

# ON THE NATURE OF SOURCES OF PULSATING RADIATION IN WEAKLY INVERTED MEDIA

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The conditions for the occurrence of a periodic sequence of pulses of coherent radiation in a weakly inverted two-level medium are investigated. It is studied the dependence of the pulse period and amplitude on the inversion pumping level and radiation losses. It is shown that an increase in the size of the radiating system leads to the growth of the total radiation intensity and the pulse repetition period. This dependence is consistent qualitatively with the observed characteristics of the cosmic sources of pulsed radiation.

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## INTRODUCTION

As known, a two-level system demonstrates the possibility of generation of both spontaneous and induced (stimulated) emission when the initial population inversion is sufficiently large [1]. Usually, the term spontaneous emission denotes the emission of oscillator (or other emitter) which not forced by external field of the same frequency. By induced or simulated emission is usually meant the emission produced because of an external field action on the emitting source at the radiation frequency [2 - 4].

In the earlier paper [5], the authors have discovered a threshold of induced radiation at a certain critical value of population inversion. The specific feature of this threshold is that it follows from the condition that the initial value of the population inversion is equal to the square root of the total number of states. Above this threshold, the number of photons begins to grow exponentially with time. It was shown that the threshold corresponds to the case when the intensity of the spontaneous and stimulated coherent radiation become equal. In further work [6], it was demonstrated that the pulse of stimulated radiation with a characteristic profile is formed when the initial population inversion slightly exceeds the threshold. The leading edge of the pulse is very sharp due to the exponential growth of the field, and the trailing edge is rather broadened. Further threshold overriding with growing of the population inversion leads to the growth of the ratio of the pulse trailing-edge time to the leading-edge time.

If there is exist a recovery mechanism for the population inversion, the pulsating mode of stimulated emission generation becomes possible. The integral radiation intensity at this may be increased several times. This approach can be used for analysis of the cosmic radiation that might help explain a great variety of pulsating radiation sources in space. In present work, we investigate the characteristics of the periodic pulse generation depending on the initial inversion, the pumping level and the absorption rate.

## 1. THE MODELS FOR DESCRIPTION OF A TWO-LEVEL SYSTEM

A two-level system with transition frequency  $\varepsilon_2 - \varepsilon_1 = \hbar\omega_{12}$  can be described by following set of

equations:

$$\partial n_2 / \partial t = -(u_{21} + w_{21} \cdot N_k) \cdot n_2 + w_{12} \cdot N_k \cdot n_1, \quad (1)$$

$$\partial n_1 / \partial t = -w_{12} \cdot N_k \cdot n_1 + (u_{21} + w_{21} \cdot N_k) \cdot n_2,$$

where the sum of level populations  $n_1 + n_2 = N$  remains constant,  $u_{21}n_2$  is the rate of change in the number density of atoms due to spontaneous emission. The rates of change in level population due to stimulated emission and absorption are  $w_{21}N_k n_2$  and  $w_{12}N_k n_1$  correspondingly. The number of quanta  $N_k$  on the transition frequency  $\omega_k$  is governed by the equation

$$\frac{\partial N_k}{\partial t} = (u_{21} + w_{21} \cdot N_k) \cdot n_2 - (w_{12} \cdot N_k) \cdot n_1. \quad (2)$$

The energy losses in active media are caused mainly by radiation outcome from a resonator. These radiative losses can be calculated by imposing the correct boundary conditions on the field. Thus, they can be estimated in rather common form with the following parameter:

$$\delta = \frac{\iint_S \frac{\partial \omega}{\partial \vec{k}} \frac{1}{4\pi} \vec{E} \times \vec{H} ds}{\iiint_V \frac{\partial [\omega \varepsilon(\omega, \vec{k})]}{\partial \omega}} \times \times \frac{1}{8\pi} (|\vec{E}|^2 + |\vec{H}|^2) dv, \quad (3)$$

i. e. as the ratio of the energy flow outside the system should be divided by the total field energy within the system. It is important, that the characteristic size of the system  $L$  should be much less than the characteristic time of field variation  $\tau \sim |\vec{E}|^2 (\partial |\vec{E}|^2 / \partial t)^{-1}$  multiplied by the group velocity of oscillations  $|\partial \omega / \partial \vec{k}|$ . In this case, the radiative losses can be replaces by distributed losses within the volume. The threshold of instability leading to exponential growth of the stimulated emission in this case is defined by condition  $\mu_0 > \mu_{TH1}$  (see, for example [6], where

$$\mu_{TH1} = \delta / w_{21}. \quad (4)$$

Equations (1)-(2) can be rewritten with consideration of the energy losses  $\delta$  and steady pumping rate  $I$ , caused for example by permanent heating of the system:

$$\partial n_2 / \partial \tau = -n_2 - \mu \cdot N_k + I, \quad (5)$$

$$\partial \mu / \partial \tau = -2n_2 - 2\mu \cdot N_k + I, \quad (6)$$

$$\partial N_k / \partial \tau = n_2 + \mu \cdot N_k - \delta \cdot N_k, \quad (7)$$

where  $\tau = w_{21} \cdot t$ ,  $u_{21} = w_{21} = w_{12}$ .

It can be assumed, at least qualitatively, that the terms in r.h.s. of Eq. (1)-(2) proportional to  $N_k$  correspond to the coherent processes, as well as the photons, which number  $N_k$  is incorporated in these terms, will be assumed coherent. With these general principles in mind, we expand the total number of photons into two components  $N_k = N_k^{(incoh)} + N_k^{(coh)}$ , where  $N_k^{(incoh)}$  is the number of quanta, corresponding to the spontaneous emission, and  $N_k^{(coh)}$  is the number of quanta, corresponding to the stimulated emission. Then Eqs. (1)-(2) can be rewritten as follows [6]:

$$\partial n_2 / \partial \tau = -n_2 - \mu \cdot N_k^{(coh)} + I, \quad (8)$$

$$\partial \mu / \partial \tau = -2n_2 - 2\mu \cdot N_k^{(coh)} + 2I, \quad (9)$$

$$\partial N_k^{(incoh)} / \partial \tau = n_2 - \delta \cdot N_k^{(incoh)}, \quad (10)$$

$$\partial N_k^{(coh)} / \partial \tau = \mu \cdot N_k^{(coh)} - \delta \cdot N_k^{(coh)}, \quad (11)$$

where  $N = n_1 + n_2$  is a total number of emitters and  $n_2 = (N + \mu) / 2$ .

It was shown in the paper [5] that the scenario of the process changes, if the initial value of the inversion  $\mu_0$  is more or less than the threshold value:

$$\mu_{TH2} = 2N^{1/2}. \quad (12)$$

The suppression of the exponential growth of the photon number, when  $\mu_0 < \mu_{TH2} = 2N^{1/2}$  demonstrates not only the changes in scenario of the process, but suggests that the stimulated emission is suppressed by preferential growth of the spontaneous emission.

Let discuss the reasons why it makes sense to use a qualitative system of equations (8)-(11) near the threshold (12).

Within the framework of the classical description, the total intensity of the spontaneous emission of an ensemble of particles-oscillators, whose phases are distributed randomly and uniformly, can be found as a sum of individual intensities produced by each particle-oscillator being in an excited state. As for the stimulated emission, the radiation field strength is so great that synchronizes the phase both of the emitting and absorbing oscillators. Thus the sign of the population inversion  $\mu = n_2 - n_1$  determines is the stimulated field will increase or decrease. Note, that the characteristic time of this process is inversely proportional to  $\mu$ . However, if the coherent field is absent, the oscillators in the excited state will emit only spontaneously, because their phases are not synchronized. The absorption of the spontaneous field by the unexcited particles-oscillators can be ignored since they are placed in a random rapidly alternating field, which averaged effect is negligible.

In the quantum case, the traditional model (5)-(7) includes the term  $\mu \cdot N_k$  which is responsible for the stimulated processes of excitation and absorption. But it has no physical meaning below the threshold (12), since in this case there is no intense stimulated field, which is able to synchronize the emission of many particles. In this case Eq. (9) takes the form

$$\partial \mu / \partial \tau = -N - \mu + 2I. \quad (13)$$

In the steady state  $\mu_{st} = I - N < \mu_{TH2}$  the intensity of the radiation source will be determined only by the spontaneous emission  $\delta \cdot N_k^{(incoh)} \approx N / 2$ . The term  $N$  in

Eq. (13) determines the effect of the spontaneous emission on the inversion. But when the threshold (12) is exceeded  $\mu > \mu_{TH2}$ , the term  $\mu \cdot N_k$  in r.h.s of Eqs. (5)-(7) and (8)-(11) plays an important role, providing an effect of the stimulated processes.

Near the threshold (12), it is reasonable to use namely a qualitative system of equations (8)-(11). Then the steady state of the spontaneous emission is determined by the value  $\delta \cdot N_k^{(incoh)} \approx N / 2$ , but the energy flow of the stimulated emission is equal to  $\delta \cdot N_k^{(incoh)}$ . In order to describe the behavior of a two-level system in presence of the radiation losses and continuous external pumping and neglecting the small values of the order of  $\mu_0^{-1}$ , Eqs. (9)-(11) can be rewritten in a convenient form:

$$\partial M / \partial T = -N_0 - 2M \cdot N_c + 2I_0, \quad (14)$$

$$\partial N_{inc} / \partial T = N_0 / 2 - \theta \cdot N_{inc}, \quad (15)$$

$$\partial N_c / \partial T = M \cdot N_c - \theta \cdot N_c, \quad (16)$$

where  $N_{inc} = N_k^{(incoh)} / \mu_0$ ,  $N_c = N_k^{(coh)} / \mu_0$ ,  $M = \mu / \mu_0$ ,  $M = \mu / \mu_0$ ,  $T = w_{21} \cdot \mu_0 \cdot t = \mu_0 \cdot \tau$ ,  $I_0 = I / \mu_0^2$ , and the only convenient for analysis free element is  $N_0 = N / \mu_0^2$ . Let specify the initial values as follows:

$$M(T=0) = 1,$$

$$N_{inc}(T=0) = N_{inc} / \mu_0 = 3 \cdot 10^4 / \mu_0;$$

$$N_c(T=0) = N_c / \mu_0 = 3 \cdot 10^4 / \mu_0;$$

$$N_1(T=0) = N_k / \mu_0 = 3 \cdot 10^4 / \mu_0.$$

The energy losses are taken into account by the parameter  $\theta = \delta / \mu_0$ , where  $\delta$  is defined by Eq. (3).

In the case, when  $\mu > \mu_{TH2}$  and  $M(0) > \theta$ , the relaxation oscillations appear in the system resulting in a stationary state  $N_{cst} = (I_0 - N_0) / 2\theta$ ,  $M_{st} = \theta$ . The total radiation flow outside the system in assumed terms is equal to

$$\theta \cdot N_{cst} + \theta \cdot N_{incst} = (I_0 - N_0) / 2 + N_0 / 2 = I_0 / 2.$$

Note, that in presence of an external mechanism, which provides an exceedance of the inversion over its stationary value  $M_{st} = \theta$ , the Eq. (14) can be supplemented by the driving term

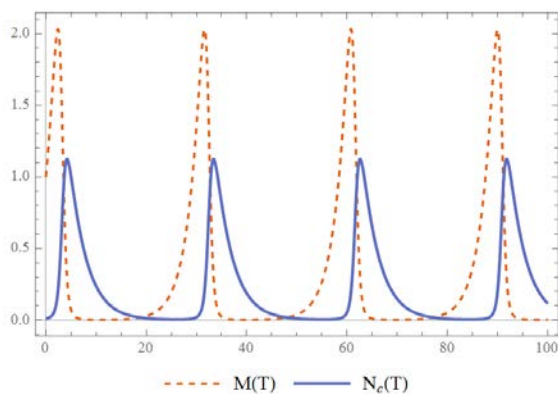
$$\partial M / \partial T = \Gamma M - N_0 - 2M \cdot N_c + 2I_0. \quad (17)$$

The Eqs. (16), (17) are similar to so-called Statz-DeMars equations [7], which describe the relaxation oscillations in a two-level media in the presence of the pump and energy losses. The only difference in equation (17) is the first term in r.h.s. that provides the maintenance of the population inversion. Namely this term changes the characteristics of pulse generation from relaxation to periodic.

The Eqs. (15)-(17) have a solution in a form of periodical sequence of coherent pulses (Figure) against a background of the mean radiation flow

$$\theta \cdot N_{cst} + \theta \cdot N_{incst} = (\Gamma \theta + I_0) / 2. \quad (18)$$

Pulse repetition rate is  $\sqrt{\theta \cdot \Gamma}$ . The integral radiation intensity on the pulse peak can exceed the background value in several times.



The repetition pulse train, resulting as a solution of Eqs. (16), (17) for  $\Gamma = 0.4$  and  $\theta = 0.3$

It should be noted that the radiation losses of the field energy  $\theta$  in open systems is defined as the ratio of the energy flux from the object to the energy in its volume, and therefore this parameter decreases with increase of the radius of the system  $R$  as  $c/R$ , where  $c$  is the speed of light. This means that an increase in size  $R$  reduces the losses  $\theta$ , which in turn, as shown in [6], provides a higher intensity of the stimulated emission. That is, at the same parameters of the system, the larger objects should generate more intense pulses but with less repetition rate.

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## О ПРИРОДЕ ИСТОЧНИКОВ ПУЛЬСИРУЮЩЕГО ИЗЛУЧЕНИЯ В СЛАБОИНВЕРСНЫХ СРЕДАХ

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Рассмотрены условия возникновения периодической последовательности импульсов когерентного излучения в слабоинвертированной двухуровневой среде. Изучена зависимость периода и амплитуды возникающих импульсов от уровня накачки инверсии и радиационных потерь. Показано, что с увеличением размеров излучающей системы растет интегральная интенсивность излучения и увеличивается период генерации импульсов. Такая зависимость качественно совпадает с наблюдаемыми характеристиками космических источников пульсирующего излучения.

## ПРО ПРИРОДУ ДЖЕРЕЛ ПУЛЬСУЮЧОГО ВИПРОМІНЮВАННЯ В СЛАБОІНВЕРСНИХ СЕРЕДОВИЩАХ

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Розглянуто умови виникнення періодичної послідовності імпульсів когерентного випромінювання в слабоінвертованому дворівневому середовищі. Досліджено залежність періоду і амплітуди виникаючих імпульсів від рівня накачки інверсії та радіаційних витрат. Показано, що зі збільшенням розмірів випромінюючої системи зростає інтегральна інтенсивність випромінювання і збільшується період генерації імпульсів. Така залежність якісно збігається з характеристиками космічних джерел пульсууючого випромінювання.