic (AF) ordering in applied magnetic field observed in the high- T_c cuprates from neutron scattering [23] (and supporting picture from STM measurements [24]) is intensively discussed topic (see, e.g., Refs. 25–28). According to Refs. 26–28, the AF ordering exists in a wide temperature range below and higher the temperature T_c of the superconducting transition. Many authors paid attention to the crystallographic structural changes in cuprates near T_c [29–32]. Evidently, these structural changes effect on the AF ordering, and the magnetic properties of YBCO samples appear to be different for the normal and superconducting states.

To follow the influence of these phenomena on the ac susceptibility, we measured the $\chi'(T)$ dependence in a wide temperature range (see Fig. 5). The susceptibility $\chi'(T)$ changes nonmonotonically for all samples prepared from tapes SF12050 and SF12100. When decreasing the temperature from 160 to 110 K, $\chi'(T)$ decreases, and then increases in the temperature interval 90 K < *T* < 110 K. When the temperature becomes less than T_c , a sharp decrease in $\chi'(T)$ is observed.

The monotonic change of $\chi'(T)$ at T > 110 K can be related to the thermal expansion of the coil or the tape substrate. The nontrivial observation is the *increase* in χ' , i.e., the *increase* of the ac magnetic flux in the tape when decreasing the temperature from 110 to 90 K. This increase is not related to the magnetic properties of the hastelloy substrate. Indeed, we measured $\chi'(T)$ for the bare tape substrate and found that χ' decreases monotonically with the



Fig. 5. (Color online) The temperature dependence of χ' for the tape SF12100 in a wide temperature range. In the inset: the $\chi''(T)$ dependence in the vicinity of T_c .

temperature decrease in the whole interval of measurements. Thus, we conclude that the observed peculiarities of $\chi'(T)$ and $\chi''(T)$ behavior reflect some magnetic restructuring in the YBCO layer in the vicinity of T_c . The interesting and very important unsolved problem is a question if the superconducting transition causes the discussed magnetic restructuring in the YBCO layer, or vice versa, the change in the AF ordering is a reason of the superconducting transition.

4. Conclusions

In this paper, we have experimentally studied the temperature dependence of the ac magnetic susceptibility of the superconducting tapes of the second generation in the parallel ac and dc magnetic fields for the both Meissner and vortex states. For the vortex state, we have observed two maxima in the temperature dependence of the imaginary part χ'' of the ac susceptibility. The first (left) maximum is caused by the size effect which corresponds to the approximate equality of the ac penetration depth and the halfthickness d/2 of the superconducting layer. Under conditions of our experiment, when the London penetration depth is comparable or exceeds the thickness d, the nonlocal effects should be taken into account. Our measurements of the position and height of the first maximum correspond to the predictions of the nonlocal theory. Actually, our results are the first direct justification of the nonlocal theory.

One of the most important our results consists in the observation of the sharp maximum in the $\chi''(T)$ dependence close to T_c . We have shown that this maximum cannot be explained within the BCS theory or by the theory of the normal skin effect. Moreover, the increase in the conductivity due to the fluctuation mechanism also cannot be responsible for the observed peak in the $\chi''(T)$ dependence. We consider that this maximum can be interpreted as the response of the *magnetic subsystem* of our samples to the external ac field.

Here we call attention to the fact that similar maxima were observed by many authors in the microwave frequency range for different high- T_c cuprate superconductors: YBCO [16–18], $Bi_2Sr_2CaCu_2O_{8+\delta}$ [19], and $Tl_2Ba_2CuO_{6+\delta}$ [20]. Mainly, these maxima are interpreted as a result of the increase in the conductivity due to the fluctuations. However, according to our estimations, this phenomenon cannot be a reason for the $\chi''(T)$ maximum observed near T_c in our measurements. Therefore, a very interesting question arises: do the maxima in the temperature dependence of the surface resistance observed in the microwave experiments are caused by the fluctuating conductivity, or the common mechanism, e.g., related to the magnetic subsystem, is responsible for them in both microwave and low-frequency cases? The answer to this question may be important for the clarification of the origin of the high- T_c superconductivity.

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