BUILDUP OF PLASMA OSCILLATIONS DURING MODELING THE ELECTROMAGNETIC WAVE PROPAGATION

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A plane wave formation in plasmas is studied numerically by Finite Difference Time Domain (FDTD) solver. The electromagnetic wave is incident normally to the plane boundary between vacuum and isotropic homogeneous plasma. Different harmonics of the plane wave need different time to build up the oscillations in plasma. Therefore a vacuum waveform differs from plasma waveform not due to a difference in dispersions only. The formation of the plane wave in plasma is studied numerically for different points of the dispersion curve known from a plasma steady state theory. The study has to issue finally a numerical model of the plane wave source in plasmas to use it in the problems of plasma wave scattering, tunnelling and conversion.

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INTRODUCTION

Actual problems of the radiolocation require extensive numerical modelling. One of the problems is a calculation of the radar cross sections of various objects when the scattered field is compared with the incident one. In this case a good numerical model has to describe accurately an incident signal, a propagation of the signal through different (probably dispersive and anisotropic) media, a reflection of the signal from a scatterer (applying the boundary conditions) and an easy separation (as a result of analysis) of the scattered signal from the incident one. This can be realized successfully by Finite Difference Time Domain (FDTD) method [1, 2] and is well developed for wave propagation through vacuum or dielectric media. In this case the waveform is easily defined by the antenna signal. The wave propagation through a dispersive media represents a more difficult case. For example, in plasmas, the incident wave generates the plasma currents which need time to reach a saturation. There is a transient time to build up the plasma oscillations and to form a plasma waveform known from plasma equilibrium theory. All this should be taken into account in the problems where the waveform of the numerical model of the wave source is important for the analysis.

FDTD method is well developed to model the source of plane wave [3, 4], wave propagation with the dispersion compensation due to a difference between analytical and numerical dispersions [5], near-to-far-field transformation using Huygens surface [6] and different types of the boundary conditions including the Perfectly Matched Layer [7]. It works well for nondispersive media with soft anisotropy (the permittivity tensor is a diagonal one). We should like to develop the method for plasmas.

For plasmas, the Maxwell's equations have to be solved together with the plasma current equations. Both analytical and numerical dispersion equations give the relations between wave frequency and wave vector in plasmas at steady state [8, 9]. But they don't answer the questions: how long does it take to build up the plasma currents when the electromagnetic wave is incident from vacuum on the plasmas and how long does it take to form the waveform of plane wave in the plasmas. The numerical model of the plane wave source in plasma is interesting in particular for the problems of the wave transmission and reflection through/from an evanescent layer.

1. NUMERICAL TREATMENT OF THE PROBLEM

A standard Yee cell algorithm [1] is applied to solve the set of Maxwell's equations. It means that electric $\vec{E}$ and magnetic $\vec{B}$ fields are calculated at different space points shifted from each other by a half of the spatial numerical grid step. Moreover, the different components of the fields are calculated at different spatial points. It allows to represent the curl numerically in convenient form (in the sense of Yee cell application). Discretization model has been chosen with the components of electric field at the edges of the Yee cell and the components of the magnetic field at the faces of the Yee cell. It is convenient for boundary condition treatment.

The time discretization is built also in the way when the electric $\vec{E}$ and magnetic $\vec{B}$ fields are calculated at different time moments shifted from each other by a half of the time numerical grid step.

In such a way the cold magnetized plasma is described by the discretized set of Maxwell's equations coupled by the equations for current density $\vec{J}_\alpha$ of the plasma species α. The FDTD scheme has been built with $\vec{E}$ - $\vec{J}_\alpha$ collocation in time. It is explicit for the Maxwell-Faraday equation but mixed explicit/implicit for the Maxwell-Ampere and current equations:

$$\begin{cases} -\dfrac{1}{c}\dfrac{\partial \vec{B}^{(n+1/2,n-1/2)}}{\partial t} = \nabla\times\vec{E}^{(n)}, \\ \dfrac{1}{c}\dfrac{\partial \vec{E}^{(n+1,n)}}{\partial t} = \nabla\times\vec{B}^{(n+1/2)} - \dfrac{4\pi}{c}\sum_\alpha \vec{J}_\alpha{}^{(n+1,n)}, \\ \dfrac{\partial}{\partial t}\vec{J}_\alpha{}^{(n+1,n)} = \dfrac{\omega_{p\alpha}^2}{4\pi}\vec{E}^{(n+1,n)} + [\vec{J}_\alpha\vec{\Omega}_\alpha]^{(n+1,n)}, \end{cases} \quad (1)$$

here $\omega_{p\alpha}$ and $\vec{\Omega}_\alpha$ are plasma and cyclotron frequencies of the plasma species α respectively. The superscripts indicate the moments of time when corresponding terms are calculated. Although the initial scheme (1) is mixed, it was resolved analytically to deal numerically with absolutely explicit scheme.

The numerical code is 3D but here it is used to consider 1D problem. In the model, the vacuum electromagnetic wave is incident normally onto a sharp boundary between vacuum (x<0) and isotropic homogeneous plasma (x>0). Simulation of the plane wave propagation with arbitrary waveform through vacuum by FDTD method is well described in the literature [2]. A plane antenna or a Huygens surface [10] could be a source of the wave and numerical modelling of the waveform is straightforward when the wave propagation is grid-aligned. When propagation is not grid-aligned then the waveform modelling is not so trivial but it is also well described in the literature [4, 11]. Therefore modelling of the waveform in vacuum is out of discussions in the paper. The processes of the plasma waveform formation are under consideration only.

2. PHYSICS OF THE PROBLEM

The vacuum plane wave of a single harmonic ($\propto \exp(-j\omega t + jkx)$, where ω is a wave frequency and $k = \omega / c$ is a vacuum wave number) is studied here since the plasma theory operates by single harmonic analysis. The dispersion of the electromagnetic wave with the frequency ω in plasma is well known:

$$k_{pl} = \sqrt{\varepsilon} k \, , \tag{2}$$

where $\varepsilon = 1 - \frac{\omega_{pl}^2}{\omega^2}$ is a plasma permittivity, $\omega_{pl}^2 \equiv \sum_{\alpha} \omega_{\alpha}^2$. The wave processes are characterized by the phase and group velocities (Fig. 1). The wavelength is always larger than the vacuum one, the phase velocity is larger and the group velocity is smaller than the speed of light. But these values are calculated for steady state when plasma (as a medium) is already adapted to the external electromagnetic action at the frequency ω. Initially (at t=0), when the vacuum wavefront reaches the plasma boundary, the plasma is similar to vacuum from an electrodynamics point of view: the plasma currents are zero, the permittivity is a unity, the phase and group velocities, as a sequence, are equal to the speed of light. Under the action of the electromagnetic wave the plasma currents begin to build up. After enough time, the plasma currents saturate and the wave propagates in plasma according to the plasma theory of steady state. There is a time/space range of the transient processes within which the wavelength, phase and group velocities change from vacuum to plasma values. Of course, in this range the meanings of “the wavelength”, “phase and group velocities” lose their sense. Instead of them we can speak about “the distance between the local maxima of the oscillations”, “local speed of phase transfer” and “local speed of the power transfer”.

Duration and location of the transient processes are possibly not interesting for the plasma theory which operates the steady state media but it is important for numerical modelling in time domain.

For example, the wave plasma diagnostics (similarly to the radiolocation) is based on comparison of the incident signal with the scattering one. But an attempt to produce numerically some given waveform in initially quiet plasma meets the problem of the transient processes. Also an attempt to restrict a scatterer by a box with Huygens surface [2, 10] which could decrease drastically the time of simulations requires knowing the analytical waveform on this surface in the medium. It works in vacuum but it is difficult to be realized in plasmas due to the transient processes again.

On the other hand, the Courant stability criterion for arbitrary medium is known as:

$$\Delta t \leq \frac{\Delta x}{v_{ph}} \, , \tag{3}$$

where v_{ph} is phase velocity in the medium. If it is valid, then the FDTD scheme is stable in plasmas for smaller time step values since the plasma phase velocity is always larger than the vacuum one. But the wavefront of the wave just launched into plasma propagates with speed of light.

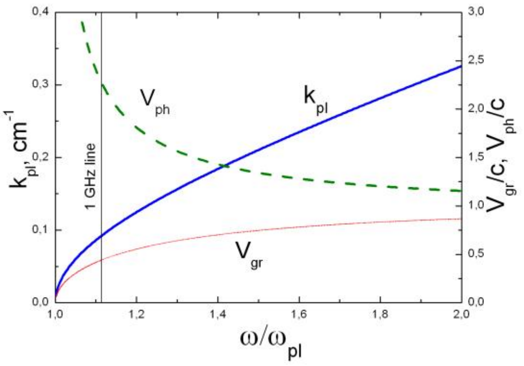

Fig. 1. Wave vector, phase velocity and group velocity of the electromagnetic wave in hydrogen plasma with the density of 10^{10} cm^{-3}

3. NUMERICAL STUDY

The numerical study operates with the plasma density of 10^{10} cm^{-3}. The frequencies are chosen to provide an essential difference between the vacuum and plasma dispersions. The incident electromagnetic wave is reflected from the plasma boundary according to the plasma theory [8] therefore the amplitude of the incident wave is chosen to provide a unity of the amplitude in plasma.

The electric field distribution is calculated as a function of coordinate and time. The spatial step is 1/50 of the wavelength in vacuum and time step is less than the Courant limit, $\Delta t = \Delta x / (2 \cdot v_{ph})$, to provide good enough numerical resolution.

Additionally, a power analysis is carried out to identify the steady state. The Poynting flux along a normal to the vacuum-plasma interface is calculated and averaged over the wave period. In such a way, the steady state can be identified locally if the averaged Poynting flux does not change anymore in time.

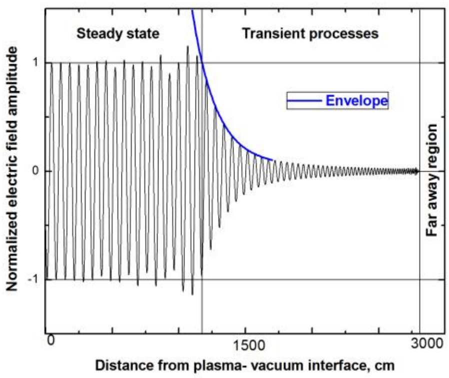

Fig. 2. Spatial distribution of electric field inside plasma with the density of 10^{10} cm^{-3} after 10^{-7} sec of the electromagnetic wave propagation. The wave frequency is 1 GHz. The vacuum-plasma interface is at coordinate 0

The frequency 1 GHz is chosen for simulations since the plasma wavelength in this case (68.17 cm) differs essentially from the vacuum one (30 cm). The plasma phase and group velocities are 2.274*c* and 0.1934*c* respectively (see Fig. 1).

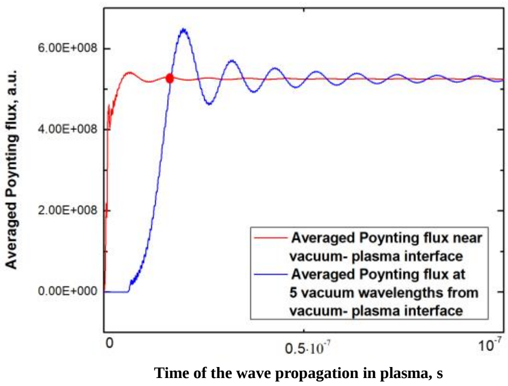

Fig. 3. Averaged Poynting flux evolution in time. Red line denotes the flux through cross section near the vacuum-plasma interface, blue line – the same at distance in 5 vacuum wavelengths inside plasma. Plasma density and wave frequency are the same as in Fig. 2

The goal is to study the waveform formation in plasma. A way of analysis can be seen from Fig. 2. The waveform in plasma can be separated into three regions: 1) the plasma oscillations are mainly saturated (the amplitude has reached a unity and the distance between the peaks is equal to the wavelength in plasma); 2) the region of the transient processes where the amplitude is less than a unity and the distances between the peaks are larger than vacuum wavelength but less than plasma one; 3) the region which is not reached by the wavefront yet. The envelope of the transient waveform is built for the second region to find its intersection with a unity. This point is used to calculate the depth and speed (Figs. 3 and 4) of the plasma oscillation buildup in plasma.

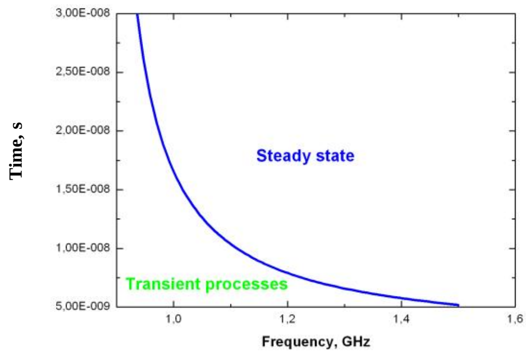

Fig. 4. Time of the steady state formation in plasma (5 vacuum wavelengths from the vacuum–plasma interface) as a function of the wave frequency. Plasma density is 10^{10} cm^{-3}

The steady state analysis of the oscillations differs from the power analysis and is less precise. Therefore the numerical analysis of the averaged Poynting flux is carried out (see Fig. 3). One curve is averaged Poynting flux near the vacuum-plasma interface there, the other one is averaged Poynting flux at distance in 5 vacuum wavelengths from the vacuum-plasma interface inside plasma. Analysis of the Fig. 2 identifies the time moment marked by a red circle as a steady state but the power analysis shows still the power oscillations around a real equilibrium power level. Therefore the power steady state can be defined as a moment when power oscillations become less than few percents. It is more precise than the field oscillation analysis but the acceptable level of the power oscillations is under discussion.

Finally we should like to know how deep in plasma the plasma oscillations are saturated. The answer is given by Fig. 4 which plots time of the plasma oscillation buildup in 5 vacuum wavelengths from the vacuum-plasma interface as a function of frequency. For large frequencies it is almost linear dependence which gives a speed of steady state penetration into plasma. But the dependence becomes nonlinear for the frequencies near plasma one. Probably the power analysis of steady state is more reasonable to build Fig. 4. But acceptable level of power oscillations for steady state should be discussed additionally.

CONCLUSIONS

The buildup of the plasma oscillations during transient processes which probably are not so important for the plasma theory are important for the problems of the numerical simulations in time domain. Space/time distribution of the waveform in plasma should be predictable to separate the steady state and transient processes.

The time of the plasma oscillation buildup is estimated for the different ranges of the dispersion curve. The field oscillations and power analysis are developed to control the saturation of the plasma oscillations. The power analysis is more precise than the field oscillations one but the acceptable level of the power deviation from equilibrium value is still under discussion.

The time of the plasma oscillations buildup increases drastically when the frequency of the wave approaches the plasma frequency. It is well predictable from the plasma theory but should be taken into consideration each time in the numerical simulations if the time of the buildup is crucial one to terminate the time of the calculations.

As a result the different harmonics of the plane wave require different time to build up the corresponding plasma oscillations. This time should be taken into consideration each time when the plane wave is launched into plasma with given waveform since plasma waveform can differ essentially from the vacuum one.

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РАСКАЧКА ПЛАЗМЕННЫХ КОЛЕБАНИЙ ПРИ МОДЕЛИРОВАНИИ РАСПРОСТРАНЕНИЯ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ

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Формирование плоской волны в плазме изучается численно методом конечных разностей во временной области (КРВО). Электромагнитная волна падает перпендикулярно на плоскую границу раздела вакуума и однородной незамагниченной плазмы. Различным гармоникам плоской волны требуется различное время на раскачку плазменных колебаний. Поэтому различие между формами волны в вакууме и в плазме определяется не только различием в дисперсионных соотношениях. Формирование плоской волны в плазме изучается численно для различных точек дисперсионной кривой, которая известна из теории стационарной плазмы. Изучение проводится с целью создания численной модели источника плоской волны в плазме для её дальнейшего использования в задачах рассеяния, туннелирования и конверсии волн.

РОЗКАЧКА ПЛАЗМОВИХ КОЛИВАНЬ ПІД ЧАС МОДЕЛЮВАННЯ ПОШИРЕННЯ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

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Формування пласкої хвилі в плазмі вивчається числовим методом скінченних різниць у часовій області (СРЧО). Електромагнітна хвиля падає перпендикулярно на пласку межу поділу вакуума та однорідної незамагніченої плазми. Різні гармоніки пласкої хвилі потребують різного часу на розкачку плазмових коливань. Тому різниця між формами хвилі у вакуумі та в плазмі визначається не лише різницею в дисперсійних співвідношеннях. Формування пласкої хвилі в плазмі вивчається числовими методами для різних точок дисперсійної кривої, яка є відомою з теорії стаціонарної плазми. Дослідження проводиться з метою створення числової моделі джерела пласкої хвилі в плазмі для подальшого її використання в задачах розсіяння, тунелювання та конверсії хвиль.