SHORT-TIME PARTICLE MOTION IN STRONG STANDING ELECTROMAGNETIC WAVE

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The particle motion in the standing electromagnetic wave was investigated on the times much shorter than the wave period. Cases of circular and linear polarizations are considered. The first nonvanishing coefficients in the series expansion of the particle momentum and particle energy are obtained. The inefficiency of energy gain in the "magnetic" region of LP wave was demonstrated.

PACS: 52.65.Rr, 52.38.Ph

INTRODUCTION

Dramatic development of laser technologies during the last few years gave an opportunity to reach in the laboratory conditions electromagnetic fields with intensities exceeding $I=10^{22}$W/cm^2 [1]. Projects to achieve even higher intensities also exist [2 - 4] (ELI and XFEL facilities). With all this going on the investigation of such intense laser fields with matter becomes important from the fundamental and applied point of view.

One of the problems that are actively discussed is intensive laser pulse interaction with a solid target or atomic cluster [5 - 12]. Fields with the intensities that were mentioned above exceed the atomic ionization threshold by the orders of magnitude, and so matter can be considered as plasma. What may be interesting there are the efficiency of laser energy absorption by the matter and radiation that the matter itself produces. Both these processes are determined by the particles motion in the field of pulse.

Another interesting problem is quantum electrodynamics (QED) cascades development [13 - 15], which may occur for example in the field of two colliding laser pulses. QED cascades base on two quantum processes – high-energy photon emission by electron and photon decay with electron-positron pair (e^+e^-) creation. Both these processes are possible in the presence of external electromagnetic field only and become essential with the intensities over $I=10^{23}$W/cm^2. The sequence of such processes may lead to an avalanche-like pair creation with an electron-positron plasma drop origination as a result. The efficiency of these processes depends on the particles trajectories.

Thus, the problem of exploration of particle motion in different laser field configuration is relevant. Let us note some peculiarities of this problem. Firstly, generally speaking, the particle motion in such an intense field is relativistic (in some cases ultra relativistic approximation is acceptable). Secondly, in such conditions radiation recoil is essential. At the intensities $I=10^{22}\ldots10^{23}$W/cm radiation may be considered classically, and radiation recoil may be taken into account during the inserting of radiation friction force (in the Landau-Lifshits form, for example) in the equations of motion. At the higher intensities the process of photon emission is essentially quantum, photon takes away considerable part of the electron energy, and the description during the force is invalid.

For some configurations, such as plane wave, exact analytical solution may be found in close formed [16]. Quite recently the analytical solution for a particle motion in the plain with the radiation friction force taken into account was found [17]. But the case of standing wave, which is also of a practical interest, turns out to be more complicated. It is impossible to find an exact analytical solution for a particle motion in this case. The results of numerical solution show that trajectories of a charged particle in the standing electromagnetic wave are very complex.

However, for some purposes it is enough to know the characteristics of particle motion during the small time intervals, small in comparison with the wave period - QED cascade is the example of it. There electron and positron motion between two acts of photon radiation may be regarded as classical. Since the mean radiation time is much smaller than the wave period, and since the particle may lose an essential part of the gained energy, the comprehension of particle dynamics on the short times is important in this problem.

1. PARTICLE MOTION

For the sake of convenience we will use dimensionless variables. Dimensionless time t is normalized to a laser period $2\pi/\omega_L$ (ω_L is the laser frequency), dimensionless coordinate is normalised to a laser wavelenght λ. We will normalise momentum and energy by mc and mc^2, respectively. Field strength is normalized to $\omega_L mc/e$, so the maximum normalized field value is $a=eE_{max}/(\omega_L mc)$. Equations of motion for electron then have the form:

$$\frac{d\mathbf{p}}{dt} = -\mathbf{E} - \left[\frac{\mathbf{p}}{\gamma}\times\mathbf{H}\right], \qquad (1)$$

$$\frac{d\mathrm{r}}{dt} = \frac{\mathrm{p}}{\gamma}, \qquad (2)$$

where γ is the particle.

We state the problem as follows. Since there are shot times under consideration, it is possible to expand all the dynamical variables in Tailor series with respect to a time interval δt. Our main aim will be the first nonvanishing terms in the expansions of $\mathbf{p}$ and γ. Coefficients of this expansions are functions of initial time instance t_0 and initial space point x_0. This functions for the first nonvanishing term will be obtained.

1.1. ELECTRON DYNAMICS IN CIRCULARLY POLARIZED STANDING WAVE

In the CP case normalized fields have the form

$$\mathbf{E} = a(0, \cos x \cos t, -\cos x \sin t), \quad (3)$$

$$\mathbf{H} = a(0, -\sin x \cos t, \sin x \sin t). \quad (4)$$

Here electric and magnetic fields are parallel to each other: B=sE, where $s=-tgx_0$.

Let us suppos for simplicity that the electron is initialy at rest at $x=x_0$ and $t=t_0$. We choose axis y along the field direction $E_0=E(x=x_0,\ t=t_0)=sH(x=x_0,\ t=t_0)$, so that $E_z=H_z=0$ at $x=x_0$ and $t=t_0$. We can expand the fields near the $x=x_0$ and $t=t_0$ into the serie

$$E_y \approx E_0 + (\partial_t E_y)\delta t + (\partial_x E_y)\delta x, \quad (5)$$

$$E_z \approx (\partial_t E_z)\delta t + (\partial_x E_z)\delta x, \quad (6)$$

where $\delta t<<1$ and $\delta x<<1$. We can also expand the electron momentum components:

$$p_x \approx \frac{1}{2} p_x'' \delta t^2, \quad (7)$$

$$p_y \approx \gamma \approx p_y' \delta t, \quad (8)$$

$$p_z \approx \frac{1}{2} p_z'' \delta t^2, \quad (9)$$

where it is taken into account that the electron first moves along y-axis so that $p_x,\ p_z<<p_y$ and $p_x \approx p_z \approx 0$.

Making of use the equations of motion Eqs. (1), (2) we can derive

$$p_x'' \delta t \approx \frac{(p_y' \delta t)(s \delta t \partial_t E_z) - \left(\frac{1}{2} p_z'' \delta t^2\right) s E_0}{p_y' \delta t}, \quad (10)$$

$$p_y \approx \gamma \approx E_0 \delta t, \quad (11)$$

$$p_z'' \delta t \approx (\partial_t E_z)\delta t + \frac{\left(\frac{1}{2} p_x'' \delta t^2\right) s E_0}{p_y' \delta t}, \quad (12)$$

where H=sE is used and the terms, which are proportional to δx, are neglected as $v_x=p_x/\gamma<<1$ and $\delta x<<\delta t$. The solution of Eqs. (10)-(12) is

$$p_x \approx \frac{s}{4+s^2}(\partial_t E_z)\delta t^2, \quad (13)$$

$$p_y \approx \gamma \approx E_0 \delta t, \quad (14)$$

$$p_z \approx \frac{2(2+s^2)}{4+s^2}(\partial_t E_z)\delta t^2. \quad (15)$$

The equation for gamma-factor of the electron can be written as follows

$$\frac{d\gamma}{dt} = a(\mathrm{p} \cdot \mathrm{E}), \quad (16)$$

which leads to the next expression:

$$\gamma = a k_\gamma \delta t, k_\gamma^2 = \cos^2 x_0. \quad (17)$$

1.2. ELECTRON DYNAMICS IN LINEARLY POLARIZED STANDING WAVE IN THE "ELECTRIC" REGION (|E|>|H|)

In the LP case normalized fields have the form

$$\mathbf{E} = a(0, \cos x \cos t, 0), \quad (18)$$

$$\mathbf{H} = a(0, 0, \sin x \sin t). \quad (19)$$

The normalized by $a^2/2$ relativistic invariant $\mathsf{F}=2(E^2-H^2)/a^2$ takes the form

$$\mathsf{F}(x,t) = \cos 2t + \cos 2x. \quad (20)$$

It is clear from the Eq. (20) that at every instant there exist spatial regions with |E|>|H| and spatial regions with |H|>|E|. The particle dynamics is different in this two types of regions, so it's more convenient to consider them separately.

First we consider the space-time region where |E|>|H|. It is convenient to treat the problem in another reference frame, namely in the "electric" frame, where at $t=t_0$ $H'(x'_0,\ t'_0)=0$ (accent marks quantities in the "electric" reference frame). The appropriate boost's velocity is given by

$$V_E = \frac{H_z(x_0, t_0)}{E_y(x_0, t_0)}, \quad (21)$$

where $H_z(x_0,t_0)$ and $E_y(x_0,t_0)$ are the electric and magnetic fields in the laboratory reference. Nearby the $(x'_0,\ t'_0)$ field component can be expanded up to the first order:

$$E'_y \approx E'_0 + (\partial_{t'} E'_y)\delta t' + (\partial_{x'} E'_y)\delta x', \quad (22)$$

$$H'_y \approx (\partial_{t'} H'_z)\delta t' + (\partial_{x'} H'_z)\delta x'. \quad (23)$$

It should be noted that for the field derivatives we can write

$$(\partial_{t'} E'_y) = -(\partial_{x'} H'_z), \quad (24)$$

$$(\partial_{x'} E'_y) = -(\partial_{t'} H'_z), \quad (25)$$

which follows from the Maxwells' equations.

Equation of the electron motion Eq. (1) can be solved with expansion in time series $a^{-1}<<\delta t<<1$:

$$p'_y \approx -E'_0 \delta t' - \frac{(\delta t')^2}{2}(\partial_{t'} E'_y), \quad (26)$$

$$p'_x \approx \frac{(\delta t')^2}{2}(\partial_{x'} E'_y), \quad (27)$$

where the terms of the zeroth order on $1/a$ are kept and the terms, which are proportional to δx are neglected because $\delta x' \sim a^{-1}<<\delta t'$.

It follows from Eqs. (26) that $\gamma' \approx |p'_y| \approx E'_0 \delta t'$. Expressing it in terms of the laboratory-frame quantities and substituting the field component for LPd standing wave we obtain

$$\gamma \approx a k_\gamma \delta t, \quad (28)$$

$$k_\gamma = \mathsf{F}(x_0, t_0). \quad (29)$$

1.3. ELECTRON DYNAMICS IN LINEARLY POLARIZED STANDING WAVE IN THE "MAGNETIC" REGION (|H|>|E|)

Let us now consider the space-time region where |H|>|E|. It is again convenient to treat the problem in another reference frame, namely in the "magnetic" frame where at $t=t'_0$ $E'_0(x'_0,t'_0)$ (accent marks quantities in the "magnetic" reference frame). The appropriate boost velocity and the boost gamma-factor are given by

$$V_H = \frac{E_y(x_0,t_0)}{H_z(x_0,t_0)}, \quad (30)$$

$$\gamma_H = \frac{H_z(x_0, t_0)}{\sqrt{H_z^2(x_0, t_0) - E_y^2(x_0, t_0)}}, \quad (31)$$

where $H_z(x_0,t_0)$ and $E_y(x_0,t_0)$ are the electric and magnetic fields in the laboratory reference. For simplicity we will consider region, where $H_y>2^{1/2}E_z$, so that $\gamma_H\sim 1$ and $v_H\gamma_H<1$. Nearby (x'_0,t'_0) the field component can be expanded up to the first order:

$$E'_y \approx \left(\partial_{t'}E'_y\right)\delta t' + \left(\partial_{x'}E'_y\right)\delta x', \quad (32)$$

$$H'_z \approx H'_0 + \left(\partial_{t'}H'_z\right)\delta t' + \left(\partial_{x'}H'_z\right)\delta x', \quad (33)$$

The field derivatives obey Eqs. (22), (23). We suppose that in the laboratory reference frame the electron is at rest at the initial moment of time $t=t_0$ so that in the magnetic frame $\gamma'_0=\gamma_B$, $p'_{z,0}=p'_{y,0}=0$. Assuming again that $\delta t'<<1$ and keeping the leading terms, the equation of the electron motion (2) can be rewritten in the non-relativistic limit as follows

$$\left(\partial_{t'}p'_x\right) = -H'_0\, p'_z, \quad (34)$$

$$\left(\partial_{t'}p'_y\right) = -\delta t'\left(\partial_{t'}E'_y\right) - H'_0\, p'_x. \quad (35)$$

The derived equations describe Larmor rotation of the electron in the magnetic field with growing electric field. The solution takes a form:

$$p'_x = p'_{x,0}\cos\left(H'_0\,\delta t'\right) - \delta t'\frac{\left(\partial_{t'}E'_y\right)}{H'_0}, \quad (36)$$

$$p'_y = -p'_{x,0}\sin\left(H'_0\,\delta t'\right) + \frac{\left(\partial_{t'}E'_y\right)}{\left(H'_0\right)^2}. \quad (37)$$

The terms proportional to $\delta x'$ in the field expansion are neglected Eqs. (32), (33) because it follows from Eqs. (37) (by it's integrating) that

$$\delta x'\sim p'_{x,0}/a+\delta t'^2/2<<\delta t',$$

where an estimation $H'_0\sim\partial_t E'_y\sim a$ is used.

Making of use Eqs. (36), (37) and the inverse Lorentz transformation the electron energy gain can be derived in the laboratory frame

$$\gamma = \gamma_H\sqrt{\gamma_H^2 + 2b_2b_3 + d_3^2} - p_H(b_2 - b_3), \quad (38)$$

$$b_2 = p_H\cos\left(H'_0\,\delta t'\right), \quad (39)$$

$$b_3 = \delta t'\frac{\partial_{t'}E'_y}{H'_0}. \quad (40)$$

Expressing it in terms of the laboratory-frame quantities and substituting the field component for linearly polarized standing wave, we obtain

$$\gamma = \gamma_H\sqrt{\gamma_H^2 + 2k_H p_H\delta t\cos\left(\omega_H\delta t\right) + \delta t^2 k_H^2}, \quad (41)$$

$$- p_H^2\cos(\omega_H\delta t) + k_H p_H\delta t, \quad (42)$$

$$\gamma_H = \frac{2^{1/2}\sin x_0\sin t_0}{\sqrt{-\mathrm{F}(x_0,t_0)}}, \quad (43)$$

$$p_H = \frac{2^{1/2}\cos x_0\cos t_0}{\sqrt{-\mathrm{F}(x_0,t_0)}}, \quad (44)$$

$$\omega_H = \frac{H_0}{\gamma_H^2} = a\frac{-\mathrm{F}(x_0,t_0)}{2\sin x_0\sin t_0}, \quad (45)$$

$$k_B = \frac{\partial_{t'}E'_y}{\gamma_H H'_0} = \frac{\sin 2x_0}{-\mathrm{F}(x_0,t_0)}. \quad (46)$$

Here $H_0=H_z(x_0,t_0)$. It follows from Eq. (41) that $\gamma\sim 1$ and thus there is no significant electron acceleration in contrast to the "electric" region where $\gamma\sim a\delta t$ (see Eq. (29)).

CONCLUSIONS

In the present work the charged particle dynamics in the field of standing electromagnetic wave is investigated. We have considered the cases of linear and circular polarizations. Our attention was mainly concentrated on the particle momenta and Lorentz-factor. We have calculated the first nonvanishing coeficients in the series expansion with respect to the time interval δt. Also it was demonstrated that a particle in the "magnetic" region of linearly polarized wave doesn't become relativistic even though the accelerating electric field exists there.

This results may be useful for the problem of cascade development treatment. From the results of Subsection 2.3. we can conclude that particles in the "magnetic" region do not regain the energy, which is also important for the cascade development.

ACKNOWLEDGEMENTS

This work has been supported by the Government of the Russian Federation (Project №.14.B25.31.0008), by RFBR № 13-02-00886, by the Ministry of Science and Education of the Russian Federation, the Russian Federal Program «Scientific and scientific-pedagogical personnel of innovative Russia», by Russian Federation President's grant.

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Article received 10.04.2013.

ДВИЖЕНИЕ ЧАСТИЦЫ В ПОЛЕ ИНТЕНСИВНОЙ СТОЯЧЕЙ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ НА МАЛЫХ ВРЕМЕНАХ

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На временах много короче, чем период волны, исследовано движение частицы в стоячей электромагнитной волне. Рассмотрены случаи циркулярной и линейной поляризаций. Получены первые неисчезающие коэффициенты в разложении импульса и энергии частицы. Продемонстрирована неэффективность увеличения энергии в «магнитной» области волны линейной поляризации.

РУХ ЧАСТИНКИ В ПОЛІ ІНТЕНСИВНОЇ СТОЯЧОЇ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ НА КОРОТКИХ ТЕРМІНАХ

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Досліджено рух частинки в стоячій електромагнітній хвилі на часових проміжках, які набагато коротші ніж період хвилі. Розглянуто випадки циркулярної та лінійної поляризацій. Отримано перші незникаючі коефіцієнти в розкладі імпульсу та енергії частинки. Продемонстрована неефективність збільшення енергії в «магнітній» області хвилі лінійної поляризації.