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New features of magnetoresistance in the strongly anisotropic layered metals

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The angular and magnetic-field dependence of interlayer magnetoresistance is calculated in the limit of strong magnetic field and very high anisotropy, where it shows several unusual properties. The monotonic part of interlayer magnetoresistance grows with the increase of magnetic field along the current, which contradicts the standard theory. This changes the angular dependence of magnetoresistance. The Dingle temperature increases with magnetic field, which damps the magnetic quantum oscillations and changes the field dependence of their amplitudes.

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73.43.Qt Magnetoresistance;
74.70.Kn Organic superconductors;
74.72.–h Cuprate superconductors.

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1. Introduction

The investigation of the angular and magnetic field dependence of magnetoresistance provides a powerful tool of studying the electronic properties of various metals. The Fermi surface geometry of the most metals has been measured using the magnetic quantum oscillations (MQO) of magnetoresistance [1–3]. The angular dependence of magnetoresistance also gives the important information about the electronic structure and is widely used to investigate the electronic properties of layered compounds: organic metals (see, e.g., Refs. 4–7 for reviews), cuprate high-temperature superconductors [8–11], heterostructures [12] etc.

In layered quasi-2D metals, where the interlayer transfer integral t_z is considerably smaller than the in-plane electron Fermi energy, the electron dispersion is given in the tight-binding approximation by

$$\varepsilon_{3D}(\mathbf{k}) \approx \varepsilon(k_x, k_y) - 2t_z \cos(k_z d), \qquad (1)$$

where $\varepsilon(k_x, k_y)$ is the in-plane electron dispersion, k_z is out-of-plane electron momentum, and d is the interlayer spacing. If t_z is much larger than the Landau level (LL) separation $\hbar\omega_c = \hbar eB / m^* c$, the standard theory of galvanomagnetic properties [1–3] works well. This theory predicts several special features of magnetoresistance in quasi-2D metals: the angular magnetoresistance oscillations [13,14] and the beats of the amplitude of MQO [1].

In strongly anisotropic layered quasi-2D metals, when the interlayer transfer integral t_z is of the order of or less than Landau level separation $\hbar\omega_c$, many new qualitative effects show up. For example, the slow oscillations of magnetoresistance appear [15,16] and the beats of MQO of transport and thermodynamic quantities become shifted compared to each other [16,17]. These effects are not described by the standard theory [1–3] because it is valid only in the lowest order in the parameter $\hbar\omega_c / t_z$. When this parameter becomes of the order of unity, the standard theory is no longer applicable.

The monotonic part of magnetoresistance also changes when $\hbar\omega_c / t_z \sim 1$. According to the standard theory [2], external magnetic field along the electric current leads only to MQO but does not influence the monotonic (background) part of this current. However, the monotonic increase of interlayer magnetoresistance R_{zz} with the increase the magnetic field **B** perpendicular to the conducting layers has been observed in various strongly anisotropic layered metals [18–24]. This monotonic growth of magnetoresistance was attributed to the "strongly incoherent" regime, where the interlayer tunnelling described by the usual Hamiltonian term in Eq. (5) is not effective, and the new mechanisms of interlayer electron transport play the major role. For example, the variable-range electron hopping between the localized states in strong magnetic field leads to the insulating behavior and to the expo-

nential dependence of interlayer conductivity on temperature and magnetic field [25]. In another model, where the in-plane electron motion is metallic but the interlayer electron transport goes via rare local crystal defects (e.g., resonance impurities), the interlayer conductivity σ_{zz} also has metallic-type temperature dependence but decreases strongly with the increase of the out-of-plane component of magnetic field [23]. Below I show, that the monotonic growth of magnetoresistance $R_{zz} \propto \sqrt{B_z}$ appears also in the standard model described by the Hamiltonian (2)–(6) in strong magnetic field at very weak interlayer coupling: $\hbar\omega_c \gg \Gamma_0 > t_z$, where $\Gamma_0 = \hbar/2\tau_0$ and τ_0 is electron mean free time in the absence of magnetic field. This contradicts the common opinion [26] that in the "weakly incoherent" regime, i.e. at $\Gamma_0 > t_z$, the interlayer magnetoresistance does not differ from the coherent almost 3D limit $t_z \gg \Gamma_0$.

2. The model

The electron Hamiltonian in layered compounds with small interlayer coupling consists of the 3 terms:

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_I. \tag{2}$$

The first term $\hat{H}_0$ is the 2D free electron Hamiltonian summed over all layers:

$$\hat{H}_0 = \sum_{m,j} \varepsilon_{2D}(m) c^+_{m,j} c_{m,j}, \tag{3}$$

where $\{m\} = \{n, k_y\}$ is the set of quantum numbers of electrons in magnetic field on a 2D conducting layer, $c^+_{m,j}(c_{m,j})$ are the electron creation (annihilation) operators in the state $\{m\}$ on the layer j, and $\varepsilon_{2D}(m)$ is the corresponding free electron dispersion given by

$$\varepsilon_{2D}(n, k_y) = \hbar\omega_c(n + 1/2). \tag{4}$$

The second term in Eq. (2) gives the coherent electron tunnelling between two adjacent layers:

$$\hat{H}_t = 2t_z \sum_j \int dxdy[\Psi^\dagger_j(x,y)\Psi_{j-1}(x,y) + \Psi^\dagger_{j-1}(x,y)\Psi_j(x,y)], \tag{5}$$

where $\Psi_j(x,y)$ and $\Psi^\dagger_j(x,y)$ are the creation (annihilation) operators of an electron on the layer j at the point (x,y). We call this interlayer tunnelling Hamiltonian "coherent" because it conserves the in-layer coordinate dependence of the electron wave function (in other words, it conserves the in-plane electron momentum) after the interlayer tunnelling. The last term

$$\hat{H}_I = \sum_i V_i(\mathbf{r}) \tag{6}$$

is the impurity potential. The impurities are taken to be point-like and randomly distributed on the layers with volume concentration n_i and areal concentration $N_i = n_i d$ on each conducting layer. The impurity distributions on any two adjacent layers are uncorrelated. The potential $V_i(\mathbf{r})$ of any impurity located at point $\mathbf{r}_i$ is given by

$$V_i(\mathbf{r}) = U\delta^3(\mathbf{r} - \mathbf{r}_i). \tag{7}$$

We also introduce the 2D analogue of the point-like impurity potential:

$$V_i(x,y) = V_0\delta(x - x_i)\delta(y - y_i), \tag{8}$$

where $V_0 = U|\psi(z_i)|^2 \approx U/d$. This is equivalent to the smearing of the 2D impurity strength with the distribution [32]

$$D_I(V_0) = \int_0^d \frac{dz}{d}\delta(V_0 - U|\psi(z_i)|^2).$$

This smearing of the impurity strength can be performed in the final result for conductivity. It does not lead to the new qualitative effects, and, for simplicity, we neglect it.

3. The electron in-plane Green's function

In the limit, $t_z \ll \Gamma, \hbar\omega_c$, the interlayer hopping t_z must be considered as a perturbation for the disordered uncoupled stack of 2D metallic layers. The 2D metallic electron system in magnetic field in the point-like impurity potential has been extensively studied [27–33]. In the self-consistent single-site approximation the coordinate electron Green's function, averaged over impurity configurations, is given by

$$G(\mathbf{r}_1, \mathbf{r}_2, \varepsilon) = \sum_{n,k_y} \Psi^{0*}_{n,k_y}(r_2)\Psi^0_{n,k_y}(r_1)G(\varepsilon, n), \tag{9}$$

where $\Psi^0_{n,k_y}(r_1)$ are the 2D electron wave functions in perpendicular magnetic field [34], and the Green's function $G(\varepsilon, n)$ does not depend on k_y:

$$G(\varepsilon, n) = \frac{1}{\varepsilon - \hbar\omega_c(n + 1/2) - \Sigma(\varepsilon)}, \tag{10}$$

where $\Sigma(\varepsilon)$ is the electron self-energy part due to scattering by impurities.

In strong magnetic field, when $\hbar\omega_c \gg \Gamma_0 = \hbar/2\tau_0$, one can consider each Landau level separately. In the self-consistent single-site approximation [27] the Green's function is given by

$$G(E, n) = \frac{E + E_g(1 - c_i) - \sqrt{(E - E_1)(E - E_2)}}{2EE_g}, \tag{11}$$

and the density of states (DOS) on each LL is described by the well-known dome-like function [27]

$$D(E) = \frac{\sqrt{(E - E_1)(E_2 - E)}}{2\pi|E|E_g}, \tag{12}$$

where the electron energy E is counted from the last occupied LL: $E \equiv \varepsilon - \varepsilon_{2D}(n_F, k_y)$, and $E_g = N_{LL}V_0$, where the LL degeneracy per unit area is $N_{LL} = 1/2\pi l_{Hz}^2 = eB/2\pi\hbar c$. The boundaries of the DOS dome in Eq. (12) are

$$E_1 = E_g(\sqrt{c_i} - 1)^2, \; E_2 = E_g(\sqrt{c_i} + 1)^2, \qquad (13)$$

where c_i is the dimensionless ratio of the impurity concentration to the electron concentration on one LL:

$$c_i = N_i / N_{LL} = 2\pi l_{Hz}^2 n_i d. \qquad (14)$$

The function $D(E)$ in Eq. (12) is nonzero in the interval $0 < E_1 < E < E_2$ and normalized to unity: $\int D(E)dE = 1$. The LL half-width

$$\Gamma_B \equiv (E_2 - E_1)/2 = 2E_g\sqrt{c_i} \propto \sqrt{B}. \qquad (15)$$

The subscript "B" in Γ_B in Eq. (15) emphasizes that the LL broadening depends on magnetic field B. The Green's function in Eq. (11) differs considerably from the Green's function used in Ref. 26:

$$G(E,n) = \frac{1}{E - i\Gamma_0}. \qquad (16)$$

The LL broadening in Eq. (12) is much larger and depends on magnetic field. The ratio

$$\Gamma_B / \Gamma_0 \approx \sqrt{4\hbar\omega_c / \pi\Gamma_0} \gg 1 \qquad (17)$$

grows as $\sqrt{B}$ in high magnetic field. Without magnetic field $\Gamma_B \approx \Gamma_0$. To get the correct asymptotic behavior for Γ_B both in strong magnetic field and at $B = 0$, one can take the simple function

$$\Gamma_B \approx \Gamma_0\left[\left(4\hbar\omega_c / \pi\Gamma_0\right)^2 + 1\right]^{1/4}. \qquad (18)$$

More realistic models of the finite-range impurity potential, and more accurate calculation of the DOS, including the many-site corrections, lead only to the small tails of the DOS dome [28,30,33]. The number of electron states in these tails is much less than the number of states in the DOS dome and can be neglected. However, to include these tails into account and to simplify the subsequent calculation, one can take the Lorentzian DOS distribution with the same broadening:

$$D(E) \approx \frac{\Gamma_B}{\pi(E^2 + \Gamma_B^2)} = -\frac{\mathrm{Im}G_R(E)}{\pi}. \qquad (19)$$

The physical origin of large DOS broadening in Eq. (12) is not the finite lifetime τ of electron states, which is mathematically described by the imaginary part of the self-energy $\mathrm{Im}\,\Sigma = \Gamma_0 = \hbar/2\tau$, as in the 3D limit. On the 2D layers the LL broadening comes from the energy shift of each electron state, which is described by the state-dependent real part of the electron self-energy $\mathrm{Re}\,\Sigma$. The averaging of the electron Green's function in Eq. (16) over the impurity configurations is independent on each conducting layer, since the impurity distribution is assumed to be uncorrelated. Then the denominator acquires the real part of electron self energy, $\varepsilon \to \varepsilon - \mathrm{Re}\Sigma$, and the averaging over impurity configurations makes $\mathrm{Re}\Sigma$ to be distributed with the DOS function $D(\mathrm{Re}\Sigma)$:

$$G(E,n) = \int dE\, D(E)\frac{1}{\varepsilon - E - \hbar\omega_c(n + 1/2) - i0}.$$

Substituting the DOS distribution from (19), one can easily perform the integration over E and obtain

$$G(E,n) = \frac{1}{\varepsilon - \hbar\omega_c(n + 1/2) - i\Gamma_B}. \qquad (20)$$

This Green's function differs from Eq. (16) by the increase of the imaginary self-energy part: $\Gamma_0 \to \Gamma_B$ with Γ_B given by Eq. (18). The more rigorous calculations of the electron Green's function gives the same result as in Eq. (20). This Green's function will be used in the next section to calculate the interlayer conductivity.

4. Calculation of conductivity

The interlayer conductivity σ_{zz}, associated with the Hamiltonian (5), can be calculated using the Kubo formula and the formalism, developed for the metal–insulator–metal junctions [35]. In analogy to Eq. (44) of Ref. 26,

$$\sigma_{zz} = \frac{4e^2 t_z^2 d}{\hbar L_x L_y} \times$$

$$\times\left\langle \int d^2\mathbf{r} d^2\mathbf{r}' \int \frac{d\varepsilon}{2\pi} \mathrm{Im}G_R(\mathbf{r},\mathbf{r}',j,\varepsilon)\mathrm{Im}G_R(\mathbf{r}',\mathbf{r},j+1,\varepsilon)[-n_F'(\varepsilon)]\right\rangle, \qquad (21)$$

where $n_F(\varepsilon) = 1/(1 + \exp[(\varepsilon - \mu)/T])$ is the Fermi distribution function, μ is the chemical potential and T is temperature. The angular brackets in Eq. (21) mean averaging over impurity configurations. Since the impurity distributions on each layer is uncorrelated with other layers, one can perform this averaging separately for each spectral function independently, which gives

$$\sigma_{zz} = \frac{4e^2 t_z^2 d}{\hbar L_x L_y} \times$$

$$\times \int d^2\mathbf{r} d^2\mathbf{r}' \int \frac{d\varepsilon}{2\pi} \left\langle \mathrm{Im}G_R(\mathbf{r},\mathbf{r}',j,\varepsilon)\right\rangle \left\langle \mathrm{Im}G_R(\mathbf{r}',\mathbf{r},j+1,\varepsilon)\right\rangle [-n_F'(\varepsilon)]. \qquad (22)$$

The averaged Green's (or spectral) functions are translational invariant: $\left\langle G_R(\mathbf{r},\mathbf{r}',j,\varepsilon)\right\rangle = \left\langle G_R(\mathbf{r}-\mathbf{r}',j,\varepsilon)\right\rangle$.

In the magnetic field perpendicular to the conducting layers the coordinate dependence of the electron Green's function on the adjacent layers is the same. Then the inte-

gration over **r** for the Green's function of the form (9) is very simple and gives

$$\sigma_{zz} = \frac{4e^2 t_z^2 d\, N_{LL}}{\hbar} \int \frac{d\varepsilon}{2\pi} \left[-n_F'(\varepsilon)\right] \sum_n \left|\mathrm{Im}G_R(\varepsilon, n)\right|^2 . \qquad (23)$$

Substituting Eq. (10) we obtain

$$\sigma_{zz} = \frac{4e^2 t_z^2 d\, N_{LL}}{\hbar} \int \frac{d\varepsilon}{2\pi} \sum_n \frac{\left[-n_F'(\varepsilon)\right] \left|\mathrm{Im}\Sigma(\varepsilon)\right|^2}{\left[(\varepsilon - \varepsilon_n - \mathrm{Re}\Sigma(\varepsilon))^2 + \left|\mathrm{Im}\Sigma(\varepsilon)\right|^2\right]^2} . \qquad (24)$$

The sum and integral in Eq. (24) is calculated in a standard way, transforming the sum over LL into the harmonic sum by applying the Poisson summation formula [36]:

$$\sum_{n=n_0}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_a^{\infty} \mathrm{e}^{2\pi i k n} f(n)\, dn, \qquad (25)$$

where $a \in (n_0 - 1; n_0)$. Then, performing the integrations, we obtain

$$\sigma_{zz} = \sigma_0 \sum_{k=-\infty}^{\infty} (-1)^k \int d\varepsilon \left[-n_F'(\varepsilon)\right] \exp\left(\frac{2\pi i k [\varepsilon - \mathrm{Re}\Sigma(\varepsilon)]}{\hbar\omega_c}\right) \times$$

$$\times \exp\left(\frac{-2\pi |k| \left|\mathrm{Im}\Sigma(\varepsilon)\right|}{\hbar\omega_c}\right) \left(\frac{\Gamma_0}{\left|\mathrm{Im}\Sigma(\varepsilon)\right|} + \frac{2\pi |k| \Gamma_0}{\hbar\omega_c}\right), \qquad (26)$$

where

$$\sigma_0 = \frac{e^2 t_z^2 \nu_F d}{\hbar \Gamma_0}, \qquad (27)$$

$\nu_F = N_{LL} / \hbar\omega_c$ is the DOS at the Fermi level in the absence of magnetic field.

5. Results

At $\hbar\omega_c \gg \Gamma_0$, substituting Eq. (20) into Eq. (26) and performing the integration over ε we obtain in analogy with Ref. 39

$$\sigma_{zz} = \sigma_0 \frac{\Gamma_0}{\Gamma_B} \sum_{k=-\infty}^{\infty} (-1)^k \exp\left[\frac{2\pi k (i\mu - \Gamma_B)}{\hbar\omega_c}\right] \times$$

$$\times \frac{2k\pi^2 T / \hbar\omega_c}{\sinh(2k\pi^2 T / \hbar\omega_c)} \left[1 + \frac{2\pi |k| \Gamma_B}{\hbar\omega_c}\right]. \qquad (28)$$

Equation (28) outwardly resembles Eqs. (17)–(21) of Ref. 37, derived in the Born approximation in the limit of well-defined 3D electron dispersion. It also resembles Eqs. (12), (15) of Ref. 38 and Eqs. (10), (15) of Ref. 16 in the limiting case of $t_z \to 0$. However, there is the important difference between Eqs. (28), (27) and the previous results, which comes out because Eqs. (28), (27) are derived beyond the traditionally used Born approximation. This difference consists in the replacement of the field independent quantities Γ_0 and Γ_ε by Γ_B, which has the strong monotonic dependence on magnetic field given by Eq. (18).

Let us now compare how strongly the field dependence of interlayer conductivity given by Eqs. (28), (27) differs from the previous results (see, e.g., Eqs. (17)–(21) of Ref. 37). In Fig. 1 we compare the field dependence of magnetoresistance $R_{zz}(B) = 1/\sigma_{zz}$, calculated using Eq. (28) with Γ_B given by Eq. (18) (solid line, corresponding to the new result) and with $\Gamma_B = \Gamma_0$ (dashed line, corresponding to Ref. 37). The strong difference is evident. First, the interlayer magnetoresistance shows the monotonic growth with the increase of magnetic field, directed along conductivity and perpendicular to the conducting layers. This monotonic growth is observed in all experiments on interlayer magnetoresistance in strongly anisotropic metals (see, e.g., Refs. 15, 19) but was not explained before and, therefore, was not used to extract any information about compounds. Equations (18), (27) allow to use this dependence for the alternative estimate of the Landau level broadening $\Gamma(B)$ from the experimental data. Second, the new result takes into account the magnetic field dependence of the Dingle temperature, which leads to the weaker field dependence of MQO amplitude and predicts smaller amplitude of MQO as compared to the result of Refs. 37, 40.

The angular dependence of interlayer magnetoresistance is also considerably modified by the replacement $\Gamma_0 \to \Gamma_B$. In tilted magnetic field the calculation of conductivity, performed in Ref. 26, can be applied with the new magnetic-field-dependent value Γ_B instead of Γ_0, which gives [39] (compare to Eq. (1) of Ref. 26)

$$\sigma_{zz} = \sigma_0(B_z) \left\{ [J_0(\kappa)]^2 + 2 \sum_{\nu=1}^{\infty} \frac{[J_\nu(\kappa)]^2}{1 + (\nu\omega_c \tau_B)} \right\}, \qquad (29)$$

where $\kappa \equiv k_F d \tan\theta$ is the same as in Ref. 26, but $\sigma_0(B_z)$ and τ_B acquire the field and angular dependence: $\sigma_0(B_z)$ is given by Eq. (27) and

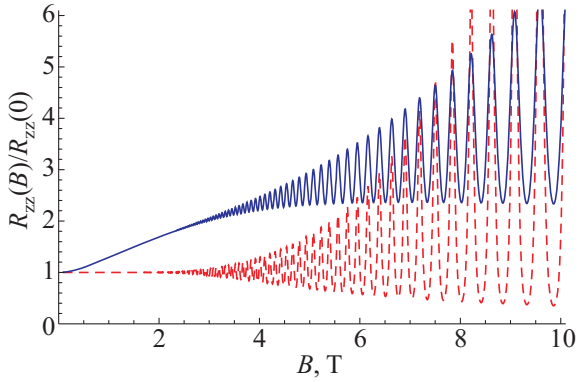

Fig. 1. The MQO of resistivity $R_{zz}(B) = 1/\sigma_{zz}$, calculated using Eq. (28) with Γ_B given by Eq. (18) (solid line) and with $\Gamma_B = \Gamma_0$, that corresponds to Ref. 37 (dashed line). The parameters are $m^* = m_e$, $\Gamma_0 = 0.8$ K and $T = 0.6$ K.

$$\tau_B = \tau_B(B_z) = \hbar / 2\Gamma_B = \tau_0(\Gamma_0 / \Gamma_B). \qquad (30)$$

In high magnetic field $\omega_c\tau \gg 1$ and in the "weakly incoherent limit" $t_z < \Gamma_B$, $\sigma_0(B_z) \propto \tau_B \propto 1/\sqrt{B\cos\theta}$.

There are two main differences between the new angular dependence of magnetoresistance given by Eq. (29) and Eq. (1) of Ref. 26. First, the sharp peaks of magnetoresistance at the Yamaji angles become smoother in the new formula. This change is due to the higher harmonics $[J_\nu(\kappa)]^2$ in AMRO, which are less damped in Eq. (29) as compared to Eq. (1) in Ref. 26 because of the smaller value of $\tau_B = \tau_0(\Gamma_0 / \Gamma_B) \propto 1/\sqrt{B_z}$. These higher harmonics also slightly shift the positions of the Yamaji angles. Second, the monotonic part of the angular dependence of magnetoresistance changes because of the additional angular dependence of the prefactor $\sigma_0(B_z)$ given by Eq. (27), which in strong field $\propto 1/\sqrt{B\cos\theta}$.

To illustrate these differences, in Fig. 2 we plot the angular dependence of magnetoresistance $R_{zz}(\theta) = 1/\sigma_{zz}(\theta)$ given by Eq. (29) with $\tau_B = \hbar / 2\Gamma_B = \tau_0(\Gamma_0 / \Gamma_B)$ (solid line) and $\tau_B = \tau_0$ (dotted line). For simplicity, we take the axially symmetric case, i.e. the symmetric in plane electron dispersion. One can see that in the minima of conductivity, i.e. at the Yamaji angles, the replacement $\tau_0 \to \tau_B$ is very important. The predicted value of magnetoresistance at the Yamaji angles with τ_B given by Eq. (30) is much smaller than with $\tau_B = \tau_0$ (see Fig. 2). This difference grows with the increase of magnetic field. The positions of the conductivity minima, i.e. the Yamaji angles, also slightly shift after the replacement $\tau_0 \to \tau_B$ in Eq. (29) (see Fig. 2). For the first Yamaji angle at $B = 5$ T this shift $\Delta\theta_{\text{Yam}} \approx 0.5^\circ$ (see Fig. 2). The angular-dependent prefactor $\sigma_0(B_z)$ in Eq. (29) considerably changes the ratio $\sigma_{zz}(\theta = \pi/2)/\sigma_{zz}(\theta = 0)$, which according to Eqs. (27) and (29) becomes larger by the factor Γ_B / Γ_0 (see Eq. (17) and Fig. 2).

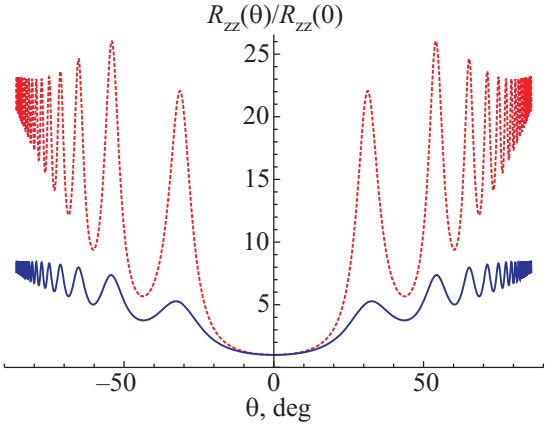

Fig. 2. The angular dependence of magnetoresistance $R_{zz}(\theta)/R_{zz}(0)$, calculated using Eq. (29) with τ_B given by Eq. (30) (solid line) and with $\tau_B = \tau_0$ (dotted line). The parameters for this plot are $k_F d = 4$, $m^* = m_e$, $B = 5$ T, $\Gamma_0 = 1$ K, which gives $\omega_c\tau \approx 1.74$.

6. Discussion

Let us formulate the main difference of the present approach to the calculation of interlayer conductivity in the weakly incoherent regime compared to the previous methods, developed in Refs. 16, 37, 38, 40, 41 to calculate the MQO of conductivity. In these papers the impurity potential is considered as a small perturbation on the background of a free electron gas with well-defined 3D electron dispersion given by Eq. (1). Hence, the impurity scattering was taken into account only by the imaginary part of the electron self-energy, which was calculated in the Born approximation. Even less accurately the impurities are treated in Ref. 26, where the constant electron mean-free time has been used to include the interaction with impurities. The Born approximation can be applied only in the 3D coherent limit, when the interlayer transfer integral is much larger than the LL broadening. In the "weakly incoherent" regime, when $t_z < \Gamma, \hbar\omega_c$, this is incorrect, because for a 2D electron system in magnetic field the impurity potential has much stronger effect than in 3D. Qualitatively, in a 3D electron system the electrons after scattering by an impurity move away in the interlayer direction and never return to this impurity. Therefore, this impurity only leads to the single scattering of this electron into some other state, which is well described by the Born approximation or even by a constant electron mean-free time τ_0, or equivalently, by the constant imaginary part Γ_0 of the electron self-energy. In 2D electron system in magnetic field, the electrons after scattering return to the same impurity after the cyclotron period. Therefore, the impurity has permanent influence on the electron state, considerably shifting the electron energy and modifying the electron states. Hence, in the weakly incoherent regime, when $t_z < \Gamma, \hbar\omega_c$, the interlayer hopping term (5) in the Hamiltonian (2), rather the impurity potential (6), must be considered as a small perturbation. Therefore, to calculate the interlayer conductivity, we start from the stack of isolated 2D disordered conducting layers in magnetic field, where the effect of impurity potential is considered much more accurately, at least in the self-consistent single-site ("noncrossing") approximation. This allows us to go beyond the Born approximation, incorrectly applied in Refs. 37, 40, 41. Then we substitute the obtained electron Green's functions to the Kubo formula for the tunnelling conductivity between adjacent conducting layers, which does not require the 3D electron dispersion. The effect of impurities in the final results turned out to be much stronger than in the previous approaches. Phenomenologically, this difference can be taken into account by the replacement of the initial level broadening Γ_0 in Eq. (16) by the larger value given by Eq. (18).

The oscillations of the chemical potential, which appear to be strong in the artificial layered compounds as hetero-

structures [42] and are also observed in some other materials [43], may considerably affect the MQO of thermodynamic and transport quantities [44,45]. For example, they may lead to the mixing of the MQO frequencies even in the de Haas–van Alphen effect [44]. However, in the natural layered compound, even in the extremely anisotropic almost 2D layered system as β″-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$, the oscillations of the chemical potential turned out to be negligibly small, as was experimentally confirmed by analyzing the shape of the magnetization oscillations [46]. This shape turned out to be the same as in the 2D theory of magnetization oscillations with fixed chemical potential (see Fig. 3 of Ref. 46). In anisotropic 3D metals as beryllium, where the magnetic quantum oscillations are very strong, the oscillations of the chemical potential are also damped by more than ten times [47]. The absence of the chemical potential oscillations was explained by the observation of MQO of the sample volume, which lead to the oscillations of the electron concentration and cancel the oscillations of the chemical potential [47]. The observation of strong MQO of the metallic sample volume is not surprising, because the delocalized electrons give the main contribution to the modulus of elasticity of metals [48]. The role of this magnetostriction on the damping of the MQO of chemical potential is somewhat analogues to the electron reservoir, which can be simply included to the theory of MQO [37,45]. The main result of the present paper, that the monotonic field dependence of the LL broadening strongly affects the magnetic field and angular dependence of the interlayer magnetoresistance, is not sensitive to the oscillations of the chemical potential.

The electron–electron interaction, neglected in the above calculations, is more important in the layered strongly anisotropic compounds than in usual 3D metals and may modify the quantitative behavior of magnetoresistance. For example, the superconducting fluctuations may considerably change the properties of cuprate high-temperature superconductors even above the superconducting transition temperature [49]. The quantum corrections to conductivity are small, but may also become detectable when magnetic field is tilted toward the conducting plane [50]. However, all these effects are much weaker than the gross qualitative effects predicted in the present work.

The proposed analysis considers only the limiting case $\hbar\omega_c \gg \Gamma_0 \gg t_z$, when Γ_B is given by Eqs. (15) or (18), but it is not accurate at $\hbar\omega_c \sim \Gamma_0$, where the crossover from weak to strong magnetic field regime takes place. The phenomenological formula (18) gives only a qualitative dependence $\Gamma_B(B_z)$ in this region. To study the opposite limit $\Gamma_0 \gg \hbar\omega_c \gg t_z$ one can expand Eq. (26) in the small parameter $R_D = \exp(-2\pi\Gamma_0/\hbar\omega_c)$. In the Born approximation in the second order in R_D this expansion gives no correction to the monotonic part of σ_{zz} [51]. This suggests that the crossover from weak to strong magnetic field is rather sharp, similar to that observed in Refs. 21, 24 and studied in Ref. 25. Evidently, the limit $\hbar\omega_c \gg \Gamma_0 \gg t_z$ and the crossover region need further theoretical study. Note, that the one-particle approximation, used in the above calculations, may violate at zero field.

7. Summary

We consider interlayer magnetoresistance R_{zz} of strongly anisotropic layered metals in the so-called "weakly incoherent" regime, when the interlayer transfer integral t_z is much less than the Landau level separation $\hbar\omega_c$ and broadening Γ_0 due to the interaction with impurities. The angular and field dependence of interlayer conductivity in this regime is calculated. We obtain that both these dependencies in strong magnetic field, $\hbar\omega_c \gg \Gamma_0$, considerably differ from those in the limit $t_z \gtrsim \hbar\omega_c, \Gamma_B$. This contradicts the previous theoretical results [26,37]. The background interlayer conductivity σ_{zz} decreases with the increase of magnetic field B_z according to Eq. (27) with Γ_B approximately given by Eq. (18), while in the standard "coherent" theory [37] it remains constant (see Fig. 1). The Dingle temperature of MQO also increases with magnetic field $\propto \Gamma_B$, which modifies the field dependence of MQO amplitude (see Fig. 1 for illustration). Meantime, in the "weakly incoherent" regime the angular oscillations of background magnetoresistance are not damped as in the completely incoherent mechanisms of the interlayer electron transport, considered, e.g., in Refs. 23, 25, 52–56. However, the angular dependence of interlayer magnetoresistance in the "weakly incoherent" regime, given by Eqs. (29), (30), considerably differs from that in the standard "coherent" theory [57] and from the dependence obtained in Ref. 26 for the same "weakly incoherent" regime (see Fig. 2 and the discussion after Eqs. (29), (30)). This difference includes both the monotonic and oscillating parts of the angular dependence of magnetoresistance. Phenomenologically, the differences between the coherent and weakly incoherent regimes can be taken into account by the replacement of the electron mean free time τ_0 by the new smaller value $\tau_B \approx \tau_0/[(8\omega_c\tau_0/\pi)^2+1]^{1/4}$ in all formulas for the field and angular dependence of interlayer magnetoresistance.

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