

# **INTERACTION OF RELATIVISTIC PARTICLES WITH CRYSTALS AND MATTER**

## **INFLUENCE OF MULTIPLE SCATTERING ON DYNAMICAL EFFECT MANIFESTATION IN COHERENT X-RAY RADIATION BY RELATIVISTIC ELECTRON**

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Coherent X-ray radiation by a beam of relativistic electrons crossing a single-crystal plate in Bragg scattering geometry is considered. In the present work, the initial divergence and multiple scattering of electrons on atoms in the target are taken into account. The manifestation possibility of dynamic diffraction effects in the conditions of multiple scattering of electrons in the beam is studied.

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### **INTRODUCTION**

As known PXR appears due to the scattering of a relativistic electron Coulomb field on a system of parallel crystal atomic planes [1 - 3]. When a charged particle crosses the crystal plate surface, the transition radiation (TR) takes place [4, 5]. TR appearing on the border diffracts then on a system of parallel atomic planes of the crystal that forms DTR in a narrow spectral range [6 - 7].

Multiple scattering of relativistic electron on atoms in a single crystal can result the spectral-angular characteristics of PXR and DTR generated by a beam of relativistic electrons.

The natural width of PXR spectrum is defined by the number of inhomogeneities with whom the electron interacts. As it was shown in the experiment on study of PXR of relativistic electrons in a single crystal target represented in the work [8], the measured width of PXR spectrum considerably exceeds the spectral width of PXR calculated for an electron moving rectilinearly in the crystal. In [9] on basis of kinematic theory it was shown that multiple scattering has considerable influence on the spectral width of "back" PXR in crystal. The averaging of spectral-angular density of the radiation in [9] was carried out on the basis of functional integration method. The contribution of diffracted bremsstrahlung (DB) and DTR in [9] were not considered. Traditionally the influence of the relativistic electron multiple scattering on the PXR characteristics is taken into account by averaging of PXR cross-section over the expanding beam of the rectilinear trajectories of the radiating particles. Nevertheless in the row of experimental works [10, 11] the noncoincidence of theory in which the averaging over the beam of rectilinear trajectories of radiating particles are used and of obtained experimental data was pointed. Evidently, in frame of considered approach the contribution of diffracted bremsstrahlung is lost. In the work [12] a theory of PXR in unlimited crystal was developed within the scope of dynamical theory of diffraction without taking into account of DTR but correctly taking account the influence of multiple scattering of radiating electron on the PXR characteristics. In that work ([12]), it was shown based on rigorous kinematic approach in the av-

eraging of the radiation cross-section over all possible trajectories of radiating particles that contribution of DB can be high-considerable. In [12] the expression describing the spectral-angular characteristics of total yield of coherent radiation have been obtained without separation on the mechanisms PXR and DTR that allowed to estimate only relative contributions of these radiation mechanisms. In this work the conditions of significance of the diffracted bremsstrahlung contribution into total yield of the radiation were obtained.

A theory of coherent X-ray radiation of relativistic in the crystal were developed in the network of two wave approximation of dynamical theory of diffraction of X-ray waves in the works [13 - 18]. In the works [13 - 15] the coherent X-ray radiation was treated in special case of symmetric reflection, when the reflecting system of atomic planes of the crystal is situated parallel to the target surface (in the case of Bragg scattering geometry) or perpendicular (in the case of Laue scattering geometry). In the works [16 - 18] the dynamic theory of coherent X-ray radiation of relativistic electron in crystal was developed for the general case of asymmetric to relate of the crystal surface reflection of the electron coulomb field, when a system of parallel reflecting atomic planes in the target can be situated at arbitrary angle to the target surface. These works showed that by changing the symmetry of reflection of the coulomb field of electron on the atomic planes in crystal by changing of the angle between the target surface and system of diffracting atomic planes, one can considerably increase the spectral-angular density of PXR and DTR. The present work is dedicated to development of dynamical theory of coherent X-Ray radiation of relativistic electron crossing a monocrystalline plate in Bragg scattering geometry with accounting of the multiple scattering of the electrons by the atoms of the target. To account the multiple scattering, we have used a traditional averaging method of spectral-angular and angular densities of the radiation over the rectilinear trajectories of electrons in the widening beam. Let us note that the rigorous kinetic approach described in [12] don't allow to consider the radiation process from the target of limited width and to separate the contributions of PXR and DTR mechanisms. Nevertheless the use of

obtained in [12] criteria of the significance of the DTR contribution to the radiation yield allows to consider the conditions under which the contribution of DTR is absent and therefore the traditional accounting method of multiple scattering in characteristics of radiation is fully justified.

## 1. RESULTS AND DISCUSSION

### 1.1. GEOMETRY OF THE EMISSION PROCESS

Let us consider a beam of relativistic electrons crossing a monocrystalline plate (Fig. 1).

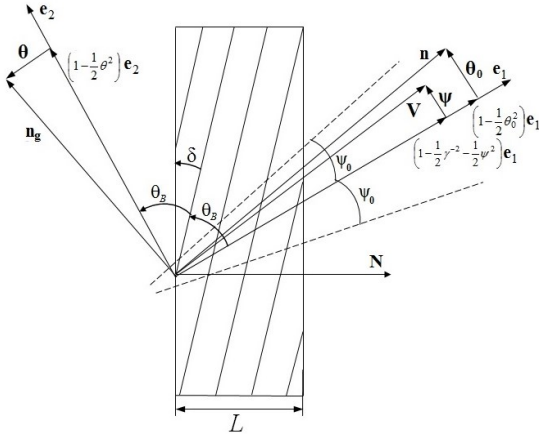


Fig. 1. Geometry of the emission process

Let us involve the angular variables  $\psi$ ,  $\theta$  and  $\theta_0$  in accordance with the definition of relativistic electron velocity  $\mathbf{V}$  and unit vectors in direction of momentum of the photon radiated in the direction near electron velocity vector  $\mathbf{n}$  and in the of Bragg scattering direction  $\mathbf{n}_g$ :

$$\begin{aligned} \mathbf{V} &= \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \quad \mathbf{e}_1\boldsymbol{\psi} = 0, \\ \mathbf{n} &= \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \quad \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B, \\ \mathbf{n}_g &= \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \quad \mathbf{e}_2\boldsymbol{\theta} = 0, \end{aligned} \quad (1)$$

where  $\boldsymbol{\theta}$  is the radiation angle, counted from direction of axis of radiation detector  $\mathbf{e}_2$ ,  $\boldsymbol{\psi}$  is the incidence angle

of an electron in the beam counted from the electron beam axis  $\mathbf{e}_1$ ,  $\theta_0$  is the angle between the movement direction of incident photon and axis  $\mathbf{e}_1$ ,  $\gamma = 1/\sqrt{1-V^2}$  is Lorentz-factor of the particle. The angular variables are decomposed into the components parallel and perpendicular to the figure plane:  $\boldsymbol{\theta} = \boldsymbol{\theta}_{\parallel} + \boldsymbol{\theta}_{\perp}$ ,  $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0\parallel} + \boldsymbol{\theta}_{0\perp}$ ,  $\boldsymbol{\psi} = \boldsymbol{\psi}_{\parallel} + \boldsymbol{\psi}_{\perp}$ .  $\psi_0$  is the divergence parameter of the beam of radiating electrons.

### 1.2. SPECTRAL-ANGULAR DENSITY OF PXR AND DTR IN A THIN CRYSTAL

We will consider the asymmetric reflection of electron coulomb field relative to surface of a crystal plate being as target with such a thickness that the length of electron path in the plate  $L_e = L/\sin(\theta_B + \delta)$  would be more than extinction length  $L_{ext}^{(s)} = 1/\omega|\chi_g'|C^{(s)}$  of X-ray waves in the crystal:

$$b^{(s)} = \frac{L_e}{2L_{ext}} \gg 1. \quad (2)$$

The relation (2) is a condition of dynamic effects manifestation in the radiation. To study of the dynamic effects "per se" let us rid of possible influence of absorption effect for photons in the crystal by means of additional condition that the maximal length of electron path in the target  $L_f = L/\sin(\theta_B - \delta)$  must be much more than the extinction length  $L_{abs} = 1/\omega\chi_0''$ :

$$2\frac{b^{(s)}\rho^{(s)}}{\varepsilon} = \frac{L_f}{L_{abs}} \ll 1. \quad (3)$$

If we perform the analytical procedures similar to those used in [19, 20] we will obtain the expressions for the spectral-angular density of PXR and DTR for the propagation direction of the emitted photon  $\mathbf{k}_g = k_g\mathbf{n}_g$  (see Fig. 1) taking into account the direction deviation of the electron velocity  $\mathbf{V}$  relative to the electron beam axis  $\mathbf{e}_1$ :

$$\omega \frac{d^2 N_{\text{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\Omega^{(s)2}}{(\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2 - \chi_0')^2} R_{\text{PXR}}^{(s)}, \quad (4a)$$

$$R_{\text{PXR}}^{(s)} = \frac{\left(\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}\right)^2 \sin^2 \left( \frac{b^{(s)}}{2} \left( \frac{\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon} - \sigma^{(s)} \right) \right)}{\xi^{(s)2} - \varepsilon + \varepsilon \sin^2 \left( \frac{b^{(s)} \sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon} \right) \left( \frac{\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon} - \sigma^{(s)} \right)^2}, \quad (4b)$$

$$\begin{aligned} \omega \frac{d^2 N_{\text{DTR}}^{(s)}}{d\omega d\Omega} &= \frac{e^2}{\pi^2} \Omega^{(s)2} \times \\ &\times \left( \frac{1}{\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2} - \frac{1}{\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2 - \chi_0'} \right)^2 R_{\text{DTR}}^{(s)}, \end{aligned} \quad (5a)$$

$$R_{DTR}^{(s)} = \frac{\varepsilon^2}{\xi^{(s)^2} - (\xi^{(s)^2} - \varepsilon) \coth^2 \left( \frac{b^{(s)} \sqrt{\varepsilon - \xi^{(s)^2}}}{\varepsilon} \right)}, \quad (5b)$$

where

$$\begin{aligned} \Omega^{(1)} &= \theta_{\perp} - \psi_{\perp}, \quad \Omega^{(2)} = \theta_{\parallel} + \psi_{\parallel}, \quad \sigma^{(s)} = \frac{1}{|\chi'_g| C^{(s)}} \left( \gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2 - \chi_0' \right), \quad \varepsilon = \frac{\sin(\theta_B - \delta)}{\sin(\theta_B + \delta)}, \\ L_{ext}^{(s)} &= \frac{1}{\omega |\chi'_g| C^{(s)}}, \quad \xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1 + \varepsilon}{2\nu^{(s)}}, \quad \eta^{(s)}(\omega) = \frac{2 \sin^2 \theta_B}{|\chi'_g| C^{(s)}} \left( 1 - \frac{\omega(1 - \theta_{\parallel} \cot \theta_B)}{\omega_B} \right), \quad b^{(s)} = \frac{1}{2 \sin(\theta_B + \delta)} \frac{L}{L_{ext}^{(s)}}, \\ \nu^{(s)} &= \frac{\chi'_g C^{(s)}}{\chi_0'}, \quad C^{(1)} = 1, \quad C^{(2)} = |\cos 2\theta_B|. \end{aligned} \quad (6)$$

The functions  $R_{PXR}^{(s)}$  and  $R_{DTR}^{(s)}$  describe the spectra of PXR and DTR. Since the inequality  $2 \sin^2 \theta_B / |\chi'_g| C^{(s)} \gg 1$  is fulfilled in the range of X-ray frequencies,  $\eta^{(s)}(\omega)$  is a fast function of frequency  $\omega$ , and it is convenient for the further analysis of the properties of the PXR and DTR spectrum to consider  $\eta^{(s)}(\omega)$  as a spectral variable. Parameter  $b^{(s)}$  characterizing the thickness of the crystal plate is the ratio of half of the path of the electron in the target  $L_e = L / \sin(\theta_B + \delta)$  to the extinction length  $L_{ext}^{(s)} = 1 / \omega |\chi'_g| C^{(s)}$ . Equations (4), (5) under  $s = 1$  describes the fields of  $\sigma$ -polarization, and under  $s = 2$  the fields of  $\pi$ -polarization.

Under a fixed value of  $\theta_B$  the value  $\varepsilon$  defines the orientation of crystal plate in relation to the system of diffracting atomic planes (Fig. 2).

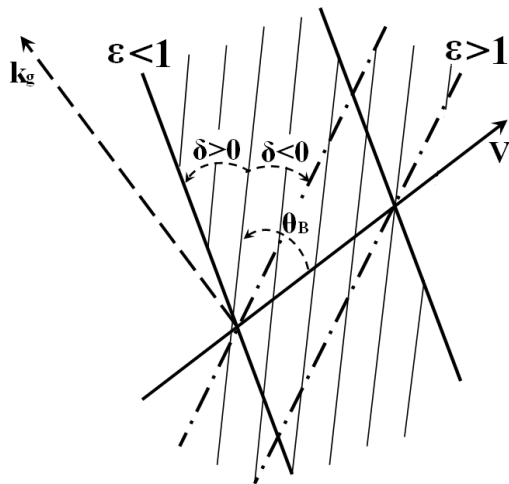


Fig. 2. Asymmetric ( $\varepsilon > 1$ ,  $\varepsilon < 1$ ) reflections of radiation from a crystal plate. The case  $\varepsilon = 1$  ( $\delta = 0$ ) corresponds to symmetric reflection

When the angle of electron incidence on the target surface  $\theta_B + \delta$  decreases the value of  $\delta$  parameter can become negative and then will increase in magnitude (in extreme case  $\delta \rightarrow -\theta_B$ ) that leads to increase of  $\varepsilon$ . On the contrary, when the angle of electron incidence decrease the value of  $\varepsilon$  decrease (in extreme case  $\delta \rightarrow \theta_B$ ).

The expressions (4), (5a), (5b) describe the spectral-angular density of PXR and DTR of the relativistic electron crossing a crystal plate at an angle  $\Psi(\psi_{\perp}, \psi_{\parallel})$  relative to the axis of the electron beam  $\mathbf{e}_1$  and their interference.

### 1.3. ACCOUNT OF MULTIPLE SCATTERING OF THE ELECTRON BEAM ON THE ATOMS OF THE TARGET

Because the multiple scattering of electrons on the atoms of the medium can lead to generation of bremsstrahlung, which then may diffract on the system of the parallel atomic planes in crystal in the direction of Bragg scattering  $\mathbf{k}_g$ , we will consider the conditions of significance of contribution of the diffracted bremsstrahlung (DB) into total yield of the coherent radiation. The investigation of relative contribution of DTR in total yield of the radiation was done in the work [12].

Let illustrate the conditions of DTR contribution in total yield of the radiation in the presence of multiple scattering of relativistic electrons in the crystal. Let consider the quantity  $\gamma_{LP}^{-2} = \psi_s^2 l_c$  that is a mean square of the scattering angle of an electron over the length of the bremsstrahlung forming  $l_c = 2\gamma^2 / \omega$ , where

$$\psi_s^2 = \frac{E_s^2}{m^2 \gamma^2} \frac{1}{L_R} \quad \text{is mean square of angle of multiple scattering of an electron on the unit of the length,}$$

$$\psi_s^2 = \frac{E_s^2}{m^2 \gamma^2} \frac{1}{L_R}, \quad L_R \text{ is the radiation length. At electron}$$

energies  $\gamma > \gamma_{LP} = \sqrt{e^2 \omega_B L_R / 8\pi}$  well-known effect of Landau – Pomeranchuk reveals itself in bremsstrahlung [21], i.e. when  $\gamma > \gamma_{LP}$  the angle of multiple scattering of an electron at forming length of the radiation considerable exceeds the characteristic angle of the radiation by the relativistic particle  $\gamma^{-1}$ , therefore the field of bremsstrahlung quant and the coulomb field of electron will split off on the path shorter than radiation forming length  $l_c$ . On other hand in the region of electron energy  $\gamma > \gamma_{TM} = \omega_B / \omega_0$ , ( $\omega_0$  is plasma frequency), the bremsstrahlung can be suppressed as a result of Ter-Mikaelyan effect [22]. In case of the conditions  $\gamma_{LP} < \gamma < \gamma_{TM}$  the suppression of DB is absent and it can introduce considerable contribution in to total yield of

the radiation. So, when one of the condition  $\gamma > \gamma_{TM}$  or  $\gamma < \gamma_{LP}$  will be satisfied the contribution of DB in total yield of the radiation one can do not account and use the traditional method of averaging of spectral-angular characteristics of radiation over the expanding beam of straight electron trajectories are used to account multiple scattering.

Let us average the PXR and DTR angular densities over function of angular distribution of electrons in the beam, which is changed on the length of path in the target  $t$  because of multiple scattering of the electrons:

$$f(\psi, t) = \frac{1}{\pi(\psi_0^2 + \psi_s^2 t)} \cdot e^{-\frac{\psi^2}{\psi_0^2 + \psi_s^2 t}}, \quad (7)$$

where  $\psi_0$  is initial divergence of the electron beam. The expression describing spectral-angular densities of PXR and DTR averaged over an expanding beam of rectilinear trajectories of radiating electrons on the length of electron path in the target  $L_e$  have a following view:

$$\left\langle \omega \frac{d^2 N_{\text{PXR}}^{(s)}}{d\omega d\Omega} \right\rangle = \frac{e^2}{\pi^3 \psi_s^2 L_e} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\psi_{\perp} d\psi_{\parallel} \left( \frac{\Omega^{(s)2}}{(\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2 - \chi_0')^2} R_{\text{PXR}}^{(s)}(\psi_{\perp}, \psi_{\parallel}) \int_{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2 + \psi_s^2 L_e}}^{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2}} \frac{e^{-x}}{x} dx \right), \quad (9a)$$

$$\left\langle \omega \frac{d^2 N_{\text{DTR}}^{(s)}}{d\omega d\Omega} \right\rangle = \frac{e^2}{\pi^3 \psi_s^2 L_e} R_{\text{DTR}}^{(s)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\psi_{\perp} d\psi_{\parallel} \times \left( \Omega^{(s)2} \left( \frac{1}{\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2} - \frac{1}{\gamma^{-2} + (\theta_{\perp} - \psi_{\perp})^2 + (\theta_{\parallel} + \psi_{\parallel})^2 - \chi_0'} \right)^2 \int_{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2 + \psi_s^2 L_e}}^{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2}} \frac{e^{-x}}{x} dx \right), \quad (9b)$$

$$\left\langle \frac{dN_{\text{PXR}}^{(s)}}{d\Omega} \right\rangle = \frac{e^2 \varepsilon^2 b^{(s)}}{8\pi^2 \sin^2 \theta_B |\chi_g'| C^{(s)} \psi_s^2 L_e} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\psi_{\perp} d\psi_{\parallel} \left( \frac{\Omega^{(s)2}}{\sigma^{(s)2}} \frac{(\sigma^{(s)2} \varepsilon - 1)}{\left( \frac{\sigma^{(s)2} \varepsilon - 1}{2\sigma^{(s)}} \right)^2 + \varepsilon \sin^2 \left( \frac{\sigma^{(s)2} \varepsilon - 1}{2\sigma^{(s)} \varepsilon} b^{(s)} \right)} \int_{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2 + \psi_s^2 L_e}}^{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2}} \frac{e^{-x}}{x} dx \right), \quad (10a)$$

$$\left\langle \frac{dN_{\text{DTR}}^{(s)}}{d\Omega} \right\rangle = \frac{e^2 \chi_0'^2 \varepsilon \sqrt{\varepsilon} \pi}{2\pi^3 \sin^2 \theta_B |\chi_g'| C^{(s)} \psi_s^2 L_e} \tanh \left( \frac{b^{(s)}}{\sqrt{\varepsilon}} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\psi_{\perp} d\psi_{\parallel} \left( \frac{\Omega^{(s)2}}{\sigma^{(s)2} \left( \chi_g' |C^{(s)} \sigma^{(s)} + \chi_0' \right)^2} \int_{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2 + \psi_s^2 L_e}}^{\frac{\psi_{\perp}^2 + \psi_{\parallel}^2}{\psi_0^2}} \frac{e^{-x}}{x} dx \right). \quad (10b)$$

The expressions (9a), (9b) and (10a), (10b) are the main results of the present work. These expressions are obtained in the framework of dynamic theory of diffraction and allows to investigate the manifestations of the effects of dynamical diffraction in PXR and DTR. It is necessary to note that in the condition of considerable contribution of diffracted bremsstrahlung  $\gamma_{LP} < \gamma < \gamma_{TM}$  the expression (9a), (9b) and (10a), (10b) also remain to be right. And in this case it is necessary only to consider the DB contribution separately.

#### 1.4. INFLUENCE OF REFLECTION ASYMMETRY ON SPECTRAL-ANGULAR DENSITY OF THE RADIATION

Let us use the expressions obtained in this work for investigation of manifestation of dynamic diffraction effects in RXR and DTR generated by the beam of relativistic electrons multiply scattered in a single crystal target. Let us consider the effects caused by variation of

$$\left\langle \omega \frac{d^2 N_{\text{PXR,DTR}}^{(s)}}{d\omega d\Omega} \right\rangle = \quad (8a)$$

$$= \frac{1}{\pi L_e} \int_0^{L_e} dt \int \int d\psi_{\perp} d\psi_{\parallel} \frac{e^{-\frac{\psi^2}{\psi_0^2 + \psi_s^2 t}}}{\psi_0^2 + \psi_s^2 t} \omega \frac{d^2 N_{\text{PXR,DTR}}^{(s)}}{d\omega d\Omega},$$

$$\left\langle \frac{dN_{\text{PXR,DTR}}^{(s)}}{d\Omega} \right\rangle = \quad (8b)$$

$$= \frac{1}{\pi L_e} \int_0^{L_e} dt \int \int d\psi_{\perp} d\psi_{\parallel} \frac{e^{-\frac{\psi^2}{\psi_0^2 + \psi_s^2 t}}}{\psi_0^2 + \psi_s^2 t} \frac{dN_{\text{PXR,DTR}}^{(s)}}{d\Omega}.$$

Using the formulas describing spectral-angular and angular densities PXR and DTR derived for a thin crystalline plate (4), (5a), (5b) and (8a), (8b) we will obtain the expression, describing the spectral-angular and angular densities of PXR and DTR with taking into account of electron multiple scattering on atoms of the medium:

reflection asymmetry of the relativistic electron coulomb field in relation to the target surface (the change of asymmetry parameter  $\varepsilon$ ).

We will make the numerical calculations for the beam of relativistic electrons of energy  $E=255$  MeV with initial divergence  $\psi_0=0.1$  mrad crossing the single-crystal plate of tungsten W(110). In this case the path of the electron in the target ( $L_e=10$   $\mu\text{m}$ ) exceeds considerably the extinction length ( $L_{\text{ext}} \approx 1.7$   $\mu\text{m}$ ) of the x-ray waves in the crystal. We will make these calculation for  $\sigma$ -polarized X-ray waves ( $s=1$ ) under the condition  $\theta_{\parallel} = 0$ .

At first, let us consider the dynamic effect of change the PXR spectrum width under changing of asymmetry of the electron Coulomb field reflection relative to the target surface i.e. under the change the parameter  $\varepsilon = \sin(\theta_B - \delta) / \sin(\theta_B + \delta)$ . The effect of spectrum width change was predicted and studied for case of a

separate moving rectilinearly electron in the work [17]. The increase of the width of PXR spectrum straight follows from the formula (4,b) because under increase of value  $\varepsilon$  the resonance condition

$$\sigma^{(s)} + \left( \xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon} \right) / \varepsilon = 0, \quad (11)$$

will depend on function  $\xi^{(s)}(\omega)$  weaker and also on  $\omega$  value. The equation (11) defines the frequency  $\omega_*$  in whose vicinity the PXR spectrum is concentrated under the fixed observation angle.

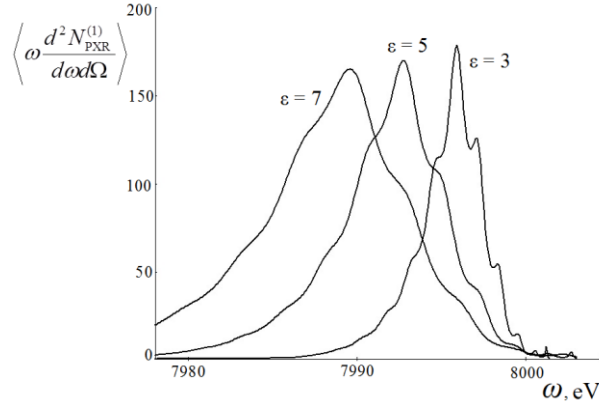


Fig. 3. Asymmetry effect on PXR spectrum:  $W(110)$ ,  $\theta_B = 20.5^\circ$ ,  $\gamma_{TM} \approx 99$ ,  $\gamma_{LP} \approx 196$ ,  $\gamma = 500$  ( $E = 255$  MeV),  $L_e = 10 \mu\text{m}$ ;  $\varepsilon = 3$  ( $\delta \approx -10.6^\circ$ ),  $\varepsilon = 5$  ( $\delta \approx -14^\circ$ ),  $\varepsilon = 7$  ( $\delta \approx -15.7^\circ$ );  $\theta_\perp = 10$  mrad

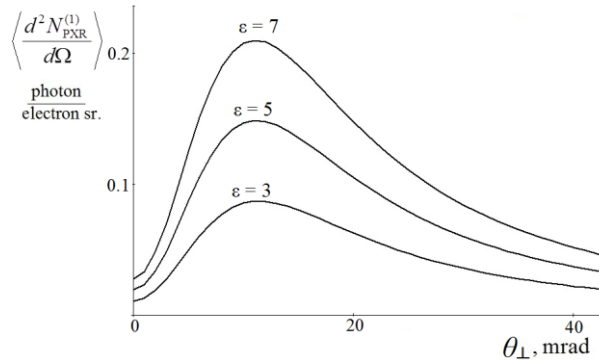


Fig. 4. Asymmetry effect in PXR angular density. All parameters have the same values as in Fig. 3

In Fig. 3 the curves calculated by the formula (9a) describe the spectral-angular density of PXR under fixed observation angle in maximum of PXR angular density  $\theta_\perp = \sqrt{\gamma^{-1} - \chi_0'} \approx 10$  mrad. The curves are plotted for different asymmetry (for different values of angle  $\delta$ ) under fixed values of electron path in the target  $L_e = 10 \mu\text{m}$  and Bragg angle  $\theta_B = 20.5^\circ$ . Under such conditions the target thickness  $L$  have different value for different value  $\delta$ : under  $\delta \approx -10.6^\circ$ ,  $L \approx 1.7 \mu\text{m}$ , under  $\delta \approx -14^\circ$ ,  $L \approx 1.1 \mu\text{m}$  and under  $\delta \approx -15.7^\circ$ ,  $L \approx 0.8 \mu\text{m}$ . Let us note that for considered small values of thickness the electron path ( $L_e = 10 \mu\text{m}$ ) is enough long to manifest in heavy-weight tungsten the considerable multiple scattering. In Fig. 3 one can see considerable change of PXR spectrum width under change of asymmetry. Under fixed angle  $\theta_B$  between the beam

axes  $e_1$  and diffracted system of atomic planes of crystal i.e. under increase of parameter  $\varepsilon$  the spectral width of PXR considerably grows.

As the result of PXR spectrum widening there is a considerable increase of angular density under grow of reflection asymmetry (under increase of  $\varepsilon$ ). The curves plotted by formula (10a) demonstrate this fact in Fig. 4.

In Bragg scattering geometry, the frequency range of total external absorption (extinction) of pseudo photons of coulomb field of relativistic electron in single crystal exists that was known for real X-ray photons. In this region, the incident wave vector takes on a complex value even in absence of absorption and as the result all the photons reflect. If absorption is absent the expression for the wave vector lengths have such a view:

$$k^{(1,2)} = \omega \sqrt{1 + \chi_0'} + \frac{\omega |\chi_g' C^{(s)}|}{2\varepsilon} \left( \xi^{(s)}(\omega) \pm \sqrt{\xi^{(s)}(\omega)^2 - \varepsilon} \right). \quad (12)$$

The region of total reflection is defined by following inequality:

$$\begin{aligned} & -\sqrt{\varepsilon} < \xi^{(s)}(\omega) < \sqrt{\varepsilon}, \\ & \text{or } -\sqrt{\varepsilon} - \frac{1 + \varepsilon}{2\nu^{(s)}} < \eta^{(s)}(\omega) < \sqrt{\varepsilon} - \frac{1 + \varepsilon}{2\nu^{(s)}}, \\ & \text{or } -\frac{\omega_B |\chi_g' C^{(s)}|}{2(1 - \theta_{||} \cot \theta_B) \sin^2 \theta_B} \left( \sqrt{\varepsilon} - \frac{1 + \varepsilon}{2\nu^{(s)}} \right) < \omega < \\ & \text{or } \frac{\omega_B |\chi_g' C^{(s)}|}{2(1 - \theta_{||} \cot \theta_B) \sin^2 \theta_B} \left( \sqrt{\varepsilon} + \frac{1 + \varepsilon}{2\nu^{(s)}} \right), \end{aligned} \quad (13)$$

which shows that the width of this range is defined by value of  $\frac{\sqrt{\varepsilon} \omega_B |\chi_g' C^{(s)}|}{2(1 - \theta_{||} \cot \theta_B) \sin^2 \theta_B}$ . It can be shown as fol-

lows from (5,b) the width of region of total reflection practically coincides with the DTR spectrum width. The curves plotted by formula (9b) describe the spectral-angular density of PXR at angular density maximum  $\theta_\perp = \gamma^{-1} = 2$  mrad under the condition of multiple scattering of relativistic electron on atoms of the medium. The graphics in Fig. 5 demonstrate considerable dependence of DTR spectral-angular density on reflection asymmetry of electron coulomb field in relation to the target surface (on asymmetry parameter  $\varepsilon$ ). When asymmetry parameter  $\varepsilon$  increase the amplitude and width of DTR spectrum strong grow that lead to growth of DTR angular density (Fig. 6).

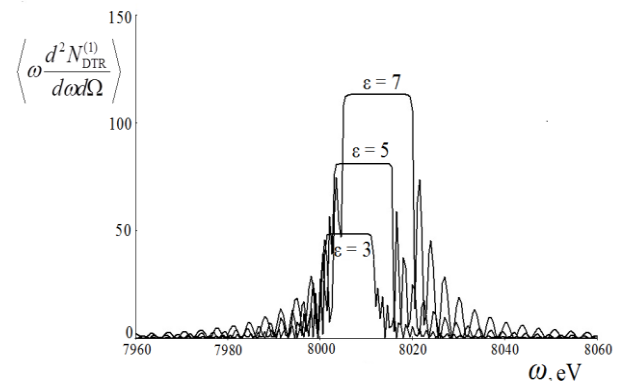


Fig. 5. Influence of reflection asymmetry on DTR spectrum. All parameters are the same as in Fig. 3, excluding  $\theta_\perp = 2$  mrad

We would remind you that the electron path in the target is the same for different value of asymmetry parameter and absorption of the radiation is negligibly small, i.e. the observed effects are not connected with these characteristics. All the numerical calculations have been carried on under the condition  $\gamma > \gamma_{TM}$  ( $\gamma=500$ ,  $\gamma_{TM} \approx 99$ ), i.e. under conditions of total suppression of bremsstrahlung because of Ter-Mikhaelyan effect.

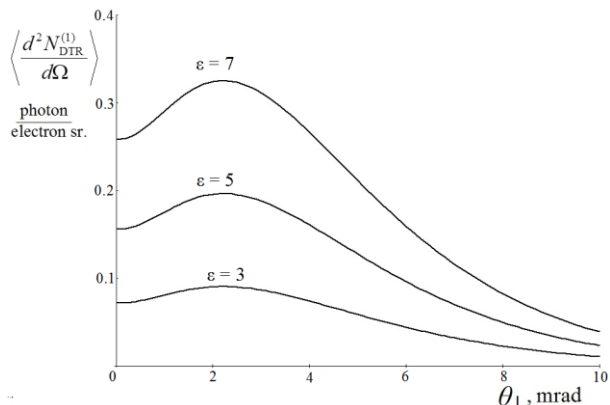


Fig. 6. Influence of reflection asymmetry on DTR angular density. All the parameters are the same as in Fig. 3

So, in the present work the expressions (9a, 9b)-(10a, 10b) have been obtained which describe spectral-angular and angular distributions of PXR and DTR generated by a beam of relativistic electrons in a single-crystal target in conditions of multiple scattering of the electrons on the atoms of the medium. All the obtained expressions are normalized on the number of electrons in the beam. These expressions have allowed to demonstrate the effects of dynamical diffraction in PXR and DTR generated by the beam of relativistic electrons multiple scattered on the atoms of the target medium.

## CONCLUSIONS

In the framework of two-wave approximation of dynamic theory of diffraction the analytical expressions are derived for spectral-angular densities of parametric X-ray radiation and diffracted transition radiation in the condition of multiple scattering of radiating relativistic electrons. The expressions describing spectral-angular characteristic of PXR and DTR have been derived based on the two-wave approximation of diffraction theory taking into account the deviation of electron velocity vector from the electron beam axis direction. The traditional method of cross section averaging over expanding beam of straight electron trajectories are used to account multiple scattering. In the present work the conditions of significance of contribution diffracted bremsstrahlung in the total yield of the radiation and are shown the applicability condition of traditional method of total yield description of the radiation generated by a beam of relativistic electrons in a monocrystal. The manifestation possibility of dynamic diffraction effects in the conditions of multiple scattering of electrons in the beam is studied.

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### **ВЛИЯНИЕ МНОГОКРАТНОГО РАССЕЯНИЯ НА ПРОЯВЛЕНИЕ ЭФФЕКТОВ ДИНАМИЧЕСКОЙ ДИФРАКЦИИ В КОГЕРЕНТНОМ РЕНТГЕНОВСКОМ ИЗЛУЧЕНИИ РЕЛЯТИВИСТСКОГО ЭЛЕКТРОНА**

*С.В. Блажевич, И.В. Колосова, Н.А. Коренькова, А.А. Мазілов, А.В. Носков*

Рассматривается когерентное рентгеновское излучение пучка релятивистских электронов, пересекающих монокристаллическую пластинку в геометрии рассеяния Брэгга. В данной работе учитывалось начальное расхождение и многократное отражение электронов на атомах мишени. Исследуется возможность проявления эффектов динамической дифракции в условиях многократного рассеяния электронов пучка.

### **ВПЛИВ БАГАТОКРАТНОГО РОЗСІЮВАННЯ НА ПРОЯВ ЕФЕКТІВ ДИНАМІЧНОЇ ДИФРАКЦІЇ В КОГЕРЕНТНОМУ РЕНТГЕНІВСЬКОМУ ВИПРОМІНЮВАННІ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОНА**

*С.В. Блажевич, І.В. Колосова, Н.А. Коренькова, О.О. Мазілов, А.В. Носков*

Розглядається когерентне рентгенівське випромінювання пучка релятивістських електронів, що перетинають монокристалічну платівку в геометрії розсіяння Бреґга. У даній роботі враховувалися початкова розбіжність і багаторазове відбиття електронів на атомах мішені. Досліджено можливість вияву ефектів динамічної дифракції в умовах багатократного розсіяння електронів пучка.