TRANSFORMATION RATIO INCREASE AT WAKEFIELDS EXCITATION IN THE DIELECTRIC STRUCTURE BY A SHAPED SEQUENCE OF RELATIVISTIC ELECTRON BUNCHES

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The energy transformation ratio is proportional to the number of bunches under the formulated condition validity at the excitation of wakefield in the dielectric resonator by sequence of short relativistic electron bunches with linear growth of charge. Shaped sequence of relativistic electron bunches charges of which increase approximately linearly, is derived on the linear accelerator "Almaz-2 M". The pulse duration (number of bunches) of shaped sequence can be adjusted from 0.4 (1200 bunches) to 0.75 μs (2250 bunches). The experiments were performed on the excitation of wakefield in dielectric structures by this sequence of electron bunches.

PACS: 29.17.+w; 41.75.Lx

INTRODUCTION

The main advantage of particle acceleration by wakefield [1] is a large accelerating gradient, which can reduce in $10^2 \ldots 10^3$ times the dimensions of accelerators and colliders [2]. Thus in the considered two beam acceleration (a driver-beam and a witness-beam) the transformation ratio (TR) is important. TR is defined as the ratio of energy, received by witness particles, to energy, lost by driver-particles.

In the simplest 1D collinear case of two bunches (driver and witness), the length of which is less than wakefield (WF) length [3], TR is equal to:

$$T_w = \frac{\Delta W_2}{W_1} \le \left(2 - \frac{N_2}{N_1}\right). \qquad (1)$$

ΔW_2 is the energy gain of particle of witness-bunch, W_1 is the energy of the driver-bunch particles, N_1 and N_2 are the number of particles of driver-bunch and of particles of witness-bunch. T_w cannot exceed two (Wilson theorem). Taking into account that the lengths along which driver-bunch loses its full energy and witness-bunch gains energy, are the same, TR can be expressed by the ratio of the maximum accelerating WF to maximum decelerating WF into the bunch

$$T_E = \frac{E^{+}_{z\,\max}}{E^{-}_{z\,\max}} \le 2 - \frac{N_2}{N_1}, \qquad (2)$$

E_1 is the WF of a single charge.

One can overcome the restriction, determined by the Wilson theorem, by shaping of long driver-bunch [4]. In [5-7] it has been concluded that one long bunch, whose density increases linearly along it, can provide TR

$$T_E = 2\pi \frac{L_b}{\lambda}, \qquad (3)$$

L_b is the bunch length, λ is the wavelength.

Another possibility to increase TR occurs at WF excitation by bunch sequence. In 1D case this possibility has been investigated at WF excitation by bunch sequence in plasma [3, 8]. In this problem in [9] for not ultra-relativistic bunch the effect of phase shift of bunches in the excited WF has been taken into account.

For a typical case of maximum accelerating field excitation the sequence is used in which the distance between bunches equals to excited wave length, i.e. for WF excitation the repetition frequency of bunches equals to excited wave frequency. This provides possible to coherently add the excited field and to increase the WF amplitude in M times (M is the number of identical bunches in the sequence) in comparison with WF of a single bunch [1, 3]. In particular, it allows to replace one bunch of large charge by sequence of small bunches of identical total charge [10, 11]. However, TR does not increase so fast with bunch number increase. This is due to the fact that bunches of sequence being in different WF amplitude from previous bunches lose their total energy on different lengths. If the 1st bunch lose their energy and transforms it to witness-bunch, which is accelerated over a length L, the 2nd, 3rd, ... M-th bunches lose their energy over lengths L/3, L/5,…..L/(2M-1). The witness-bunch is accelerated over different intervals by different WF and as a result, receives less total energy in comparison with single driver-bunch with identical total charge. TR equals

$$T_{WM} = \sum_{k=1}^{M} \frac{2}{2k-1} - \frac{N_2}{N_1}. \qquad (4)$$

Using the Euler's formula for sum of the first n terms of harmonic series

$$S_n = \ln n + C + \varepsilon_n,$$

C=0.5772.... is the Euler's constant, and $\varepsilon_n \to 0$ at $n \to \infty$, one can find that asymptotically for considered case TR increases logarithmically with the number of bunches.

$$T_{WM} \propto \ln M. \qquad (5)$$

Unlike the case of two bunches in multi-bunch scheme energy TR and field TR do not equal to each other $T_{WM} \neq T_{EM}$.

A significant increase of T_M with the number of bunches in sequence can be achieved [3, 4, 12], if all bunches are putted in phases, where the WF amplitude from previous bunches equals zero. Then all bunches are decelerated by field, equal to half of own WF and therefore all bunches lose their energy at the same length. The 2nd bunch should follow through ¼ WF wavelength. WF adding the first two bunches does not lead to a doubling of the field, as in the previous case, but to increase it only in $\sqrt{2}$ times i.e. acceleration rate is smaller. The repetition frequency of next bunches is selected according to phase of M-th bunch

$$\vartheta_M = \sum_{n=2}^{M} \mathrm{arctg}\left(\frac{1}{\sqrt{n-2}} \right). \quad (6)$$

TR at this shaping equals

$$T_M = 2\sqrt{M} - \frac{N_2}{N_1}. \quad (7)$$

Previous cases concerned the point bunches. In [12, 13] on the basis of finite length of bunches, using the shaped sequence, method of increase of accelerating gradient and TR has been proposed and researched, which are proportional to the number M of bunches. It is necessary that length of bunches to be equal to half the wavelength, they are located through wavelength and half. Then bunches are in the same decelerating fields as the 1st bunch. The charges of the bunches should grow as 1:3:5:7: … Each bunch makes the same contribution to the WF, as the 1st bunch. Thus the decelerating gradient equals

$$E_M = ME_1, \quad (8)$$

E_1 is the WF of the 1st bunch.

TR equals

$$T_M = 2M. \quad (9)$$

However, this is achieved by increasing the total charge sequence

$$Q_M = M^2 Q_1, \quad (10)$$

Q_1 is the charge of the 1st bunch.

One can derive even more TR at the WF excitation in the dielectric resonator by sequence of electron bunches at their charge shaping not only along sequence but along each bunch [14].

The TR increase at the WF excitation in the dielectric resonator by sequence of relativistic electron bunches at their charge shaping according to 1:3:5:7:… (Fig. 1) is investigated theoretically and experimentally in this paper. The bunch parameters correspond to experimental ones of the accelerator "Almaz-2M". Namely, the length of the bunches equals 1.7 cm, their energy is 4.5 MeV, the charges of the bunches grow linearly to a maximum 0.26 nC, the diameter of the bunches is 1cm, the period of the bunch injection is $3.6\cdot10^{-10}$s.

1. RESONATOR CONCEPT OF TRANSFORMATION RATIO INCREASE

Because WFs of a small number of bunches are added in the waveguide of finite length, we consider the injection of bunch sequence, the charges of which increase as 1:3:5:7: …, in dielectric resonator along its axis. The length of the resonator equals wavelength L=λ. WF pulse of length $L(V_0/V_g$-1), excited by M-th bunch, moves with the group velocity V_g from the bunch injection boundary to the opposite resonator end. Then the pulse is reflected from the end and moves back to the injection boundary, from which it also is reflected. Here V_0 is the bunch velocity. M-th bunch is injected into the resonator when the trailing edge of the WF pulse, excited by previous bunches, is on the injection boundary (condition (11)). M-th bunch leaves the resonator when the front edge of the WF pulse, excited by previous bunches, is at the end of the resonator. Then coherent accumulation of WF is realized. The ratio of bunch charges WFs from which are coherently added, is equal to 1:2M_0+3:, … (3:4M_0+3:, …). M_0 is equal to number of bunches, after which the bunch M+M_0 follows. WF of bunch M+M_0 is coherently added to WF of the bunch M. When M_0=1 the sequence of the bunches, charges of which increase according to 1:3:5:7: …, is divided into two sequences, charges of which increase according to 1:5: … and to 3:7: …, from which WFs are coherently added separately (see Fig. 1).

To ensure a large T_E, decelerating WF for all driver-bunches should be small, but they can be inhomogeneous along bunches and along the sequence (a periodical beatings).

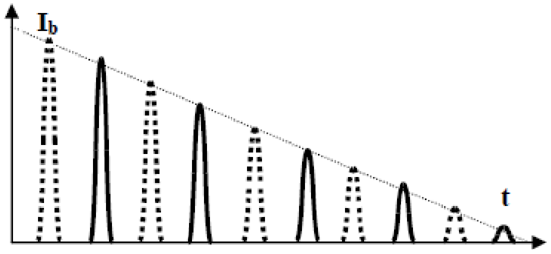

Fig. 1. The charge distribution of shaped sequence of short bunches, which excite WF. The solid line shows bunches, charges of which increase according to 1:5 ... and from which WFs are coherently attached. The dotted line shows bunches, charges of which increase according to 3:7: ... and from which WFs are coherently attached

For achieving a large TR several conditions should be satisfied. Namely, we choose the length of the resonator L, the group velocity V_g, the bunch repetition frequency $\omega_{rep} = 2\pi V_0 / L_0$ and the wave frequency ω_0, which satisfy the following equalities

$$\frac{2L}{V_g} = \frac{2\pi n}{\omega_{rep}}, \; n = 1,\, 2,\, ..., \quad (11)$$

$$\frac{2L}{V_g} \; V_0 - V_g \; = q\lambda + L_b, \; q = 1,\, 2,\, ..., \quad (12)$$

L_b=L_0/6.26 is the bunch length, L_0 is the distance between bunches. The expression (11) is determined by the requirement that the bunch is injected into the resonator at a time when the trailing edge of the WF pulse is on the bunch injection boundary. The expression (12) is determined by requirement that excited longitudinal decelerating wakefield E_z for all bunches should be small. The growing total wakefield provides a large TR.

From (11) one can derive M_0

$$M_0 = n - 1 = \left(\frac{2L}{\lambda} \right) \left(\frac{V_0}{V_g} \right) \left(\frac{\omega_{rep}}{\omega_0} \right) - 1. \quad (13)$$

It is important that to the end of the resonator the subsequent bunch cannot overtook the rear edge of the pulse from the previous bunch. For that the inequality should be correct

$$(1-V_g/V_0)L/\lambda \le (V_g/V_0)(\omega_0/\omega_{rep}). \qquad (14)$$

From (11)-(13) one can derive

$$(1/2-1/n)q \le (L/\lambda)(1-L_b/nL_0). \qquad (15)$$

To many electrons from the fronts of bunches do not get into the accelerating phases, the inequality should be correct

$$q<nL_0/2L_b-1/2. \qquad (16)$$

For the case $L/\lambda=1$, we derive $n=2$, $N_0=1$, $q=3$ and

$$\omega_0/\omega_{rep}=1.63,\ V_g/V_0=0.61. \qquad (17)$$

Then from (11) one can obtain

$$L/V_g=2\pi/\omega_{rep}, \qquad (18)$$

the distance between bunches L_0 equals $L_0=(3\lambda+L_b)/2$. That is period between bunches equals to flight time of bunch to the end of the resonator. Then the next bunch is injected into the resonator when the previous bunch leaves the resonator.

Formula (12), (17) means that the distance between the bunches, from which WFs are added coherently, equals the sum of the bunch width and multiple wavelength. Computer calculation (parameters are shown in Table) shows that, when this condition is correct, in the case of longitudinal unlimited waveguide the next spatial distribution (along the axis) of WF is realized (Fig. 2).

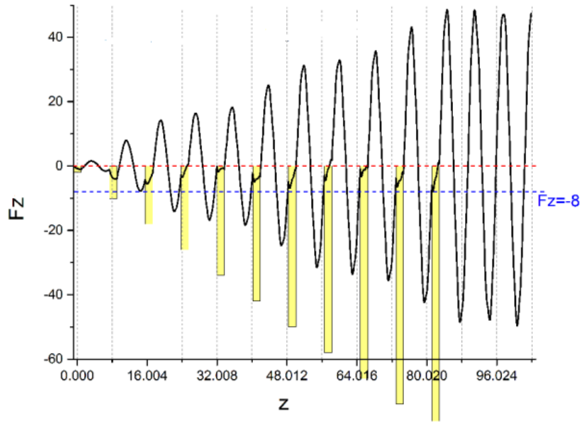

Fig. 2. The spatial distribution in longitudinal unlimited dielectric waveguide (along the axis) of WF, which is excited by sequence of short relativistic electron bunches with linear growth of charge 1:5:9: …

Structure	ω/ω_{rep}	Charge shaping of bunch sequence	a, cm	T_E
resonator	1.63	1:3:5: …	3.2	0.4M
waveguide	1.26	1:5:9: …	3.2	0.8M

One can see that bunches are decelerated by small periodic field and the WF amplitude after the bunches grows. Thus, TR increases with the bunch number M increase. Conditions for large

$$T_E \approx 0.4M, \qquad (19)$$

are satisfied in the case when the resonator radius in the dielectric channel equals a=3.2 cm and the external radius equals b=4.25 cm (see Table).

2. EXPERIMENTAL INSTALLATION

Producing method of a shaped sequence of bunches on the basis of the linear resonant electron accelerator "Almaz-2M" is proposed in this paper. In normal regime, the electron bunches with the energy of 4.5 MeV and charge 0.26 nC are created by a deep current modulation of the electron gun by generator microwave field of duration 2 μs. The number of bunches equals N=6000, the repetition frequency of the bunches equals f_{rep}=2805 MHz, the size of bunches: σ_z=17 mm, the radius r_b=5 mm, the duration of the bunch τ_b=60 ps, repetition interval of bunches T=360 ps.

The current pulses of the accelerated beam, which is a sequence of relativistic electron bunches, are measured by a Faraday cup, placed after exit foil of accelerating section, and by oscilloscope GDS-840 C.

The modulator of the electron gun generates a voltage pulse to the gun cathode of the duration 4 μs and amplitude 80 keV. Pulse has flat top 2.5 μs and flat wave-fronts. Forming lines of master generator and amplifier klystron provide rectangular pulses of duration 2 μs.

All three modulators are triggered by trigger pulses of the generator with an adjustable delay for each channel. In normal operation of accelerator, the pulse delay, triggering klystron modulator and the master generator, are the same and such that a power microwave pulse gets to the flat part of the gun pulse. In this case, a rectangular pulse of duration 2 μs is formed on the accelerator exit. Shifting pulses of master generator and of klystron KIU-2M relative to each other one can control the duration of RF power pulse, supplied to the input of the accelerating section and, accordingly, the duration of current pulse at the output of the accelerator. When a microwave power pulse gets to the leading edge of the gun current pulse the triangular pulse of the beam current is formed at the accelerator output, Fig. 3.

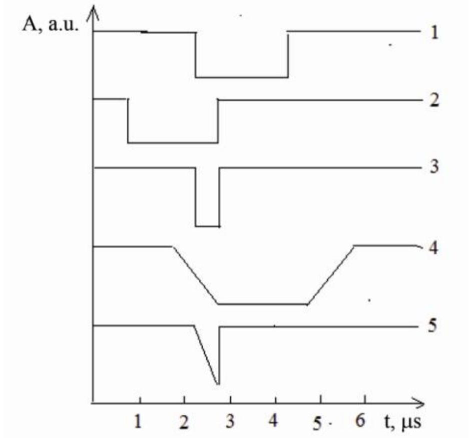

Fig. 3. The time dependences of driving parameters of key accelerator components:
1 – the envelope of the microwave pulse of the master generator; 2 – the voltage pulse on the klystron;
3 – the microwave pulse at the input of the accelerating section; 4 – the beam current pulse of the electron gun;
5 – the envelope of the beam current pulse at the output of the accelerator

3. EXPERIMENTAL RESULTS

As it has been shown above that one can change the RF power pulse duration at the input of accelerating section by shift of delays of master generator and of amplifier klystron KIU-2M. Triangular pulse of beam current is formed at the output of the accelerator, when the microwave power pulse is synchronized with the moment of the rise (leading edge) of the gun current pulse. The waveforms of thus pulses are shown in Fig. 4.

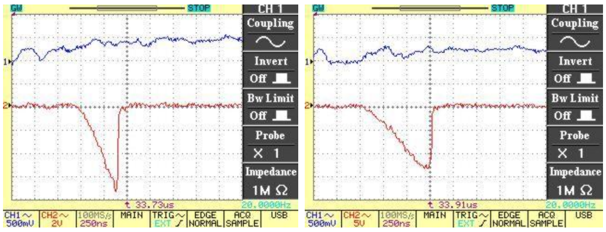

Fig. 4. The waveforms of shaped sequences of electron bunches of duration 0.4 μs (1200 bunches) and 0.75 μs (2250 bunches)

For the study of dependence of the excited wakefield on the parameters of bunches and dielectric structure it is necessary to know the value of this field. In our case, the longitudinal component of the electric field of excited wake wave is measured by a dowel short probe (dowel antenna), located at the outlet end of the dielectric structure. One can determine the field value near the probe, using value of electromotive force -ε, induced on the probe, knowing acting (effective) length of the registering probe (registering antenna). The effective length of the registering antenna ℓ_e is a factor that has the dimension of length, which connects the amplitude of the field strength at the registered point and voltage on the antenna [15]:

$$\varepsilon_{A\,max} = \ell_e E_{max} .$$

This factor is only for linear antennas and characterizes their conversion efficiency of the electromagnetic field energy into the energy of high-frequency currents. The effective length of the registering antenna depends on its length and on the amplitude and phase distribution of the current on the antenna. For short antennas the effective length is taken to be equal to half of its length [16]. In this case, the probe length (dowel antenna) is chosen from the requirement to obtain an optimal signal of amplitude for oscilloscope TDS 6154. The probe is part of the central conductor of the coaxial cable RJ-58 serving in the 2 mm outside braided cable.

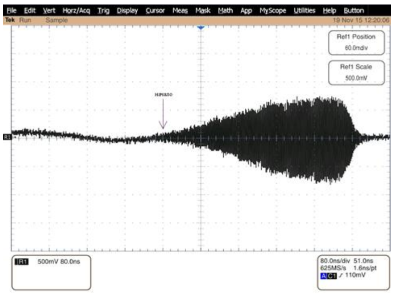

Fig. 5. Waveform of microwave signal, excited by shaped sequence of electron bunches of the duration 400 ns in the dielectric structure

Microwave signal from the probe at the injection into the structure of the current pulse of duration 400 ns (1200 bunches) with linearly increasing charge of bunches is shown in Fig. 5 and its amplitude increases during the pulse.

The excited wakefield at different times from the beginning of the microwave pulse is presented in Fig. 6. Exciting field is close to a sine wave with a frequency 2805 MGz equal to the repetition frequency of electron bunches.

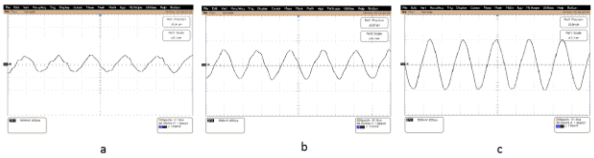

Fig. 6. Waveforms of the excited wakefield at various times from the beginning of the microwave pulse: a – 160 nc; b – 1240 nc; c – 320 nc

The amplitude of the signal from the probe reaches 1 V, which corresponds to the electric field near the probe $E_{max}=2U/\ell_e = 10^3$ V/m. This field value is less than the theoretical value of the electric field in the excited wakefield. The reason for discrepancy between the experimental data and theoretical value is, in our opinion, the excitation of cross-dipole (HEM) mode [17, 18], which is excited by bunches in the dielectric structure. The experimentally observed shift of bunches in the transverse direction from the axis of the dielectric structure indicates excitation of dipole mode. One needs note that the external focusing magnetic field was not used in these experiments.

CONCLUSIONS

It has been shown that the energy transformation ratio is proportional to the number of bunches at the formulated condition validity at the excitation of wakefield in the dielectric resonator by sequence of short relativistic electron bunches with linear growth of charge. Shaped sequence of relativistic electron bunches with approximately linearly increase charges has been derived on the linear accelerator “Almaz-2M”.

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Article received 31.03.2016

УВЕЛИЧЕНИЕ КОЭФФИЦИЕНТА ТРАНСФОРМАЦИИ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОГО ПОЛЯ ПРОФИЛИРОВАННОЙ ПОСЛЕДОВАТЕЛЬНОСТЬЮ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ В ДИЭЛЕКТРИЧЕСКОЙ СТРУКТУРЕ

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Показано, что при возбуждении кильватерного поля в диэлектрическом резонаторе последовательностью коротких релятивистских электронных сгустков с линейно нарастающим зарядом коэффициент трансформации энергии пропорционален числу сгустков последовательности при выполнении сформулированных условий. На линейном ускорителе “Алмаз-2М” получена профилированная последовательность релятивистских электронных сгустков, заряды которых нарастают примерно по линейному закону. Длительность импульсов (количество сгустков) профилированной последовательности можно изменять от 0,4 (1200 сгустков) до 0,75 мкс (2250 сгустков). Проведены эксперименты по возбуждению кильватерных полей в диэлектрических структурах этой последовательностью электронных сгустков.

ЗБІЛЬШЕННЯ КОЕФІЦІЄНТА ТРАНСФОРМАЦІЇ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОГО ПОЛЯ ПРОФІЛЬОВАНОЮ ПОСЛІДОВНІСТЮ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ У ДІЕЛЕКТРИЧНІЙ СТРУКТУРІ

Г.П. Березіна, А.Ф. Лінник, В.І. Маслов, О.Л. Омелаєнко, І.М. Оніщенко, В.І. Приступа, Г.В. Сотніков, В.С. Ус

Показано, що при збудженні кільватерного поля в діелектричному резонаторі послідовністю коротких релятивістських електронних згустків з лінійно наростаючим зарядом коефіцієнт трансформації енергії пропорційний числу згустків послідовності при виконанні сформульованих умов. На лінійному прискорювачі “Алмаз-2М” отримана профільована послідовність релятивістських електронних згустків, заряди яких наростають приблизно за лінійним законом. Тривалість імпульсів (кількість згустків) профільованої послідовності можливо змінювати від 0,4 (1200 згустків) до 0,75 мкс (2250 згустків). Проведено експерименти по збудженню кільватерних полів у діелектричних структурах цією послідовністю електронних згустків.