# DOWNSIZING FROM THE MERGING MODEL POINT OF VIEW (PRELIMINARY DISCUSSION)

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In the four-particle scattering processes with mass transfer, unlike mergers in which mass can only increase, an essential role are played the processes, when the mass of the most massive galaxies can be decreased. Elementary model describing such a process is considered. In this respect, it is supposed to explain the observed phenomenon of “downsizing” when with cosmological time the growth of characteristic mass of the heaviest galaxies is followed by its decrease.

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## INTRODUCTION

Quite unusual in terms of the paradigm of mergers, though long discussed fact that the maximum galaxy masses (Shechter parameter *M**), which grow up with decreasing of the red shift at large distances, begin to decrease with approaching the present time (Fig. 1), that seems to be in conflict with the model of mergers. Here will be shown that this is not true.

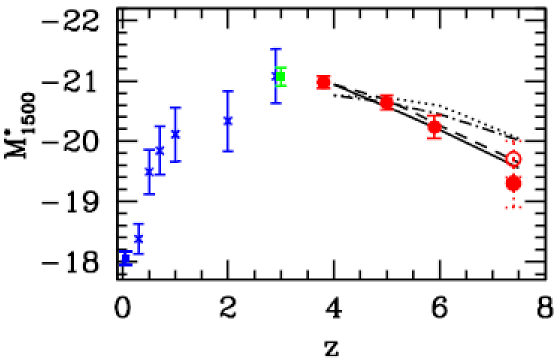


FIG. 13.— Evolution of the characteristic luminosity ($M^*$) of the $UV$ LF as a function of redshift. Determinations are from the present work (red circles) at $z \sim 4-6$, Steidel et al. (1999) at $z \sim 3$ (green square), Arnouts et al. (2005) (blue crosses) at $0.1 \lesssim z \lesssim 3$, and Wyder et al. (2005) at $z \lesssim 0.1$ (blue square).

*Fig. 1. Observational data of the Hubble ultra deep field [1] relating to the Shechter parameter M* (in our consideration it corresponds to characteristic maximum mass)*

In the model of galaxy mergers built on the basis of Smoluchowski kinetic equation (KE), only the processes of the (paired) mergers are taken into account, that is the processes involving three "particles" (App. A, Fig. 2). The resulting solutions (App. B) allow to find the slope of the mass function $f(M,t)$ in a wide range of redshifts [2, 3], satisfactorily explaining the observational data of the Hubble ultra deep field [1] (the evolution of slopes right up to limiting redshifts). However, thus arising "explosive" evolution leads to the maximum mass unlimited growth with approaching the "explosion" time $t=t_{cr}$ [2]. The "explosive" singularity in the solution manifests itself as unlimited MF growth with approaching the maximum mass. Asymptotics of solution *K(M, t)* for the modified MF (mMF, App. B) $F = M^u f$ near the singularity (outside the physical area of the power behavior) has the form (for *u*=2)

$$K(M,t) \to \frac{\beta}{\sqrt{\dfrac{1}{M} - \dfrac{1}{M_{\max}(t)}}},$$

$$M_{\max}(t) = \frac{1}{c(t_{cr}-t)}. \qquad (1)$$

The artefact, associated with use of instant δ-shaped source in the Smoluchowski KE can be avoided by the obvious physical regularization [2, 3], which meaning lies in accounting for a finite rise time of gravitational instability, leading to separation of galaxies from the general expansion of the Universe. Mathematically, this was taken into account by the δ-function blur on the right side of KE and its replacement by a Π-shaped stepping stone with a finite small width Δ. The values of MF remain finite in the region of maximum masses too. However, the very maximum mass in regularized solutions also increases infinitely when approaching the moment of explosion [2 - 4].

As in the other similar tasks with accounting for the three-particle processes (in our case – mergers of galaxies) resulting in the explosive evolution, the final results occur when the four-particle processes appear in the vicinity of singularity and, in our case describe the scattering with transfer of mass. In this case, unlike mergers (see Fig. 2), in which the mass can only increase, a significant role is played by the scattering processes in which the mass of the most massive galaxies can be decrease too (Figs. 3, 4). Below, we consider a simple model scheme describing the disaggregation. In this respect, it is supposed to explain the observed "downsizing" phenomenon (see Fig. 1), where increasing the heaviest characteristic mass over time changes into its decreasing.

## 1. KE WITH SCATTERING

As before in [2], we restrict ourselves by a differential approach, which describes the transfer of a small mass. But now the kinetic equation from the linear is converted into a non-linear (quasi-linear) one, whose most simple form consists in occurrence in the KE of the nonlinear term $-\tilde{\gamma} F \partial F / \partial M$, where the $\tilde{\gamma}$ coefficient denotes the probability of "inelastic" scattering process.

We restrict ourselves initially to a merging probability proportional to the square of the mass $M^2$. In this case of the simplest model, it is natural to choose the same mass dependence for the probability of scattering, too: $\tilde{\gamma} = \gamma M^2$. To do this, there are physical reasons not to be discussed here. By introducing variable $z = M^{-1}$ we rewrite quasilinear term $-\gamma M^2 F \partial F / \partial M$ as $\gamma F \partial F / \partial z$. Though the source in KE is quite substantial, the mentioned asymptotic expression (1) satisfies an homogeneous kinetic equation, to which we will confine ourselves.

Our problem [1] reduces to solution of the differential equation

$$\frac{\partial F}{\partial x} + g(F)\frac{\partial F}{\partial z} = 0\,, \qquad (2)$$

where $g(F) = c + \gamma F$ is linear on mass function *F*. The choice of signs is essential. Equation (2) is a generalized Hopf equation and very well studied. The solution of Cauchy problem of this KE for the mass function *F(M,t)* with the quasi-linear term having a coefficient *g(F)* [5], reduces to cubic equation[2] (*x* – time $t - t_0$, where $t_0 < t_{cr}$ is the moment of sewing together with an explosive solution, playing the role of the initial conditions of the Cauchy problem for the equation (3), $z = M^{-1}$ where *M* is the mass of galaxy):

$$\gamma(t-t_0)F^3 - \left\{[\frac{1}{M} - \frac{1}{M_0}] - c\cdot(t-t_0)\right\}F^2 + \beta^2 = 0\,. \quad (3)$$

Here $\gamma$ is a nonlinearity parameter, $c = C\Pi$ is the parameter entered from a linear KE, namely, *C* being the factor in the probability of mergers of galaxies $CM^2$, $\Pi = \int_0 dM_2 M_2 f(M_2)$ is the total mass of low-mass galaxies, $M_0 \equiv M_{max}(t_0)$ is the maximum mass of galaxies in the linear theory [2] at time $t_0$, $\beta$ is the parameter of the asymptotics of explosive solution (1), which is used as an initial condition for solving KE (2) with the nonlinear term.

At $t = t_0$, the MF $F(M,t)$, as follows from (3), satisfies the initial condition (1)

$$F^2(M,t_0) = \frac{\beta^2}{[\frac{1}{M} - \frac{1}{M_0}]}\,, \qquad (4)$$

(corresponding to the asymptotics of our explosive solution of the linear KE), and the moment $t_9$ was taken close to $t_{01}$, in order to be able to use a simple analytical form of the asymptotics of (1)). With *M*, close to $M_0$ – being the maximum mass of the explosive solution at the moment $t_9$, – it is a large value. (Excluding nonlinearity, it tends to infinity at the time approaching to the moment of explosion $t \to t_{cr}$).

## 2. SOLUTION OF KE WITH SCATTERING

We are interested in a real positive solution of the cubic equation (3) for *F(M,t)* on large times $t >> t_0$ as function of *M* in particular, the behavior of the new nonlinear "maximum mass", which is yet to be determined, and its time dependence.

We restrict ourselves to demonstrating the asymptotic solution of the cubic equation (3) for the mMF at times $t >> t_0$ and masses $M << M_0$. The free term in (3) can be neglected. For $\gamma > 0$ and $c > 0$, there is a unique solution corresponding to positive curly brackets

$$F \approx \frac{\frac{1}{M} - \frac{1}{M_0} - c\cdot(t-t_0)}{\gamma\cdot(t-t_0)} \to \frac{\frac{1}{M} - c\cdot t}{\gamma\cdot t}\,. \qquad (5)$$

And the mass is bounded from above by (vanishing curly brackets (3))

$$M < M_{\max}(t) = \frac{1}{c\cdot t}\,. \qquad (6)$$

It can be seen that the maximum mass decreases with time, thus revealing the required dawnsizing phenomenon. In the resulting solution at $t > t_0$ all the quantities are finite[3]. From the explosive evolution, only local by masses increasing of MF solution near the former peculiarity remained. It will be observed that this fact may be evidence of the explosive stage of evolution.

## 3. DISCUSSION

Thus, a complete solution is a decreasing power function, of the Schechter function kind, which, however, before the recession in large masses begins to increase at times close to the time of "explosion" [2, 3]. The interference of downsizing processes (scattering with maximum mass decrease) leads both to the MF decrease with the growth of mass which passes through the local maximum, and to the maximum mass decrease with time. This is consistent with the observed effect of downsizing.

From the $M^2$ dependence in the probability of mergers we can easily go to any $M^u$ power dependence by replacement $z = M^{1-u}/(u-1)$, i.e. by replacement $M^{-1} \to M^{1-u}/(u-1)$ in the resulting solution (cf. [2]).

We have solved an extremely simplified model problem in which the phenomenon of downsizing appears. Actually, the process of downsizing should be described by integral kinetic equation, when, as a result of galaxies scattering, the galaxies of comparable masses appear.

The author is grateful to Boris Komberg for the debates which inspired the author to discuss the problem, to Alexander Kats for participation in previous joint works on the subject, as well as to Dr. Rychard J. Bouwens and his co-authors for the kind permission to reproduce the Figure from their paper [1].

## 4. APPENDIX A

Following are the schematic drawings for explaining the above considered merger and scattering processes.

[1] We use the notation from the reference book by Zaitsev and Polyanin [5], item 12.4.2.1, point 2 (p. 271).

[2] The latter is easily verified by its direct differentiation on time and mass.

[3] With the exception of infinity introduced by the initial condition. Starting from the regularized solution (if $t < t_0$) we could get the final values. But this leads to a more cumbersome calculation.


*Fig. 2. Merging by the triple processes with mass increasing, leading to the Smoluchowski KE. At low mass transfer, the KE becomes differential [2, 3]. The processes shown in Figs. 3 and 4, subject to the low mass transfer, lead to the considered quasilinear KE (2) describing the downsizing. Through M in all figures, the most massive galaxy mass is denoted*

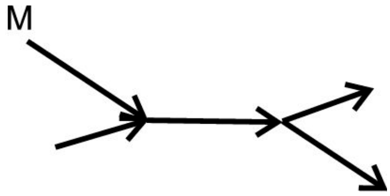


*Fig. 3. Merging with the appearance of unstable intermediate galaxy which immediately disintegrates. (Effective scattering due to the triple process in the second order). Here, the highest mass of galaxies can be decreased thus leading to downscaling*

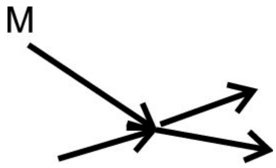


*Fig. 4. The direct scattering by quaternary processes with downsizing of galaxies. In fact, the equation we use corresponds to the mass loss during its transfer. In the absence of losses, the nonlinear equation is of somewhat different type. Note, that accounting for losses in merger process is not essential and never leads to qualitative effects*

## 5. APPENDIX B

Consider the Smoluchowski KE solutions in the differential form supposing that the main contribution is due to mergers of the low-mass galaxies with the massive ones with the corresponding merging probability,

$$U(M_1, M_2) = 0.5CM_1^u \text{ for } M_2 \ll M_1.$$

$$\frac{\partial}{\partial t} f(M,t) + C\Pi \frac{\partial}{\partial M}\left[M^u f(M,t)\right] = \phi(M,t). \quad \text{(B1)}$$

Rewriting Eq. (1) for mMF $F(M,t) = M^u f(M,t)$, as

$$\frac{\partial}{\partial t} F(M,t) + C\Pi M^u \frac{\partial}{\partial M} F(M,t) = \Phi(M,t), \quad \text{(B2)}$$

where the modified source is $\Phi(M,t) = M^u \phi(M,t)$, we restrict ourselves by the localized sourse, that allows finding the solution explicitly [2, 3]. The MF solution has the power-low part which is in good agreement with the observed data. But near the maximum mass, the MF has nonphysical singularities. Regularization [2, 3] leads to the Shechter type MF which has no singularities but the maximum mass tends to infinity when the moment of time goes to the explosion time $t_{cr}$.

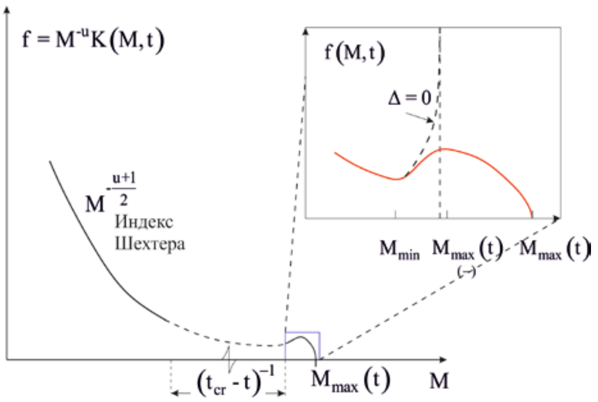


*Fig. 5. The MF established as a result of only mergers (see Fig. 2) with small mass increments. The dashed line shows the MF singularity in case of a δ-function source [2]. The $M_{max}(t) \to \infty$ when $t \to t_{cr}$*

## 6. APPENDIX C

The initial problem solution for equation (2) can be used in the parametrical form [5]

$$z = \xi + G(\xi)(x - x_0),\ G(\xi) = g(F_0(\xi)),\ F = F_0(\xi) \quad \text{(C1)}$$

where $\xi$ is the parameter, $F_0$ is the initial value of mMF: $F_0(z) = F(z, x_0)$ for $x = x_0$, where we have made the change of variables from "*M*" to "*z*" shown in the main text and labeled *t* by $x$. Thus, from (2) we have $F = \beta / \sqrt{\xi - \xi_0}$ and hence:

$$\xi - \xi_0 = \beta^2 / F^2. \quad \text{(C2)}$$

By excluding the parameter $\xi$ from (C1) and (C2) we have the cubic equation (3), using $G(\xi)$ in the form of

$$G(\xi) = C\Pi + \gamma F.$$

## REFERENCES

1. R.J. Bouwens, G.D. Illingworth, M. Franx, and H. Ford. UV luminosity functions at $z = 4.5$ and 6 from the HUDF and other deep HST ACS fields: evolution and star formation history // *Astrophys. J.* 2007, v. 670, p. 928-958.
2. A.V. Kats and V.M. Kontorovich. Merger Driven Explosive Evolution of Distant Galaxies (Minor Mergers) // *Astrophysical Bulletin*. 2013, v. 68, № 3, p. 273-284; astro-ph/1309.0957.
3. A.V. Kats and V.M. Kontorovich. Explosive evolution of galaxies at high redshifts due to minor mergers // *Advances in Astronomy and Space Physics*. 2013, v. 3, p. 131-134.
4. A.V. Kats and V.M. Kontorovich. The use of gravitational lenses in the study of distant galaxy mergers // *Radio Physics and Radio Astronomy*. 2013, v. 18, № 3, p. 220-223.
5. V.F. Zaitsev and A.D. Polyanin. Differential equations with the first order partial derivatives // *Reference book*. Moscow: "FIZMATLIT", 2003, p. 271.

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## РАЗУКРУПНЕНИЕ (DOWNSIZING) С ТОЧКИ ЗРЕНИЯ МОДЕЛИ СЛИЯНИЙ (ПРЕДВАРИТЕЛЬНОЕ ОБСУЖДЕНИЕ)

***В.М. Конторович***

При учете четырехчастичных процессов, описывающих рассеяние с передачей массы, в отличие от слияний, при которых масса может только увеличиваться, существенную роль играют процессы разукрупнения, при которых масса наиболее массивных галактик может уменьшаться. Рассмотрены простейшие модельные схемы, описывающие разукрупнение. На этом пути предполагается дать объяснение наблюдаемому явлению Downsizing, когда с течением космологического времени возрастание характерной наибольшей массы галактик сменяется ее убыванием.

## РОЗУКРУПНЕННЯ (DOWNSIZING) З ТОЧКИ ЗОРУ МОДЕЛІ ЗЛИТТІВ (ПОПЕРЕДНЄ ОБГОВОРЕННЯ)

***В.М. Конторович***

З урахуванням чотиричастинних процесів, що описують розсіювання з передачею маси, на відміну від злиттів, при яких маса може лише збільшуватися, істотну роль відіграють процеси розукрупнення, при яких маса наймасивних галактик може зменшуватися. Розглянуто найпростіші модельні схеми, що описують розукрупнення. На цьому шляху передбачається дати пояснення спостережуваного явища Downsizing, коли з плином космологічного часу зростання характерної найбільшої маси галактик змінюється на її убування.