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Problems of nonlinear optics and acousto-optics in the curriculum of “Quantum Electronics” specialization

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Abstract. We show the role of nonlinear and acousto-optical phenomena in optical and quantum electronics and the physics of semiconductor lasers. The examples of the use of these phenomena in the optical systems of information processing and the light generation by lasers are presented.

Keywords: nonlinear optical phenomena, acousto-optics, integral optics, quantum electronics, semiconductor laser.

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1. Introduction

Acousto-optical and nonlinear optical phenomena are included in the university curriculum on physical specialities, namely on “Physical electronics” speciality and “Quantum electronics” specialization.

The acquaintance with the nature of these phenomena begins in the course of general natural-scientific disciplines. The course “Optics” gives the idea about the reasons of breaking down the linearity of light’s interaction with the substance through the nonlinear polarization of the medium under the effect of light wave’s electrical field.

The next level is concerned with studying the circle of general professional and special disciplines. The general methods to study nonlinear phenomena are given in the course “Theory of wave processes,” where the nonlinear interactions of waves in electrodynamics and acoustics, generation of harmonic components, three-wave interaction, and self-interaction and cross-modulation of waves are analyzed.

2. Acousto-optics phenomena in optoelectronics

The problems of acousto-optics in the course “Optoelectronics” are considered from the side of their use for performing the informational operations over light beams [1]. One knows that, for the effective internal modulation of light directly in a radiation source, it is necessary that the lifetime of the particles at the upper operating level τ be quite short ($\tau < 1/2\pi\Delta f$, where Δf is the required modulation frequency band). It is about 10^{-9} s in the case of semiconductor injection lasers (IL), thus, in these lasers, the internal high-speed light modulation is available. In solid-state and gas lasers, the lifetime τ is much longer (about 10^{-7} s in a He-Ne laser and about 10^{-3} s in a YAG:Nd one). So long lifetimes restrict the direct modulation by so low frequencies, it loses any practical purpose. Thus, the radiation of gas and solid-state lasers is modulated externally with the use of various physical phenomena.

The acousto-optical effect is one of form of photoelasticity. Under the influence of mechanical stresses caused by an acoustic wave in the medium, there appear the alternating regions with different refractive indices traveling with the sound speed. As a result, the medium gets characteristics of an optical phase grating that has a period determined by the acoustic wavelength and a sharpness determined by the applied acoustic power. If the grating period Λ is comparable to the light wavelength λ and the linear aperture of the light beam $D >> \Lambda$, then light diffracts on the acoustic wave.

Consider two limiting diffraction modes depending on the angle between the wave vectors of the acoustic $\vec{K}_S$ and optical $\vec{k}$ waves and the ratio $\lambda \cdot l / \bar{n} \cdot \Lambda^2$, where l is the length of interaction, and $\bar{n}$ is the non-disturbed refractive index of the substance. If the length of interaction of the acoustic and light waves satisfies the relation

$$l << \bar{n} \cdot \Lambda^2 / \lambda ,$$

and the product $\vec{k} \cdot \vec{K}_S = 0$ (vector $\vec{k}$ is normally directed to the vector $\vec{K}_S$), then the Raman-Nath diffraction takes place. In this case at the acousto-optical cell output, there is observed a series of light beams (no less than three) symmetrically dispersing at the angles of Θ_m to the direction of a falling beam, and

$$\sin\Theta_m = m\lambda/\bar{n}\Lambda,$$

where $m = 0, \pm 1, \pm 2, \ldots$ is the diffraction order.

Physically, relation (1) is the condition of single light diffraction on an acousto-optical wave. Under the inverse condition

$$l >> \bar{n} \cdot \Lambda^2/\lambda,$$

the clear diffraction picture at the cell's output can be created by the light that falls on it at the angle of Θ_B satisfying the relation

$$\sin\Theta_B = \lambda/2\bar{n}\Lambda.$$

This is the case of Bragg diffraction, and Θ_B is called the Bragg angle. In contrast to the Raman-Nath diffraction, the light has multiple diffraction acts in the Bragg's mode before it leaves the interaction region. As the resulting secondary diffraction beams are mutually coherent, all the diffraction maxima, except the zero and the first ones, are mutually darkened as a result of the interference. At the cell's output are observed the zero and the first beams. The power of the diffracted (the first) beam is

$$P_1 = P\sin^2\left(\pi\bar{n}\Delta\bar{n}l/\lambda\right),$$

where $\Delta\bar{n}$ is a change of the refractive index. As one can see, if the condition

$$\bar{n}\Delta\bar{n}l = \lambda/2$$

is satisfied, the almost 100 % effectiveness of diffraction may be achieved.

From the quantum-mechanical point of view, the Bragg diffraction is explained as a result of the elastic scattering of photons on phonons [1]. The diffraction character is determined by the conservation laws of energy and momentum at the photon-phonon interaction.

Let us consider the use of the acousto-optical effect for the modulation of the intensity, frequency, and traveling direction of light beams in the construction of tunable acousto-optical filters. For the light modulation, both diffraction modes can be used. It is also worth noting that acousto-optical modulators can be constructed in the integrated-optical form. In this case, light interacts with surface acoustic waves, rather than with volume ones.

The possibility of frequency modulation of light is based on its interaction with the moving diffraction grating and on a change of the output radiation frequency due to the Doppler effect. As the acoustic frequencies are many orders less than the optical ones, it is extremely difficult to observe the effect of frequency modulation at the direct light detection. To detect the frequency-modulated light, one uses the optical heterodyne method.

Changing the acoustic wave frequency allows one to perform the spatial scanning of a light beam. It is shown that the deflector's angle resolution (the ratio between the scanning angle and the angular width of a light beam) is equal to $N = \tau\Delta f$, where $\tau = a/\upsilon$ is the time constant determining the deflector's performance speed (υ is the speed of an acoustic wave, Δf is the frequency deviation). This means that, at the given performance speed, the deflector's angle resolution can be increased only by increasing Δf. Because, at the given values of λ, $\bar{n}$, and l, the range of frequency changes is restricted by condition (1), the Raman-Nath deflector can operate in a relatively narrow frequency band. Therefore, the Bragg diffraction mode is more often used for the spatial scanning. With the help of an acousto-optical cell, one can perform not only the one-dimensional light scanning, but also the two-dimensional one.

3. Integral optics

The practical use of nonlinear and acousto-optical phenomena is considered in the special courses "Systems of semiconductor quantum electronics" [2] and "Semiconductor radiation sources in information-measurement systems," where the potentialities of the application of laser diodes in acousto-optical processors (AOPs) with the processing of the information in real time are analyzed. The radiation traveling in a planar wave guide diffracts on the phase grating that appears under the influence of acoustic waves excited in an interdigital transducer (IDT) by the source of radio-frequency signals. The cases are possible when one signal is supplied to the IDT input and its spectrum is analyzed or two signals are supplied with the further determination of their convolution or correlation. Schemes including an acousto-optical processor on planar light-guides, spectrum analyzers with spatial and time integrations, a correlator, and a convolver are presented in Figs. 1-4.

In a spectrum analyzer with spatial integration at supplying a harmonic signal $S(t) = \cos\omega t$ to the IDT input, the phase grating will be a harmonic field of elastic deformations $S(x) = \cos\omega_x t$, where the spatial frequency $\omega_x = \omega/\upsilon$ (υ is the sound speed in the material of an AOP). The lens performs a Fourier transformation of the input light field $S(x)$, and the distribution of the light intensity that is proportional to the input signal spectrum $S(\omega)$ is formed in its focal plane:

$$S(\omega) = \frac{1}{\upsilon}S(\omega_x) = \frac{1}{\upsilon}\int_{-\infty}^{\infty} S(x)e^{-i\omega_x x}dx.$$

The spatial frequencies ω_x are connected with diffraction angles Θ by the relation $\omega_x = (\lambda/\Theta)^{-1} = (\lambda f/x)^{-1}$, where x is a coordinate of the spectral component in the Fourier-lens' focal plane,

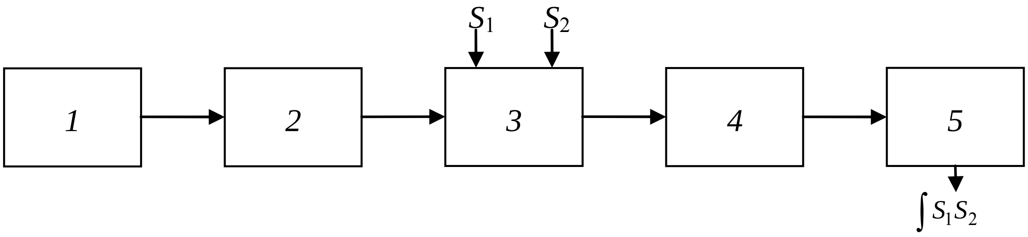

Fig. 1. General scheme of an acousto-optical processor: 1 – source of light; 2 – forming system; 3 – acousto-optical processor; 4 – spatial filter; 5 – photodetecting system.

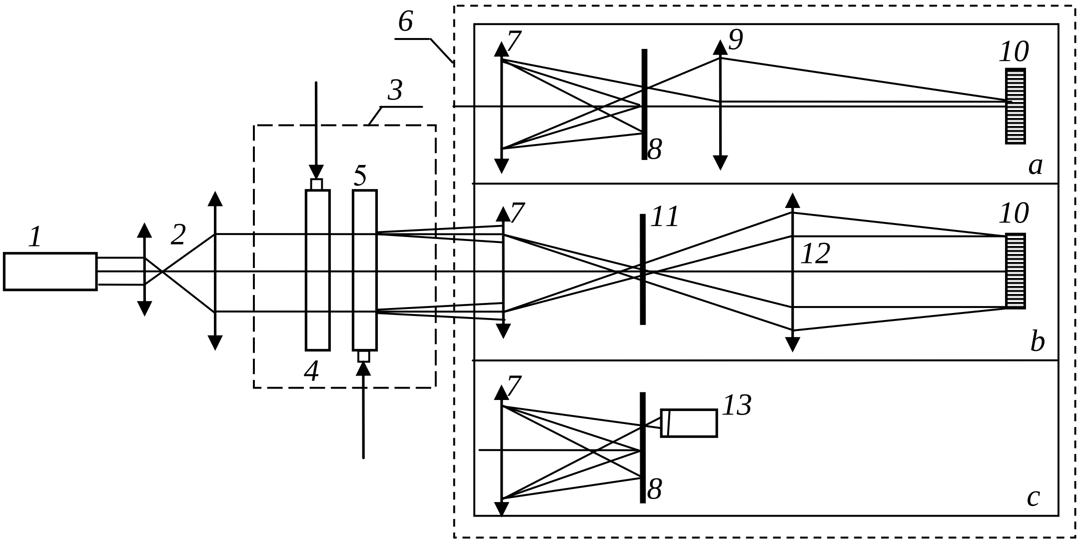

Fig. 2. Optical scheme of an acousto-optical processor with functions of the spectrum analyzer (*a*), correlator (*b*), and convolver (*c*): 1 – laser; 2 – collimator; 3 – integral acousto-optical modulator; 4, 5 – surface acoustic wave transducer; 6 – output signal converters; 7, 9, 12 – objectives; 8, 11 – spatial filters; 10 – CCD; 13 – photodetector.

and f is the lens' focal distance. Then the spectral components of an input signal are determined through the corresponding coordinates of the x-images of the light source by the following formula:

$$\omega = \omega_x \upsilon = \frac{x}{\lambda f} \upsilon .$$

By the spatial filter, one of the diffraction orders is selected and registered by a line of photodetectors. The position of every sensitive element corresponds to some definite frequency of the electrical signal.

The frequency resolution of a spectrum analyzer with spatial integration δf is limited by the size D of the input aperture of AOP's sound transducer or by the time τ of an acoustic wave passing through the optical beam aperture. At the optical beam divergence restricted by diffraction, it is equal to

$$\delta f = \frac{\upsilon}{D} = \frac{1}{\tau} .$$

In the correlator, the signals $S_1(t)$ and $S_2(t)$ are supplied towards each other to two IDT and the diffraction field $S(x, t)$ at the AOP output is determined by the total influence of these signals:

$$S(x,t) = S_1\left(t - \frac{x}{\upsilon} - \frac{D}{2\upsilon}\right) + S_2\left(t + \frac{x}{\upsilon} - \frac{D}{2\upsilon}\right).$$

Subjecting this field to the direct and inverse Fourier transformation by lenses and performing the further time integration of the optical signal by a line of photodetectors, we get that the variable component of the output signal

$$U(x) = 2\,\mathrm{Re}\int_0^{\tau} S_1\left(t - \frac{x}{\upsilon} - \frac{D}{2\upsilon}\right) S_2\left(t + \frac{x}{\upsilon} - \frac{D}{2\upsilon}\right) dt , \qquad (1)$$

where τ is the integration time.

If the input signals S_1 and S_2 are radio signals modulating the carrier frequency ω, then relation (1) can be written as

$$U(x) = \left|R_{12}(x)\right| \cos\left[2\omega \frac{x}{\upsilon} + \theta(x)\right], \qquad (2)$$

where $R_{12}(x)$ is the cross-correlation function of signals, $\theta(x)$ is a slowly changing quantity corresponding to the phase of the correlation function. From (2), one can see that the correlation function is an envelope of a high-frequency signal with the spatial

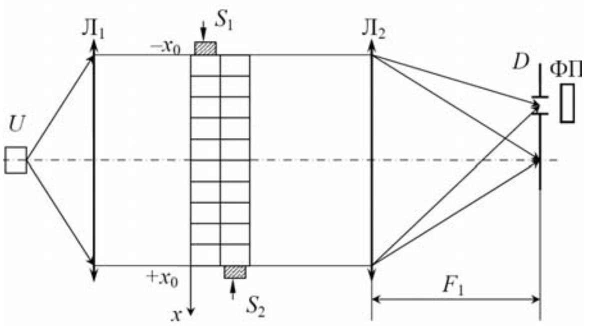

Fig. 3. Scheme of an acousto-optical convolver with spatial integration.

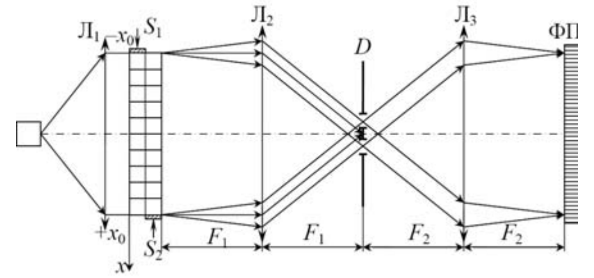

Fig. 4. Scheme of an acousto-optical correlator with time integration.

frequency equal to $2\omega_x$. The spatial filter in this scheme screens the zero-order diffraction. As a matter of fact, this scheme performs a transformation of the phase modulation of the light field to the intensity modulation and also carries the image of acoustic waves' coverage (the phase grating) in the plane of photodetector's sensitive elements.

In the convolver, the converter performs the integration of input signals over the AOP space by a lens and a photodetector. If the spatial filter selects one of the diffraction orders, then the signal proportional to the convolution function $U(t)$ of input signals $S_1(t)$ and $S_2(t)$ is formed at the photo detector's output:

$$U(t) \approx \mathrm{Re}\left[e^{-2\omega_0 t} \int\limits_{-D/2}^{+D/2} S_1\left(t - \frac{x}{\upsilon}\right) S_2\left(t + \frac{x}{\upsilon}\right) dx \right].$$

One can see that the convolution function is modulated by the doubled carrier frequency ω_0, over which the input signals are imposed. The convolution interval is restricted by AOP parameters: $t = D/\upsilon$ in time and D in space.

Moreover, the requirements to laser diodes about coherence, divergence, light polarization, facilities of effective input of the radiation into planar waveguides are considered as well.

4. Nonlinear phenomena in light-medium interaction

According to the classical approach, the light propagation through a medium is described by the Maxwell equations. If these equations are linear, then the waves travel through the medium irrespective of one another, i.e. the superposition principle is realized. This picture corresponds to the linear optics. The intensities of light fields created by lasers reach the values comparable to those of intraatomic fields (~ 10^8 V/cm). At the interaction with substance, the relation between the medium polarization P and the electrical field strength of a strong light wave becomes nonlinear:

$$P = \chi_1 E + \chi_2 E^2 + \chi_3 E^3 + \ldots \qquad (3)$$

Here, χ_1 is the linear medium susceptibility; χ_2, χ_3, … are the nonlinear susceptibilities of the first, second, and higher orders. As a result of breaking down the superposition principle, the energy exchange between different waves occurs. Most nonlinear effects are related to the quadratic and cubic terms of series (3). The quadratic term stipulates such phenomena as self-focusing, second-harmonic generation, optical detection, parametric generation, etc. Due to the cubic term of the series, there occur the third-harmonic generation, two-photon absorption, various kinds of stimulated light scattering, and others. The great attention in the "Quantum radiophysics" course is paid to the generation of the second harmonic (and higher ones) of light and to the parametric generation.

Consider the interaction between a strong harmonic light wave with frequency ω and the medium that has a polarization described by two first terms of series (3). The nonlinear polarization described by the second term on the right-hand side of series (3) contains a timely constant component that can be interpreted as a result of the "optical detection" of a strong light wave in the nonlinear medium. The time-depending polarization and, consequently, the reemitted field oscillate in time with a frequency of 2ω (the second harmonic of a pumping light wave). The light wave of the second harmonic gets the energy from radiation at the basic frequency ω through the medium polarization component with a frequency equal to 2ω. It is obvious that the energy transfer to a light wave with a frequency equal to 2ω will be efficient if the phase shift between the polarization wave with frequency 2ω and the second harmonic of the light wave keeps constant through quite long distances. If the refractive indices of the medium for the light waves with frequencies ω and 2ω are different, the phase shift between them reaches the value of π at a distance l_c that is equal to

$$l_c = \lambda / 4\left(\bar{n}_{2\omega} - \bar{n}_{\omega}\right).$$

Consequently, one cannot possibly expect the power accumulation from the second harmonic at distances more than l_c. The value l_c which is called a coherent length turns to infinity at the equality of the refractive indices for waves with the basic and doubled frequencies:

$$\bar{n}_{2\omega} = \bar{n}_{\omega}. \qquad (4)$$

Expression (4) is known as the condition of phase synchronism. Realization of condition (4) is impeded by dispersion, i.e. by the dependence of the refractive index on the frequency, $\bar{n}_\omega = \bar{n}_\omega(\omega)$. In negative uniaxial crystals, one can find such a direction, along which the refractive indices of an ordinary basic wave and an extraordinary second-harmonic wave are equal, and the coherent length turns to infinity.

The use of nonlinear optical phenomena allows one also to get the coherent radiation with a smoothly tunable frequency. This kind of processes is called the parametric light generation. Parametric generation is stipulated by the interaction between a strong pumping light wave with frequency ω_0 and two faint light waves with frequencies ω_1 and ω_2 which are induced by self-fluctuations of the polarization which are inevitably present in the active medium. As the polarization depends nonlinearly on the total field intensity in the medium, so the waves with frequencies ω_0, ω_1, and ω_2 become bound with one another. The radiation at the frequency ω_1 gets the energy from the strong pumping wave with ω_0 while it interacts with the ω_2-wave, and the wave ω_2 gets the energy due to the interaction between the ω_0- and ω_1-waves.

At the parametric generation, the synchronism condition looks as

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_0 , \tag{5}$$

where $\vec{k}_1$ and $\vec{k}_2$ are the wave vectors of amplified waves, $\vec{k}_0$ is the wave vector of a strong pumping wave. If this condition is realized, then the waves with frequencies ω_1 and ω_2 are amplified in the nonlinear medium due to the energy of the pumping wave ω_0. To attain the generation, the nonlinear medium is put into a complicated resonator that provides the positive feedback for the waves with ω_1 and ω_2. As relation (5) is vectorial, the smooth tuning of the frequencies ω_1 and ω_2 of the generated waves is attained by turning the axis of two system-composing resonators, which results in changing the direction and length of the vectors $\left|\vec{k}_1\right| = \omega_1 \bar{n}_1 / c$ and $\left|\vec{k}_2\right| = \omega_2 \bar{n}_2 / c$. It should be noted that, to get a coherent radiation with smoothly tunable frequency, one can also use a single-resonator scheme that provides an output signal of higher stability.

5. Nonlinear optics of semiconductor lasers

Some courses of the specialization "Quantum electronics" are concerned with the consideration of the nonlinear optics of semiconductor media and structures, including the optical bistability and other effects [3].

In the special course "Semiconductor lasers" [4], it is shown that, due to the strong nonlinear dependence of the amplification coefficient on the concentration of excess charge carriers, the pumped medium has characteristics of an antireflecting filter. The phenomenon of undamped pulses in semiconductor lasers is explained as the relaxation vibrations appearing due to the influence of some nonlinear mechanism resulting in an instability of the steady-state mode. The instability criterion is generally brought to the condition of positive nonlinear losses in a resonator. The following phenomena are typical of the complex dynamics of injection lasers:

1) hard self-excitation resulting in a fast increase of the light intensity at reaching the generation threshold;

2) hysteresis of the watt-ampere characteristic;

3) automodulation of the light intensity with typical times of 10^{-10} s, determining the complicated time structure of light pulses;

4) generation of single coherent pulses with the rise and fall times not more than 10^{-10} s (analog of the generation of monopulses by lasers on ruby and glasses);

5) emission of a regular sequence of short light pulses of length $\sim 10^{-10}$s and the repetition frequency of $\sim 10^9$s^{-1} at synchronizing the laser by a periodic signal;

6) self-synchronization of the longitudinal modes with the emission of a sequence of short light pulses with repetition frequencies of 10^8 Hz (with an external resonator) and 10^{11} Hz (with a resonator formed by crystal facets);

7) competition and anticompetition of modes consisting in their mutual damping and excitation.

The variety of instable phenomena in IL is closely related to its properties as a pulse system. By approaching the study of the dynamics of IL from positions of the theory of oscillations, except the basic process – the induced emission of electromagnetic waves, one should also remember two important periodic processes typical of any type of lasers. The first one is that any disturbance of the electromagnetic field in a resonator is repeated many times with the period $T = 2L/\upsilon$, where L is the resonator length and υ is the speed of light in the medium. This results in periodic oscillations of the amplification coefficient and the intensity of pulses (interaction in the "electron–photon" system). The fundamental source of these oscillations is the shot noise of a quantum transition that is responsible for the spontaneous radiation and the induced one. At a quite strong feedback and at the impact of an exterior resonance disturbance, the oscillations can become undamped (light auto-modulation).

Experiments show that, as a rule, only one mode is generated continuously. In this case, the laser beam intensity and frequency just slightly fluctuate near their average values. If several modes are simultaneously generated, then the laser radiation becomes unsteady as a result of the competition between them. In this case, the radiation consists of a set of chaotic pulses.

At the pulse excitation, the time interval from the start of generation to its end can be divided into four parts:

1) delay time, when the substance is excited and there is no generation;

2) transient mode from the start of generation to reaching the quasisteady operation;

3) quasisteady operation is the continuous generation with auto-modulation phenomena;

4) in the absence of pumping, the generation power relaxes to zero.

The facilities of the formation of a radiation pulse with extremely short length and large amplitude by laser diodes in the modes of free generation and modulation of the Q-factor of a resonator are analyzed.

The influence of saturation effects on the generation dynamics in a single-mode injection laser, including spectral and spatial burn-out dips [5], is studied. It is shown that the generation mode induces transitions between the conduction band and the valence one by breaking the quasiequilibrium in bands. The amplification coefficient stops to be a single-valued function of the concentration of carriers. The higher the density of photons in a resonator, the greater is the deviation from the equilibrium. To take this effect into account in the kinetic equations, the amplification coefficient is multiplied by the factor

$$\gamma = \left(1 + S / S_{\text{sat}}\right)^{-1},$$

where S and S_{sat} are, respectively, the density of photons in the resonator and the saturating density. This approach means that the amplification instantly follows a change in the density of photons. It is based on the fact that the time of establishing the equilibrium in the bands is much less than the times typical of the transient processes in a laser.

The radiation kinetics of semiconductor sources is studied in specialized laboratory practices [6].

The special course "Electromagnetic theory of semiconductor lasers" [7] considers the phenomena concerned with nonlinear refraction and the influence of carriers on the refractive index of a semiconductor. As known, the concentration of carriers depends on the radiation intensity. At the photoelectric absorption, the concentration increases, whereas it decreases under the induced radiation. This results in the nonlinear refraction of two types. In a laser medium, it has the properties of auto-focusing and causes non-homogeneities and instabilities. At the use of external resonators, nonlinear refraction allows one to get bistable modes, as it occurs in nonlinear resonators. But, in this case, external sources aren't necessary, and the continuous mode is possible at room temperature [3, 7].

Another important aspect of the use of nonlinear refraction consists in damping the unwanted modes in the spectral surrounding of a laser line in the single-frequency mode by the parametric interaction between modes. The space-time (dynamical) grating of medium's optical parameters, which appears at the beating of modes, causes the energy exchange between interfering modes. The process involves two or three modes that produce the beating at the total frequency Ω. At the two-mode interaction, a sharp spectral asymmetry with the transmission of energy to the long-wave mode is found. The exceptions are given by small values of $\Omega \le 1/\tau$, where τ is the relaxation time of an induced dynamical grating. In those cases, the damping of side modes on both sides of the "strong" mode prevails. If the Q-factor of a resonator provides the damping of "far" modes, then the internal mechanism of stabilizing the single-frequency mode works in a laser with such a resonator. In this case, due to the optical nonlinear scattering on the dynamical grating, the antiphase oscillations appear at the side frequency (phase inversion), which leads to the damping of oscillations and fluctuations at the side frequencies.

In a special course "Quantum-well lasers and integrated-optical elements," the effects of dimensional quantization, possibilities of optical transitions and rules of selection, nonlinear amplification effects, new laser structures including ones with optical bistability and laser amplifiers are considered [3].

In a special course "Kinetic theory of semiconductor injection lasers" for magisters [8], the dynamics and peculiarities of light generation in various types of injection lasers including the influence of nonlinear amplification effects on the transient and modulation characteristics of injection lasers, the nature of a nonlinear interaction between modes, and its influence on laser radiation dynamics are analyzed.

6. Conclusion

We have considered the questions of nonlinear and acoustic optics included in the curriculum of general and special courses for university's students specializing in quantum electronics. Along with general questions, a special attention is paid to the use of acousto-optical methods in the analysis and the processing of optical signals in integrated optoelectronics and to nonlinear optics of semiconductor injection lasers.

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