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Superconducting phase-dependent force in SNS junctions with a movable scatterer

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We calculate a quantum (Casimir-like) superconducting phase-dependent force acting on a movable scatterer in a superconductor–normal metal–superconductor (SNS) junction. Repulsive Casimir forces are predicted for a short SNS junction with nonequilibrium (inverse) populations of Andreev levels. In a long SNS junction an anomalous (nonmonotonic) temperature behavior of quantum force is found.

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03.70.+k Theory of quantized fields.

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1. Introduction

The Casimir effect [1] and its various generalizations (see, e.g., [2]) is perhaps the simplest and most spectacular manifestation of zero-point fluctuations of quantum fields in macroscopic physics. The effect predicts attractive interaction between neutral objects due to the change of energy spectrum of fluctuating fields, induced by boundary conditions (associated with the introduction of the objects).

Modern STM and AFM techniques allow manipulations of small objects on nanoscale dimensions and measurement of quantum forces acting on them. These forces could be produced by zero-point fluctuations of the electromagnetic field as in classical Casimir effect, or by quantum fluctuations of fermionic or bosonic fields in various solid state problems (see, e.g., [3]). One of the aims of these studies is to find conditions for the occurrence of repulsive Casimir-like forces which could result in levitation effects.

In Ref. 4 similarity between the physics of a ballistic SNS junction and the Casimir effect was revealed. It was shown that the Josephson current can be considered as the superconducting phase derivative of Casimir energy calculated for quasiparticle wave functions in the normal region. The corresponding Casimir-like force (coordinate derivative of Casimir energy) was studied in [5] where superconductivity-induced force oscillations in ideal (impurity-free) SNS junctions were predicted. Note that the force oscillations (as a function of the gate voltage) in the normal quantum point contact (see, e.g., [6]) disappear in the limit of a multichannel junction. In contrast in SNS junctions transverse channels contribute coherently to the Casimir energy (for a short SNS junction the thermodynamic potential is simply multiplied by the number $N_{\perp}$ of transverse channels) and the force is therefore strongly enhanced in this case.

In the present paper we consider a quantum (Casimir-like) force acting on a movable impurity inside the normal part of a SNS junction. As an example we model such scatterer by a neutral fullerene molecule inside a single wall carbon nanotube which bridges the gap between two superconducting electrodes. We show that in a short SNS junction, depending on the populations of Andreev bound states, this force could be either attractive (equilibrium population) or repulsive (nonequilibrium population). The amplitude of the quantum force is controlled by the Josephson current. In a long SNS junction we found a rather unusual (nonmonotonic) temperature behavior of the Casimir force, with a maximum value reached at a crossover temperature, where the Josephson current starts to decay exponentially.

2. Basic equations

We consider a SNS junction with a movable scatterer (impurity) in the normal region. The scatterer could be for instance a fullerene molecule inside a metallic single wall carbon nanotube (SWNT) which forms a weak link between two superconducing electrodes, or a tip of atomic force microscope (AFM). For a given position ($x = l$) of "impurity" inside a SNS junction of a length L (see Fig. 1) the equation for the Andreev energy levels, E, takes the standard form (see, e.g., [7])

$$\cos\left[2\arccos\frac{E}{\Delta_0} - \frac{2E}{\Delta_0}\frac{L}{\xi}\right] = \quad (1)$$

$$= D\cos\varphi + R\cos\left[\frac{2E}{\Delta_0}\frac{(L-2l)}{\xi}\right].$$

Here Δ_0 is the superconducting gap, $\xi = \hbar v_F / \Delta_0$ is the superconducting coherence length and $\varphi = \varphi_2 - \varphi_1$ is the superconducting phase difference, D is the junction transparency ($R = 1 - D$). We note that for the Andreev levels there is no phase difference between the right and left superconducting leads. When difference between the leads is taken into account these levels are called the Andreev–Kulik levels [8].

In the case when a neutral fullerene molecule (C_{60}) is placed inside a SWNT the effective scattering potential experienced by the electrons is produced by the hybridization of the lowest occupied molecular orbitals (LUMO states) of the C_{60} with the conducting states of the SWNT [9]. The potential was shown [10] to be smooth and therefore it does not mix electronic states in two degenerated valleys of the SWNT spectrum. This intra-valley scattering depends strongly on the chiral properties of the nanotube and it was termed "chiral tunneling" in Refs. 11, 12. Chiral tunneling is characterized by the transmission coefficient $D(\theta)$ [11,12]

$$D(\theta) = \frac{\cos^2\theta}{\cos^2(U_0\cos\theta)\cos^2\theta + \sin^2(U_0\cos\theta)}. \quad (2)$$

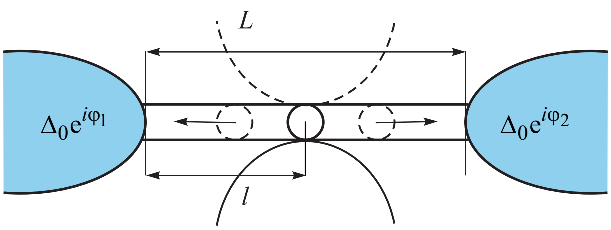

Fig. 1. A SNS junction between two superconducting electrodes (blue reservoirs on the right and left), formed by a metallic carbon nanotube with a fullerene molecule impurity which can freely move inside the tube. The movable "impurity" is attracted to superconductor leads, for equilibrium populations of the Andreev bound states and can levitate inside the tube for nonequilibrium level occupations.

Here θ is the effective chiral angle of the SWNT (the difference between the chiral angle of the SWNT and the phase of scattering potential). Both the supercurrent J through the junction and the force, F, acting on the impurity, can be evaluated as derivatives of the grand-canonical potential $\Omega(\varphi, l)$ of the SNS junction

$$J(\varphi) = \frac{e\nu_s}{\hbar}\frac{\partial\Omega}{\partial\varphi}, \qquad F(\varphi) = -\nu_s\frac{\partial\Omega(\varphi, l)}{\partial l}, \quad (3)$$

where the statistical factor $\nu_s = 2\times 2$ accounts for the spin and valley degeneracy in the SWNT. The definition of the force F in Eq. (3) means that we calculate the quantum (Casimir) force induced by the discreteness of energy levels (this force vanishes in the limit $L \to \infty$). Since in a SNS junction the bound states are given by the Andreev levels we will call the phase dependent force in a SNS junction as the Andreev force [5]. Andreev force can be analytically calculated in the limiting cases of short and long junctions.

3. Repulsive Casimir interaction in a short junction with nonequilibrium Adreev level populations

To calculate Andreev force in a short SNS junction we will use perturbation theory in the small parameter L/ξ. As it is well known in a short SNS junction there are only two Andreev bound states (see, e.g., [7]). The *l*-dependence of energies appears in the second order perturbation theory as $\delta E^{(2)} \propto \pm[1 - 2l/L]^2$. By knowing *l*-dependent shift of the energy levels it is easy to valuate the force Eq. (3). The Andreev force, acting on an impurity in a short SNS junction, calculated under conditions of equilibrium populations of the Andreev levels at a temperature T, reads as

$$F(\varphi) = 2\nu_s R\Delta_0\frac{2l - L}{\xi^2}\sqrt{1 - D\sin^2(\varphi/2)}\times$$

$$\times\tanh\left(\frac{\Delta_0\sqrt{1 - D\sin^2(\varphi/2)}}{2T}\right). \quad (4)$$

Notice that the contribution to the force from two Andreev levels is of opposite sign. Consequently this the Andreev force at finite temperatures decays in the same way as the Josephson current. The force is a linear function of l ($l = L/2$ is a point of unstable equilibrium) and it has the same dependence on the phase, φ, as the energy levels. It is clear that in the absence of reflection (both at S/N contacts and in the interior of the junction) the force vanishes ($R = 0$, i.e. the case of transparent junction). The Andreev force reaches its maximum, $F \sim (L/\xi)\Delta_0/\xi$, in the opposite limit $D \to 0$, when one can neglect the dependence of force on the superconducting phase difference.

It is well known (see, e.g., [7]) that to L/ξ order in perturbation theory not only bound states but also the continuum (scattering) states contribute to the Josephson current in a short SNS junction. The calculated force, Eq. (4), appears in this order of perturbation theory and one has to estimate the contribution of the scattering states to the Andreev force. The corresponding thermodynamic potential $\delta\Omega_c$ can be found as the integral of "continuum-states-induced-current" $\delta J^{(c)}$, over the phase difference, $\delta\Omega_c = (\hbar / e v_s)\int \delta J^{(c)} d\varphi$. The general formula for $\delta J^{(c)}$ takes the form [7]

$$\delta J^{(c)} = e v_s / \hbar D(\theta) \left(\int_{-\infty}^{-\Delta_0} + \int_{\Delta_0}^{\infty} \right) | u_0^2 - v_0^2 | \times \tag{5}$$

$$\times \frac{\sin\varphi}{\sin\alpha} \left(\frac{1}{D(E,-\alpha)} - \frac{1}{D(E,\alpha)} \right) f(E) dE,$$

where $D(E,\alpha) = u_0^4 + v_0^4 - 2u_0^2 v_0^2 \cos\left[EL/\Delta_0\xi + \alpha \right]$. Here u_0 and v_0 are the standard BCS coherence factors $2u_0^2 = 1 + (E^2 - \Delta_0^2)^{1/2}/E$ and $2v_0^2 = 1 - (E^2 - \Delta_0^2)^{1/2}/E$. The phase α in Eq. (5) is defined by the relation $\cos\alpha = D\cos\varphi + R\cos[2E(L-2l)/\Delta_0\xi]$. Straightforward calculations in perturbation theory (with order parameter L/ξ) show that continuum contribution to the force is $\delta F^{(c)} \sim R(\Delta_0/\xi)(L/\xi)^2(1-2l/L)$. We observe that force produced by the continuum spectrum is reduced by a factor $L/\xi \ll 1$ compared to the Andreev force induced by the discrete spectrum.

We note that irradiation of a short SNS junction by an electromagnetic field can lead to a nonequilibrium population of the upper level. Under conditions when the population of the upper Andreev level exceeds that of the lower level (so-called "somersault effect" [13]) the Casimir interaction in a SNS junction will be repulsive. Then the middle point $l = L/2$ becomes the point of stable equilibrium. In the limiting case when only upper level is populated the oscillation frequency of the scatterer (C_{60} molecule) around equilibrium position reads

$$\omega_A^2 = \Delta_0 R \sqrt{1 - D\sin^2\left(\frac{\varphi}{2}\right)} / M\xi^2, \tag{6}$$

where M is the mass of the C_{60} molecule. Repulsive Andreev forces could result in levitation of the fullerene molecule inside the tube at equal distances from the superconducting leads (i.e., the middle of the tube).

4. Anomalous temperature behavior of Andreev force in a long junction

Next we consider limit of a long SNS junction, i.e. $L \gg \xi$. In this case all relevant energies in the problem are smaller than the superconducting gap, i.e. $E \ll \Delta_0$, and Eq. (1) simplifies. We consider first an almost transparent junction ($D \simeq 1$) and use perturbation theory, with a reflection coefficient $R \ll 1$ in the evaluation of the energy spectrum. To zeroth order the spectrum is [8] $E_{n,j} = (\hbar v_F / 2L)[j\varphi + \pi(2n+1)]$ and the first order correction reads

$$\delta E_{n,j} = -\frac{\hbar v_F}{2L} \frac{R}{\sin(j\varphi)} \left\{ \cos\left[\frac{2E_{n,j}}{\hbar v_F}(L-2l) \right] - \cos\varphi \right\}, \tag{7}$$

where $n = 0, \pm1, \pm2, \ldots$, $j = \pm$, and the phase difference in $\delta E_{n,j}$ can not be too close to $\varphi = 0, \pm\pi$ (that is $R \ll |\sin\varphi|$). We will see below that this restriction can be omitted for the phase dependence of the total Andreev force, which is not the singular at these points.

The l-dependent part of the potential Ω, to first order perturbation in the reflection coefficient $R \ll 1$, takes the form

$$\Omega(l,\varphi) = \sum_{j,n} \frac{\delta E_{n,j}}{e^{E_{n,j}/T} + 1}. \tag{8}$$

We neglect higher-order (l-independent) contributions to $\delta E_{n,j}$. To perform summation over n we use the Poisson summation formula

$$\sum_{n=-\infty}^{\infty} F(n) = \int_{-\infty}^{\infty} F(x)\,dx + 2\,\mathrm{Re} \sum_{k=1}^{\infty} \int_{-\infty}^{\infty} F(x)\, e^{2\pi i k x} dx. \tag{9}$$

The first term in Eq. (9) does not contribute to the force and performing integration in the second term we get the desired expression for the Andreev force acting on the impurity in a long SNS junction

$$F(\varphi) = \frac{\hbar v_F R}{\pi L^2} \left(\frac{T}{T^*} \right)^2 \sum_{k=1}^{\infty} (-1)^k \frac{\sin(k\varphi)}{\sin\varphi} \times$$

$$\times \left\{ \frac{\cosh\left[T(k-a)/T^* \right]}{\sinh^2\left[T(k-a)/T^* \right]} - \frac{\cosh\left[T(k+a)/T^* \right]}{\sinh^2\left[T(k+a)/T^* \right]} \right\}. \tag{10}$$

Here $a = (1 - 2l/L)$, $T^* = \hbar v_F / 2\pi L$ is the crossover temperature which separates the low- and high-T regions. The two terms in the curly brackets in Eq. (10) correspond to the attractive forces associated with the left and the right superconducting reservoirs. After summation over the energy levels the singularities at $\varphi = 0, \pm\pi$ disappear and Eq. (10) is valid in the whole interval $-\pi \le \varphi \le \pi$.

The low- and high-temperature asymptotics of Andreev force are

$$F \simeq \begin{cases} F_0 \pi \dfrac{\sin(a\varphi)}{\sin\varphi} \left(\dfrac{\pi\cot(a\pi)}{\sin(a\pi)} - \dfrac{\varphi\cot(a\varphi)}{\sin(a\pi)} \right), & T \ll T^*, \\ F_0 \left(\dfrac{T}{T^*} \right)^2 \left(e^{-T(1-a)/T^*} - e^{-T(1+a)/T^*} \right), & T \gg T^*, \end{cases} \tag{11}$$

where $F_0 = \hbar v_F R / \pi L^2$. According to these analytical expressions the force decays exponentially at high temperatures. However numerical calculations reveal an anomalous (nonmonotonic) temperature behavior of the Andreev force (see Fig. 2). Note, that the Josephson current demonstrates an usual (monotonous) temperature behavior (it decays with increasing of temperature) without any en-

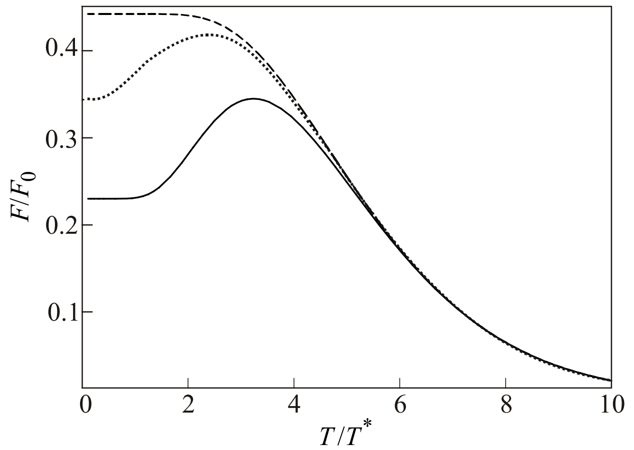

Fig. 2. The force (in units $F_0 = \hbar v_F R / \pi L^2$) as a function of normalized temperature T/T^* for $\varphi = 0$ (solid), $\varphi = 2$ (dotted), $\varphi = 3.14$ (dashed). $l/L = 0.7$.

hancement in the crossover region ($T \simeq T^*$). It is clear that adjacent energy levels carry supercurrents of opposite signs and hence the temperature, tending to equalize level populations always suppress the net (Josephson) current. The signs of partial forces associated with the pair of adjacent energy levels are not always opposite. For the same value of n, but different values of j, the partial forces $f_{n,j} \propto d\delta E_{n,j}/dl$ are not oppositely directed (as in the case of a short junction). This results in the enhancement of quantum force in the crossover region.

In Figs. 3, 4 we plot the dependence of the force on the coordinate of the mobile impurity and on the superconducting phase in a long SWNT junction. It is seen that in contrast to the case of short junction the *l*-dependence is not linear. Near the boundaries of junction the force exhibits a quadratic divergence characteristic to the Casimir force in one-dimensional systems (see, e.g., Ref. 2). The phase-dependence is also different from the case of a short junction. Now the force is enhanced (see Fig. 4) at the values $\varphi = \pi(2n+1)$, when one of the Andreev levels coincide with the Fermi level.

In the end of this section we consider long tunnel SNINS junction (superconductor–normal metal–insulator–normal metal–superconductor). In zeroth order perturbation theory in junction transparency $D \ll 1$, we have two sets of uncoupled levels (to the left E_l and to the right E_r with respect to the impurity position):

$$E_l = \frac{\hbar v_F \pi}{2l}(n+1/2), \quad E_r = \frac{\hbar v_F \pi}{2(L-l)}(n+1/2). \quad (12)$$

Here the superconductivity (in the leads) is "manifested" through doubling the quantization length caused by the Andreev reflection at the N/S boundary. The Casimir-like force for these energy levels can be readily obtained with the help of summation formula for half-integer numbers [2]:

$$\mathrm{reg}\sum_{n=0}^{\infty} F(n+1/2) = -i\int_0^{\infty} \{F(it) - F(-it)\} \frac{dt}{e^{2\pi t}+1}. \quad (13)$$

Here the symbol "reg" stands for the regular (finite) part of the sum. The main contribution to the force for $D = 0$ does not depend (as should be the case) on the superconducting phase and at $T = 0$ it takes the form

$$F_C = \frac{\hbar v_F \pi}{48L^2}\left\{\frac{1}{(1-l/L)^2} - \frac{1}{(l/L)^2}\right\}. \quad (14)$$

The calculated force in Eq. (14) is simply the difference between two Casimir forces for spinless fermions on S^1-manifold. The temperature dependence of the Casimir forces for a 1D fermion systems was calculated in Ref. 14 and it can be easily generalized to our problem. Notice that unlike the ordinary "bosonic" Casimir effect, where the low-temperature corrections are power-law-like, here the temperature dependent contributions to the force are always exponentially small even for massless fermions. Anyway

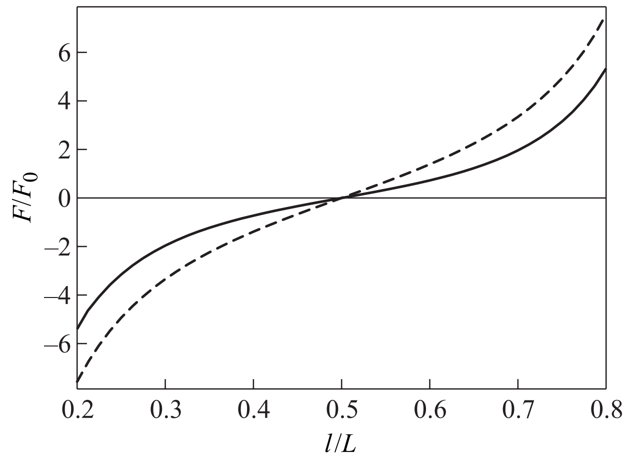

Fig. 3. The force (in units $F_0 = \hbar v_F R / \pi L^2$) as a function of the normalized coordinate l/L for two different values of phase difference: $\varphi = \pi/12$ (solid) and $\varphi = \pi$ (dashed).

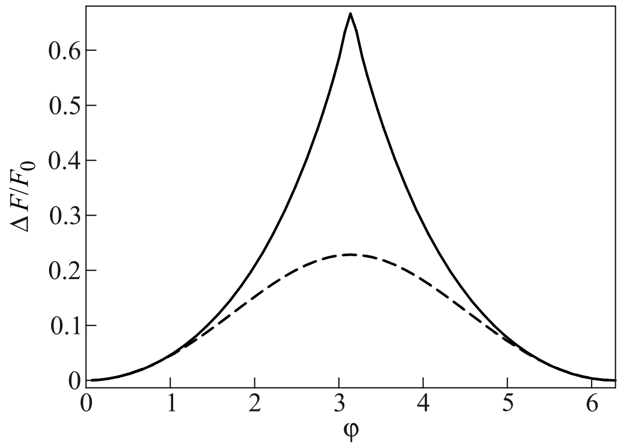

Fig. 4. The phase-dependent part of force $\Delta F(\varphi) = F(\varphi) - F(0)$ (in units $F_0 = \hbar v_F R / \pi L^2$) for two different temperatures: $T = 0$ (solid) and $T/T^* = 3$ (dashed).

the Casimir force (Eq. (14)) and the corresponding finite temperature corrections, do not depend on superconducting phase and thus they are not of interest in the contest of the present paper.

5. Conclusion

In summary, we have calculated quantum (Casimir-like) superconducting phase-dependent force acting on a movable "impurity" in a SNS junction. In a short junction the force is attractive (the superconducting leads attract the scatterer in the normal region) for equilibrium populations of the Andreev energy levels. In Ref. 13 it has been predicted that in a short SNS junction under microwave irradiation an inverse population of the energy levels takes place, accompanied by the reversal of the direction of the Josephson current ("somersault effect"). We showed that under the conditions of the "somersault effect" the Andreev force is repulsive and the movable scatterer (in our consideration a neutral fullerene molecule inside a metallic single wall carbon nanotube) can levitate at the equal distances from the leads. For a long SNS junction we predicted anomalous (nonmonotonic) temperature behavior of the Andreev force with a maximum at a crossover temperature T^* when Josephson current starts to decay. A simple estimation of the amplitude of the Andreev force for a few channel junction with $\Delta_0 \sim 10^2$ K gives $F \sim 0.1$ pN which is too small to be measured in superconducting devices. However, this value can be strongly enhanced (by two orders of magnitude) in multichannel junctions.

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