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Numerical study of the diffusive-like decay of the vortex tangle without mutual friction

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The numerical simulation of the diffusive-like decay of the vortex tangle without mutual friction was performed. The simulation was made with the use of the localized induction approximation. The early developed by authors algorithm, which is based on consideration of crossing lines, was used for vortex reconnection processes. We have determined the influence of different factors on decay of an inhomogeneous vortex tangle: a diffusion (large vortex loops break up to smaller ones which go away from initial volume), change of length owing to reconnection processes, the eliminations of small vortices below the space resolution, the insertion and removing of points to supply numerical algorithm stability. The obtained numerical results demonstrate that the vortex tangle, initially localized in the small region, is smearing into ambient space. The time evolution of vortex line density inside the initial domain satisfactory agrees with the ones, obtained from the solution of diffusion equation.

PACS: *67.30.he* Textures and vortices;
47.32.C– Vortex dynamics;
05.40.–a Fluctuation phenomena, random processes, noise, and Brownian motion.

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1. Introduction

The experiments and numerical simulations indicate that the superfluid turbulence decays at the very low temperatures (see, for example, [1–4]), when the usual mechanism of dissipation (mutual friction) is absent. Currently some approaches and ideas such as a cascade-like breakdown of the loops, Kelvin waves cascade, acoustic radiation, reconnection loss, etc., are discussed in the literature (see, e.g., Ref. 5). All these mechanisms are related to very small scales, which gave a rise to idea about the cascade-like transfer of the vortex energy, just like it happens in the classical turbulence. In order to clarify the mechanism of decay of the vortex tangle we have performed the direct numerical simulation of the evolution of the vortex tangle, originally concentrated in a restricted domain. We were carefully monitoring all mechanisms of the decrease of the total vortex length (reconnection procedure, inserting or removing of intermediate points, elimination of small loops etc.). It was found that the most effective mechanism is related with the spreading of the vortex tangle. The rate of changing of the total length is occurring in accordance with the diffusion theory of the vortex tangle developed in work [6].

2. Statement of the problem

In the absence of friction the vortex lines move according to the Biot–Savart law. We made the calculation with the using the so-called localized approximation, which neglects the nonlocal effects. The velocity of the vortex elements on the filament is defined by the local contribution arising from a curved-line element acting on itself:

$$\dot{\mathbf{S}} = \mathbf{V}_{si} = \beta \mathbf{S}' \times \mathbf{S}'' . \quad (1)$$

Hereafter $\mathbf{S}(\xi,t)$ is the radius-vector of the vortex line points, ξ is the arc length, $\mathbf{S}'$ is the derivative with respect to the arc length, $\mathbf{S}''$ is the second derivative with respect to the arc length, $\beta = c(\kappa/4\pi)\ln(R/a_0)$, $c = 1.1$, R is the local radius of curvature, $\kappa = h/m_{\mathrm{He}}$ is the quantum of circulation, h is Planck's constant, m_{He} is the mass of a ^{4}He atom. The quantity a_0 is the vortex core radius. The approximation in the time step is the Runge–Kutta method of the fourth order. The reconnection processes are introduced with the use of algorithm, which is based on consideration of crossing lines [7]. To improve the accuracy of approximation of Eq. (1) in the computational procedure the following procedure is used: a) spatial step along the vortex filament $\Delta\xi_0$ was chosen so that the numerical and analytical values of the velocities of the vortex ring coincided with good accuracy; b) if the distance between adjacent points on the loop less than $\Delta\xi_0 / 2$, then one of the points is removed; if this distance is twice $\Delta\xi_0$, then an intermediate point is added by using a circle interpolation. The loops, consisting of three or less segments, are deleted from the calculation, since the numerical study of the dynamics of ones becomes impossible to continue. Calculations are performed in an infinite volume. The initial configuration of vortex loops was taken out from the calculations obtained by us in the study of stochastic dynamics of vortex loops. This configuration corresponds to the equilibrium state of vortex tangle [8] (see Fig. 1).

In the calculation process we were monitoring the change of length due to various mechanisms such as loosing of length while the reconnection processes, changes due to the insertion and removing of points, the eliminations of small vortices, decrease of total length inside domain (spherical domain with the radius $R \sim 0.008$ cm) due to escaping of the small loops.

3. Results

The vortex configurations at various times are presented in Fig. 1. One can see that due the self-reconnection the large vortex loops generate the loops of smaller size. The size of these loops is less than the average distance between vortices. The small size loops move relatively quickly and leave the selected volume without further reconnections. Clearly, there is escaping of the vortex loops from the volume. In Fig. 2 we depicted the number of reconnections $n(t)$ for the breakdown (curve *1*) and the fusion processes (curve *2*). As it is seen the breakdown processes prevail. After some time, the reconnection processes are not practically observed. Figure 3 shows the distribution of the vortex loops over their length at the different times corresponding to the configurations in Fig. 1. It can be seen that the number of small loops increases in time.

We performed the careful monitoring of the changes in the total length of the vortex inside the initial domain. In particular, we studied all the mechanisms that lead to the loss of the length, such as the length changes due to reconnection processes, the elimination of very small loops below the space resolution, change of length due to insertion and removal of points. The results of this monitoring are shown in Fig. 4. As seen from this figure, the diffusion of vortex loops (the escaping of vortex loops) is the main mechanism leading to the dilution of the vortex tangle. Contribution of other mechanisms, listed above, is much smaller.

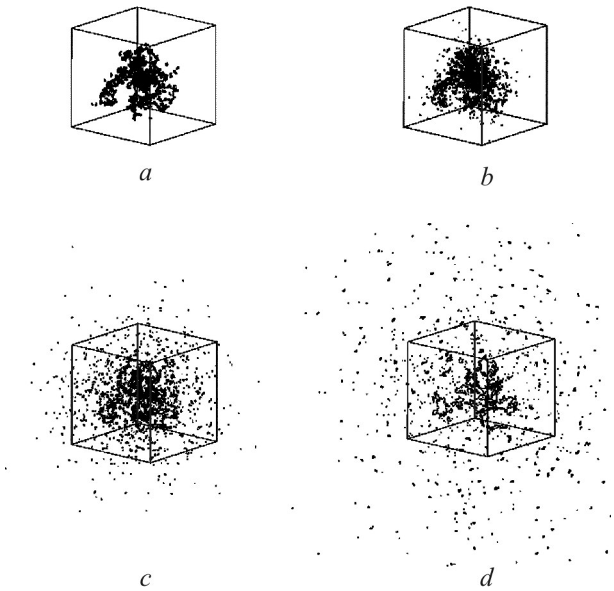

Fig. 1. The vortices at different times t, ms: 0 (*a*), 0.48 (*b*), 2.16 (*c*), 8.63 (*d*).

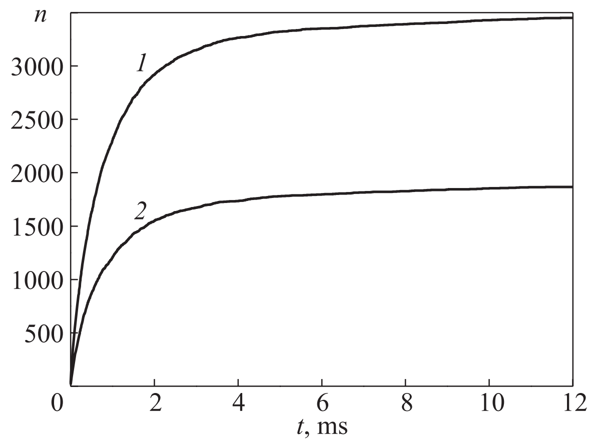

Fig. 2. Number of reconnections as a function of time for the breakdown (*1*) and fusion processes (*2*).

We compared the results of our numerical simulation with the theory of diffusion of vortex tangle [6]: The theory of inhomogeneous superfluid turbulence was developed on the basis of kinetics of merging and splitting vortex loops. It predicts that the temporal-spatial evolution of the vortex line density L obeys the diffusion equation.

$$\frac{\partial L}{\partial t} = D\nabla^2 L\,, \qquad (2)$$

where $D \approx 2.2\kappa$ is a diffusion coefficient evaluated as in work [6]. We obtained, that the evolution of the length in

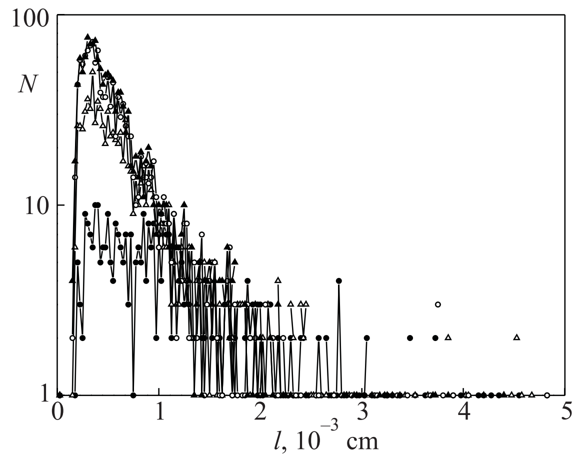

Fig. 3. The distribution of the vortex loops on their length at the time region t, ms: 0 (●), 2.16 (△), 0.48 (○), 8.63 (▲).

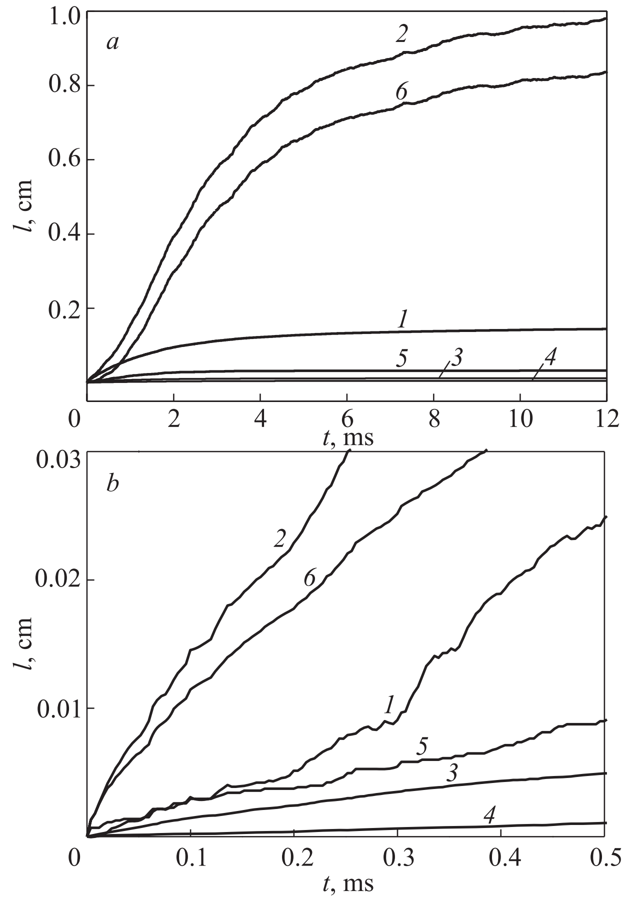

Fig. 4. Contribution of various mechanisms into decrease of total length (*b* — the initial region of curves depicted in Fig. 4,*a*). Decrease of the total length (*1*), decrease of the total length inside domain (spherical domain with the radius R ~ 0.008 cm) due to escaping of the small loops (*2*), decrease of the total length due to elimination of small loops (3 mesh sizes) (*3*), change of the total length due to artificial procedure — inserting or removing of intermediate points (*4*), change of the total length due to reconnection procedure (*5*), the total length outside domain (*6*).

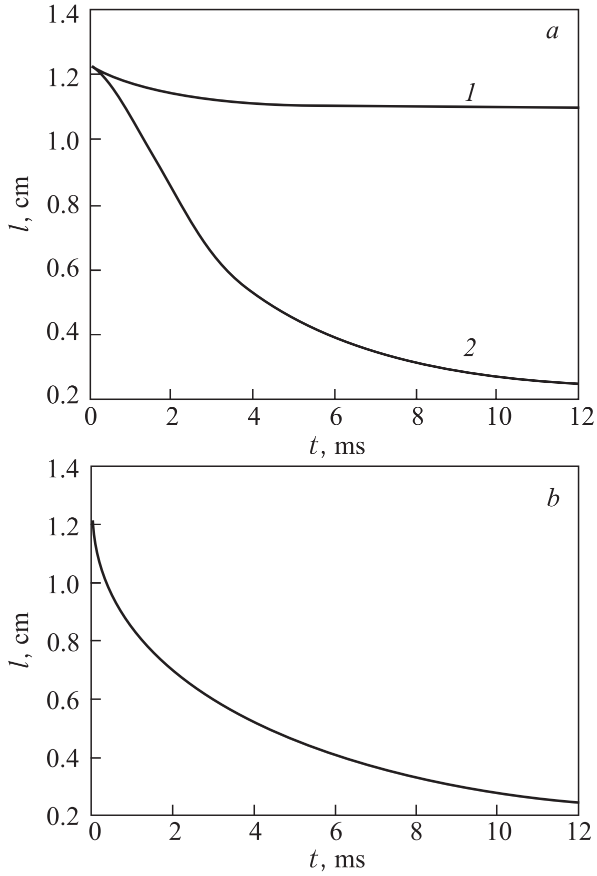

Fig. 5. The length $l(t) = \int L(r,t)dr$ obtained in numerical simulation: the total length (*1*), the length inside domain (*2*) (*a*) and calculated on the base of the diffusion equation (for spherical domain) (*b*).

the initial domain is satisfactory described by Eq. (2). The result of comparison is depicted in Fig. 5. The good agreement between the solution of Eq. (2) and numerical simulation enables us to conclude that the diffusion process plays a dominant role in the free decay of the vortex tangle in the absence of the normal component.

4. Conclusion

In the present work we study the attenuation of the vortex length (correspondingly, the vortex energy) in the direct numerical simulation. We study the various factors (appeared in the numerical experiments) and leading to the decrease of the total length. Our main conclusion is that the decay of the vortex tangle is realized, mainly, by escaping of the vortex loop from the volume. The influence of other mechanisms is negligible.

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