SKIN-EFFECT INFLUENCE ON TRANSITION RADIATION

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(Received April 3, 2015)

It is considered the transition radiation for the normal incidence of a charged particle on the boundary of plasma medium in the conditions of anomalous skin effect, at the frequencies much less than plasma frequency. The problem is solved in the assumption that electron scattering from the boundary is partially specular and partially diffusive. The spectral density of the radiated energy is obtained for the cases of uniform particle motion and of the motion with two running across the boundary.

PACS: 41.60.-m

1. INTRODUCTION

Whet a particle crosses the plasma boundary the transition radiation arises. Its characteristics are studied for different situations. In particular, the problem was solved with account of spatial dispersion [1]. Also, the characteristics of the transition radiation generated by the particle, which crosses the boundary soon after collision, are studied [2]. With aim to develop generators of transition radiation the characteristics of radiation generated by modulated beams are obtained [3], and experimental investigations of generation of wide-range transition radiation with use of pulsed accelerators of direct action are carried out [4]. Transition radiation as elementary mechanism is the base of operation of some other devices, in particular, monotron [5].

If plasma medium is metal at low temperature, the radiation may be realized in the conditions of anomalous skin effect. Its theory to the considerable extent was built in [6] and [7], where the reflection of an electromagnetic wave from the plasma with sharp boundary is considered, and the scattering of electrons from the boundary is characterized with some proportion between electrons scattered specularly and diffusely. This proportion depends on the angle of electron incidence. In [8], the problem is solved for an arbitrary such dependence, with use of expansion into Neumann series. But considerable amount of results, in particular, of exact ones, were obtained in the assumption that the proportion is constant. In particular, in such assumption the problem of normal wave incidence in maximum anomalous skin effect conditions was solved [9] and the characteristics of longitudinal field penetration into plasma in the near conditions are determined [10]. Also, in [11], the problem of normal incidence, in the assumption that distribution function of the scattered electrons is fixed up to the factor, which describes the type of electron scattering from the boundary, is solved exactly, and in [12], the explicit relationships for plasma layer are obtained.

The main object of the present work is to obtain the amplitude of radiation for normal particle incidence on the locally isotropic plasma with the sharp boundary at the frequencies much greater than collision frequency, but much less than plasma frequency, in presence of considerable spatial dispersion. In the next sections the construction of solving is described. The method, in comparing with one of [13] (where an oblique incidence of an electromagnetic wave on plasma is considered), is somewhat changed, and some designations are introduced in a different way. In the section next to the last, the question of efficiency of generation of wide-range radiation with use of the pulsed accelerators of direct action is discussed.

2. INITIAL RELATIONSHIPS

Let a particle with charge $Z_0 e_0$ moves along OZ axis with velocity $\beta_0 c\vec{e}_z$, where $\vec{e}_z$ is unit vector of OZ axis, c is the speed of light, e_0 is electron charge, $\beta_0 \in (-1,1)$, $\beta_0 \neq 0$, and the plasma medium is in the half-space $z > 0$. Maxwell equations there may be written in the form $\mathrm{rot}\vec{E} + c^{-1}(\partial/\partial t)\vec{H} = 0$, $\mathrm{rot}\vec{H} - c^{-1}(\partial/\partial t)\vec{E} - 4\pi c^{-1}(\vec{j} + \vec{j}_0) = 0$, where $\vec{j}_0 = Z_0 e_0 \delta(x)\delta(y)\delta(z - \beta_0 ct)\beta_0 c\vec{e}_z$, $\vec{j} = e_0 \int d^3\vec{v}\vec{v}f$, the perturbation, $f = f(\vec{v}, \vec{r}, t)$, of electron distribution function obeys the equation $(\partial/\partial t)f + \vec{v}(\partial/\partial\vec{r})f + (e_0/m)\vec{E}(\partial/\partial\vec{v})f_0 + \nu f = 0$, m is electron mass, ν is collision frequency, the unperturbed electron distribution function f_0 is taken for the isotropic Fermi distribution with zero temperature, $f_0 = 3n_0(4\pi v_F^3)^{-1}$ at $v < v_F$, $f_0 = 0$ at $v > v_F$, v_F is electron velocity at Fermi level, n_0 is electron density. Electron flow from the boundary

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into plasma is characterizes by the part, $p \in (0,1)$, of electrons scattered from the boundary specularly (the rest is scattered diffusely) and obeys the boundary condition $f(v_z) = pf(-v_z)$ for $v_z > 0$ at $z = 0$. It is assumed that Fourier transformation with the factor $\exp[i\omega c^{-1}(ct - k_x x - k_y y - k_z z)]$ and the integration over intervals $t \in (-\infty, +\infty)$, $x, y \in (-\infty, +\infty)$, $z \in (0, +\infty)$ is applied. Let us put

$$q_\lambda(\eta) = 3\eta^{-2}\{(2\eta)^{-1}\log[(1+\eta)/(1-\eta)] - 1\},$$

$$q_\tau(\eta) = 3\{1 - (2\eta)^{-1}\left(1-\eta^2\right) \times$$
$$\times \log[(1+\eta)/(1-\eta)]\}/(2\eta^2),$$

$$Q_\lambda(k_z) = 1 - \Omega^2 q_\lambda(\beta(k_\perp^2 + k_z^2)^{1/2}),$$

$$Q_\tau(k_z) = 1 - k_\perp^2 - k_z^2 - \Omega^2 q_\tau(\beta(k_\perp^2 + k_z^2)^{1/2}),$$

$$\Psi_\lambda(k_z) = \omega c^{-1}[k_\perp E_\perp(k_z) + k_z E_z(k_z)],$$

$$\Psi_\tau(k_z) = \omega c^{-1}[k_\perp E_z(k_z) - k_z E_\perp(k_z)],$$

$$\Phi_\lambda(k_z) = k_z I_z(k_z) +$$
$$+ Q_\lambda(k_z)[\Psi_\lambda(k_z) + p\Psi_\lambda(-k_z)], \quad (1)$$

$$\Phi_\tau(k_z) = k_\perp I_z(k_z) +$$
$$+ Q_\tau(k_z)[\Psi_\tau(k_z) - p\Psi_\tau(-k_z)]. \quad (2)$$

Here $k_\perp = |\vec{k}_\perp|$, $\vec{k}_\perp = k_x\vec{e}_x + k_y\vec{e}_y$, $\vec{e}_x$ and $\vec{e}_y$ are unit vectors of the axes OX and OY, $E_\perp$ is projection of Fourier component of electric field strength on the vector $\vec{k}_\perp$ direction, $\beta = v_F\omega[c(\omega + i\nu)]^{-1}$, $\Omega = \omega_e[\omega(\omega+i\nu)]^{-1/2}$, $\omega_e = (4\pi e_0^2 n_0/m)^{1/2}$, $I_z(k_z) = I_{z0}/(k_z - k_{z0})$, $k_{z0} = \beta_0^{-1}$, $I_{z0} = 4\pi\mathrm{sign}(\beta_0)Z_0 e_0/\omega$. The functions $\Psi_\lambda(k_z)$ and $\Psi_\tau(k_z)$ (and the functions $E_\perp(k_z)$ and $E_z(k_z)$) should be analytical in the half-plane $\mathrm{Im}k_z < 0$ and in the point $k_z = -k_{z0}$. The functions $\Phi_\lambda(k_z)$ and $\Phi_\tau(k_z)$ correspond to some linear combinations of the left hand sides of Maxwell equations written above, and they should be analytical in the half-plane $\mathrm{Im}k_z > 0$ and in the point $k_z = k_{z0}$, in connection with validity of the equations in the half-space $z > 0$. The functions $\Psi_\lambda(k_z)$ and $\Psi_\tau(k_z)$ should be bounded in the half-plane $\mathrm{Im}k_z < 0$, and also, the equalities $\Psi(\pm ik_\perp) = 0$ for the function $\Psi(k_z) = k_\perp\Psi_\lambda(k_z) - k_z\Psi_\tau(k_z)$ should be held. With the method similar to one used in [6], [14], and [15] for the problem of wave incidence on the medium, each of the equalities, (1) or (2), together with the requirements of analyticity, is reduced to Riemann-Hilbert boundary problem for the pair of functions, $\{\Phi_{\lambda,\tau}(k_z),\ \Psi_{\lambda,\tau}(-k_z)\}$ and $\{\Phi_{\lambda,\tau}(-k_z),\ \Psi_{\lambda,\tau}(k_z)\}$, analytical in the different half-planes. For the boundary (at $z \to 0+$) values of relevant field components, $\widetilde{E}_z(z)$, $\widetilde{E}_\perp(z)$, and $\widetilde{H}_\varphi(z)$ (the unit vector of φ direction is $\vec{e}_\varphi = [\vec{e}_z, \vec{k}_\perp/k_\perp]$), which were obtained with integration only with respect to t, x and y, and with the factor $\exp[i\omega c^{-1}(ct - k_x x - k_y y)]$, one has the equalities $\widetilde{E}_z(0+) = i\Psi_\lambda(\infty)$, $\widetilde{E}_\perp(0) = -i\Psi_\tau(\infty)$, and $\widetilde{H}_\varphi(0) = \lim_{u\to\infty}\{-iu[\Psi_\tau(u) - \Psi_\tau(\infty)]\}$ (the letter u, and the letter w below, sometimes is used instead of k_z as arguments of functions; the values of $\widetilde{E}_z(0+)$ and $\widetilde{E}_z(0-)$ for $p \neq 1$ may be different, in connection with existence of infinitely thin charge layer varying with time at the sharp plasma boundary [16]). Let us denote $A = (\beta\Omega)^{2/3}$. It is assumed, that $|\beta_0| \sim 1$, $v_F \ll c$, and the frequency is considered, for which $\nu \ll \omega$, $|A| \gg 1$ (so, skin effect is close to maximum anomalous), moreover, the collision frequency ν is considered as infinitesimal, and its positiveness is used only for ascertaining the rules of path tracing around singularities in the complex plane. Also, it is assumed, that $k_\perp \in (0,1)$. The waves with $k_\perp > 1$ are not emitted into the free half-space $z < 0$, as they decreases there exponentially with $z \to -\infty$. The functions $Q_{\lambda,\tau}(k_z)$ have the branching points $\pm q$, where $q = (\beta^{-2} - k_\perp^2)^{1/2}$, $\mathrm{Im}q \to 0+$ with $\nu \to 0+$. Let the functions $Q^+_{\lambda,\tau}(k_z)$ are analytical in the half-plane $\mathrm{Im}k_z > 0$ and near the point q, and the equalities $Q^+_{\lambda,\tau}(k_z)Q^+_{\lambda,\tau}(-k_z) = Q_{\lambda,\tau}(k_z)$, $Q^+_\lambda(\infty) = 1$, and $\lim_{u\to\infty}[Q^+_\tau(u)/u] = 1$ take place, and let $Q^\times_{\lambda,\tau}(k_z) = Q^+_{\lambda,\tau}(k_z)/Q^+_{\lambda,\tau}(-k_z)$. If the cut Γ in the half-plane $\mathrm{Im}k_z > 0$ from the point q to infinity is made then analytical extension of the functions $Q_{\lambda,\tau}(k_z)$ with different path tracing around the point q gives the different values of the functions, so that for the values at the different cut sides the equalities

$$Q_{\lambda,\tau}(k_z(1-i0)) - Q_{\lambda,\tau}(k_z(1+i0)) =$$
$$= -\Omega^2\Delta_{\lambda,\tau}(\beta(k_z^2 + k_\perp^2)^{1/2})$$

with $\Delta_\lambda(\eta) = -3\pi i\eta^{-3}$, $\Delta_\tau(\eta) = 3\pi i(\eta^{-3} - \eta^{-1})/2$ take place. Denoting $X_{\lambda,\tau}(k_z) = \Psi_{\lambda,\tau}(-k_z)Q^+_{\lambda,\tau}(k_z)$, $Y_{\lambda,\tau}(k_z) = \Phi_{\lambda,\tau}(k_z)/Q^+_{\lambda,\tau}(k_z)$, one comes to the equations,

$$X_\lambda(k_z) + pQ^\times_\lambda(k_z)X_\lambda(-k_z) =$$
$$= Y_\lambda(-k_z) + k_z I_z(-k_z)/Q^+_\lambda(-k_z), \quad (3)$$

$$X_\tau(k_z) - pQ^\times_\tau(k_z)X_\tau(-k_z) =$$
$$= Y_\tau(-k_z) - k_\perp I_z(-k_z)/Q^+_\tau(-k_z), \quad (4)$$

and to the requirements of analyticity of the functions $X_{\lambda,\tau}(k_z)$ and $Y_{\lambda,\tau}(k_z)$ in the half-plane $\mathrm{Im}k_z > 0$ and in the points q and k_{z0}, and also, the limiting values of the quantities $X_\lambda(k_z)$, $Y_\lambda(k_z)$, $X_\tau(k_z)/k_z$, and $Y_\tau(k_z)/k_z$ at $k_z \to +i\infty$ should be bounded.

3. THE EQUATIONS FOR LONGITUDINAL FIELD

From the equation (3), representing some terms as sums of the functions analytical in the different half-planes, $\mathrm{Im}k_z > 0$ and $\mathrm{Im}k_z < 0$, and transforming the equation in such a way that each its side is analytical in one of the half-planes and tends there to zero with $k_z \to \infty$, for $\mathrm{Im}k_z > 0$ one can get the equality

$$X_\lambda(k_z) - I_{z0}k_{z0}[(k_z + k_{z0}))Q^+_\lambda(k_{z0})]^{-1} +$$
$$+ (2\pi i)^{-1}p\int dw(w - k_z)^{-1} \times$$
$$\times [Q^\times_\lambda(w)X_\lambda(-w) - X_\lambda(\infty)] - X_\lambda(\infty) = 0. \quad (5)$$

The path of integration in (5) is symmetrical with respect to zero, goes near the real axis in its positive direction, and the points k_z, k_{z0}, and q are to be to left of the path. Replacing w with $-w$ and moving the path to the cut Γ, one can get the equation

$$X_\lambda(k_z) - I_{z0}k_{z0}[(k_z + k_{z0}))Q^+_\lambda(k_{z0})]^{-1} =$$
$$= X_\lambda(\infty) - p\widehat{K}_\lambda[k_z, w; X_\lambda(w)], \quad (6)$$

in which an action of the operator $\widehat{K}$ on a function $f(w)$ is defined with the equalities

$$\widehat{K}_{\lambda,\tau}[u,w;f(w)] = = \int_\Gamma dw(u+w)^{-1}K_{\lambda,\tau}(w)f(w),$$

$$K_{\lambda,\tau}(w) = (2\pi i)^{-1}\Omega^2[Q^+_{\lambda,\tau}(w)]^{-2} \times \times\Delta_{\lambda,\tau}(\beta(w^2+k_\perp^2)^{1/2})$$

(the designations with index τ are used below). Simple manipulations give the equation

$$X_\lambda(k_z)/k_z + I_{z0}[(k_z+k_{z0}))Q^+_\lambda(k_{z0})]^{-1} = = X_\lambda(0)/k_z + p\widehat{K}_\lambda[k_z,w;X_\lambda(w)/w], \quad (7)$$

solution of which may be given as linear combination, $X_\lambda(k_z) = X_\lambda(0)X^r_\lambda(k_z) - I_{z0}[Q^+_\lambda(k_{z0})]^{-1}X^e_\lambda(k_z)$, of the solutions of two equations,

$$X^r_\lambda(k_z)/k_z - k_z^{-1} = p\widehat{K}_\lambda[k_z,w;X^r_\lambda(w)/w], \quad (8)$$

$$X^e_\lambda(k_z)/k_z - (k_z+k_{z0})^{-1} = = p\widehat{K}_\lambda[k_z,w;X^e_\lambda(w)/w]. \quad (9)$$

From the integral equations (8) and (9), the values of $X^{r,e}_\lambda(k_z)$ at Γ may be found, and then these equations may be used as explicit formulae for $X^{r,e}_\lambda(k_z)$ in all complex plane of k_z, except of the cut, symmetrical to Γ with respect to zero.

4. THE EQUATIONS FOR TRANSVERSE FIELD

For the known $\widetilde{E}_\perp(0)$ and $\widetilde{H}_\varphi(0)$, the solution of (4) may be given with the linear combination,

$$X_\tau(k_z) = -iX(p;k_z)\widetilde{H}_\varphi(0) + +I_{z0}k_\perp[Q^+_\tau(k_{z0})]^{-1}X^e_\tau(k_z) +[ik_zX(-p;k_z) - c_\tau\Psi_{\tau 1}X(p;k_z)]\widetilde{E}_\perp(0).$$

Here $\Psi_{\tau 1} = ic_\tau^{-1}\lim_{u\to\infty}[uX(-p;u) - Q^+_\tau(u)]$, $c_\tau = \exp(-i\pi/6)A/\beta$, $X^e_\tau(k_z)$ and $X(\pm p;k_z)$ are the solutions of the functional equations

$$X^e_\tau(k_z) - pQ^\times_\tau(k_z)X^e_\tau(-k_z) = = Y^e_\tau(-k_z) + (k_z+k_{z0})^{-1}$$

and $X(\pm p;k_z) \mp pQ^\times_\tau(k_z)X(\pm p;-k_z) = Y(\pm p;-k_z)$ with the requirements of analyticity of the functions $X^e_\tau(k_z)$, $Y^e_\tau(k_z)$, $X(\pm p;k_z)$, and $Y(\pm p;k_z)$ in the half-plane $\mathrm{Im}k_z > 0$ and in the points q and k_{z0}, and the requirements $X^e_\tau(k_z) \to 0$ and $X(\pm p;k_z) \to 1$ for $k_z \to +i\infty$. The way similar to one used in deducing of the equations (6) and (7) leads to the equations

$$X^e_\tau(k_z) - (k_z+k_{z0})^{-1} = p\widehat{K}_\tau[k_z,w;X^e_\tau(w)],$$

$$X(p;k_z) - 1 = p\widehat{K}_\tau[k_z,w;X(p;w)], \quad (10)$$

$$X(-p;k_z)/k_z - X(-p;0)/k_z = = p\widehat{K}_\tau[k_z,w;X(-p;w)/w]\,. \quad (11)$$

If $1/\beta \ll |w| \ll A/\beta$ then $K_\tau(w) \approx 1/\pi$, so, the kernels in the equations for the following six functions, $X^e_\tau(u/\beta)$, $u^{-1}X(p;ic_\tau/u)$, $u^{-1}X(-p;u/\beta)$, $u^{-1}X^e_\tau(ic_\tau/u)$, $X(p;u/\beta)$, and $X(-p;ic_\tau/u)$, as the functions of u, for $1 \ll |w| \ll A$ are close to $p/[\pi(u+w)]$. The possibility to construct the solution of the equation with the kernel $p/[\pi(u+w)]$ explicitly makes it possible to use the method of semi-inversion. Let us denote $\kappa = \pi^{-1}\mathrm{arcsin}(p)$ and consider the equation

$$X(u) = f(u) + + \sin(\pi\kappa)\int_1^\infty dw[\pi(u+w)]^{-1}X(w). \quad (12)$$

Replacing u and w with $\exp(u)$ and $\exp(w)$, one transforms it to the integral equation on the interval $(0,\infty)$ with the kernel dependent on the difference $u-w$. Solving such equation with Wiener-Hopf method, one gets the equality

$$X(u) = f(u) + \int_1^\infty dwV_\kappa(u,w)f(w), \quad (13)$$

in which

$$V_\kappa(u,w) = \pi^{-1}(uw)^{-1/2}\tan(\pi\kappa) \times \times\{\sinh[\ln(u/w)(1/2+\kappa)]/\sinh[\ln(u/w)] + +\pi^{-1}\tan(\pi\kappa)\sum\nolimits_{m,n=1}^\infty [(-1)^{m+n-1}\times \times\Lambda_{\kappa,m}\Lambda_{\kappa,n}(\sigma_{\kappa,m}+\sigma_{\kappa,n})^{-1}\times \times\exp(-\sigma_{\kappa,m}\ln u - \sigma_{\kappa,n}\ln w)]\}\,,$$

$$\sigma_{\kappa,n} = n - 1/2 + (-1)^n\kappa,\ \Lambda_{\kappa,n} = \Lambda_\kappa(i\sigma_{\kappa,n}),$$

$$\Lambda_\kappa(s) = [1-\sin(\pi\kappa)]^{1/2} \times \times\prod\nolimits_{n=1}^\infty[(1-is/\sigma_{\kappa,n})/(1-is/\sigma_{0,n})].$$

The difference between the kernels of the equations for the mentioned six functions and the kernel $p/[\pi(u+w)]$ for $1 \le |w| \ll A$ depends on w and u approximately as $(u+w)^{-1}w^{-1}$. If after the limit transition $\{A\to\infty,\ \beta\to 0\}$ one considers the integral with this difference as the known function (although it contains the unknown function) and includes it into the function $f(u)$ in the equation of the type (12) then the equality (13) becomes the integral equation, the kernel of which sufficiently quickly decreases with the unbounded increase of variables, and such equation with simple change of variables may be transformed to the integral equation with the bounded kernel on the bounded interval. The functions $X^e_\tau(u/\beta)$, $u^{-1}X(p;ic_\tau/u)$, and $u^{-1}X(-p;u/\beta)$ at $1 \ll |u| \ll A$ depend on u approximately as $u^{\kappa-1}$. In the equations for the functions $u^{-1}X^e_\tau(ic_\tau/u)$, $X(p;u/\beta)$, and $X(-p;ic_\tau/u)$, at $1 \ll |u| \ll A$ free terms are relatively small, and the solutions of these equations are close to ones of relevant homogeneous equations, which depend on u at $1 \ll |u| \ll A$ approximately as $u^{-\kappa}$, in connection with the equality $\pi^{-1}\sin(\pi\kappa)\int_1^\infty dw(u+w)^{-1}P_{-\kappa}(w) = P_{-\kappa}(u)$, where P is Legendre function. After solving of relevant equations, the estimations of $X(-p;u/\beta)/X(-p;0)$ and $X(-p;ic_\tau/u)$ at $u = (i\beta c_\tau)^{1/2}$ give a possibility to calculate the value of $F_a = X(-p;0)[A\exp(i\pi/3)]^\kappa$. In connection with the equality $X(-p;0)X(p;0) = 1$ (deduced briefly in the next paragraph), the equality $X(-p;0)/X(p;0) = F_a^2[A\exp(i\pi/3)]^{-2\kappa}$ is held.

The equality $X(-p;0)X(p;0) = 1$ may be obtained from the equalities (10) and (11), which have the same kernel, $K_0(u,w) = pK_\tau(w)/(u+w)$, and the same resolvent $R(u,w;p)$ defined with the equalities $R(u,w;p) = \sum_{n=0}^\infty[K_n(u,w)p^n]$ and

$K_{n+1}(u,w) = \int_\Gamma dw' K_0(u,w')K_n(w',w)$ [17]. Writing the solutions through the resolvent, for the values of $X(p;0)$ and $X(-p;\infty)/X(-p;0)$, with use of the equality $K_\tau(u)R(u,w;p) = K_\tau(w)R(w,u;p)$, one can get the equality $X(p;0) = X(-p;\infty)/X(-p;0)$ and take into account the condition $X(-p;\infty) = 1$.

As really the type of the electron scattering from the boundary depends on electron incidence angle [8], the question arises how does this type influences on the degrees in the dependences $u^{\kappa-1}$ and $u^{-\kappa}$. Let the dependence $p(\vartheta)$, where ϑ is the angle between normal and electron motion direction, is analytical function in the interval $\vartheta \in (0, \pi/2)$, and for a sufficiently small positive a the equality $\lim_{\vartheta\to\pi/2}\{[p(\vartheta) - p(\pi/2)](\cos\vartheta)^{-a}\} = 0$ takes place. Then in the solving construction through relevant integral equations the nonzero $p(\vartheta) - p(\pi/2)$ leads to the integrals, which may be transformed to ones with the bounded kernels on the bounded intervals. As a result, the value of κ in the mentioned degrees has to correspond to the value of $p(\pi/2)$.

5. THE RADIATION AMPLITUDE

Let us put

$$F_\lambda = \beta^{-1}\lim_{u\to 0}(\partial/\partial u)\log[X^r_\lambda(u)/Q^+_\lambda(u)],$$
$$F_\tau = \beta^{-1}\lim_{u\to 0}(\partial/\partial u)\log[X(-p;u)/Q^+_\tau(u)].$$

At $\{\beta \ll 1,\ A \gg 1\}$, the values of F_a, F_λ, F_τ, and $\Psi_{\tau 1}$ are close to real numbers (dependent on p). In the paper [9], in fact, the relationship $\Psi_{\tau 1} \approx (\pi^2/48)^{1/6}[\sin(\alpha/2)/\sin(\alpha/3)]^2$ is obtained, where $\alpha = \arccos(p)$. The numerical solving of relevant integral equations by the way described above shows that the dependences of the quantities F_a, $F_\lambda(1-p)$, and $F_\tau(1-p)^{1/2}$ on p, accurate to within 1% are close to linear ones, with the values close to 1, 0.714, and 0.277 at $p = 0$ and to 0.85, 1.34, and 0.6 at $p = 1$. Limitedness of the quantities $F_\lambda(1-p)$ and $F_\tau(1-p)^{1/2}$ near $p = 1$ is connected with the analyticity of the function $V_\kappa(u,w)$ as the function of κ near the point $\kappa = 1/2$ and with the possibility to expand the resolvent of the symmetrical continuous bounded kernel of the integral equation on the bounded interval into the series in terms of eigenfunctions with coefficients, which contain in the denominators the differences between the factor at integral and relevant eigenvalue of this factor [17].

If $|k_z| \le 1$ then with use of the conditions $\Psi(\pm ik_\perp) = 0$ one gets

$$C_E\widetilde{E}_\perp(0) - \widetilde{H}_\varphi(0) \approx B_E I_{z0}, \tag{14}$$

where $C_E = ic_\tau\Psi_{\tau 1} + \beta k_\perp^2(F_\lambda - F_\tau)X^2(-p;0)$, $B_E = -\beta\Omega^{-1}k_\perp(F_\lambda - F_\tau)X(-p;0)$. The consideration of field in the half-space $z < 0$, for the given current, $\vec{j}_0 = Z_0 e_0\delta(x)\delta(y)\delta(z - \beta_0 ct)\beta_0 c\vec{e}_z$, gives

$$\widetilde{E}_\perp(0) + w_z\widetilde{H}_\varphi(0) = ik_\perp(w_z - k_{z0})^{-1}I_{z0}, \tag{15}$$

where $w_z = (1 - k_\perp^2)^{1/2}$. For the boundary values, $\widetilde{H}^r_\varphi(0-)$, $\widetilde{E}^r_\perp(0-)$, and $\widetilde{E}^r_z(0-)$, of relevant field components of the wave with wave number $k_z = -w_z$ emitted into the half-space $z < 0$, one has $\widetilde{H}^r_\varphi(0-) = \widetilde{H}_\varphi(0) + ik_\perp(k_{z0}^2 - w_z^2)^{-1}I_{z0}$, $\widetilde{E}^r_\perp(0-) = -w_z\widetilde{H}^r_\varphi(0-)$, $\widetilde{E}^r_z(0-) = -k_\perp\widetilde{H}^r_\varphi(0-)$, and the emitted energy may be given with the integral $\int_0^\infty d\omega\int_0^{\pi/2} d\theta 2\pi\sin\theta W(\omega,\theta)$, where the angle θ is connected with $k_\perp$ through the equality $k_\perp = \sin\theta$, and the function $W(\omega,\theta) = (2\pi)^{-4}c^{-1}\omega^2\cos^2\theta|\widetilde{H}^r_\varphi(0-)|^2$ gives the spectral density of the radiation into a solid angle. From (14) and (15) one can find the boundary values, $\widetilde{E}_\perp(0)$ and $\widetilde{H}_\varphi(0)$, and then get the value of $X_\lambda(0)$ and the functions $X_{\lambda,\tau}(k_z)$, $\Psi_{\lambda,\tau}(k_z)$, and $E_{\perp,z}(k_z)$, through which the field in plasma is described. One can obtain the relationships $|\widetilde{E}_\perp(0)| \ll |\widetilde{H}_\varphi(0)|$, $\widetilde{H}_\varphi(0) \approx ik_\perp[w_z(w_z - k_{z0})]^{-1}I_{z0}$, $\widetilde{E}_\perp(0) \approx \widetilde{H}_\varphi(0)/C_E$, and $C_E \approx ic_\tau\Psi_{\tau 1}$. So, the amplitude of the transition radiation in the given frequency range is close to one, corresponding to the case of ideally conducting medium in the half-space $z > 0$. The difference of the medium from ideally conducting one leads to change of the emitted wave amplitude on relatively small amount, and this change may be estimated with use of nonzero surface impedance, as it was made in [1]. The second summand in the definition of C_E is relatively small, and it corresponds to the small contribution into impedance from the existence of the component normal to the boundary in the field created at the boundary by the particle motion in the half-space $z < 0$ (such summand also appears with solving of the problem of oblique incidence of electromagnetic wave on plasma medium [13], and it is absent in the case of normal incidence). The right hand side of (14) deals with the field created with the particle motion in the half-space $z > 0$.

If $\omega \ll \omega_e$ then impedance is small, and the spectral density of the emitted energy is nearly independent on ω. But if $\omega \gg \omega_e$ then the spectral density is quickly decreases with frequency increase, and the radiated energy is the bounded quantity.

6. THE RADIATION BY THE MOTION WITH SHORT-TERM GOING OUT FROM THE MEDIUM

The difference of the given medium from the ideally conducting one may to have a considerable influence on the radiation amplitude in the case when the field created at the boundary in connection with the particle motion in the half-space $z < 0$ is small. As an example of such situation, it may be considered the case when a particle moving along OZ-axis goes out of the medium at $t = -t_0$, where $t_0 > 0$, and at $t = 0$ it changes the motion direction with opposite one without change of the absolute value of velocity, due to elastic collision. In this case, the consideration of the field in the half-space $z < 0$ gives the equalities

$$\widetilde{E}_\perp(0) + w_z\widetilde{H}_\varphi(0) = -ik_\perp I_{z2}(w_z),$$
$$\widetilde{H}^r_\varphi(0-) = \widetilde{H}_\varphi(0) - 2[w_z(\beta_0^2 w_z^2 - 1)]^{-1}\times$$
$$\times[\sin(\omega t_0\beta_0 w_z) - \beta_0 w_z\sin(\omega t_0)]\beta_0 k_\perp I_{z0},$$

where $\beta_0 > 0$ is assumed, and

$$I_{z2}(k_z) = 2I_{z0}\beta_0(1 - \beta_0^2 k_z^2)^{-1}\times$$
$$\times[\cos(\omega t_0) - \exp(i\omega t_0\beta_0 k_z) + i\beta_0 k_z\sin(\omega t_0)].$$

For the field in the half-space $z > 0$, taking the linear combination, with the coefficients $\mp\exp(\mp i\omega t_0)$, of the solutions of the problems, in which the particle with velocity $\mp\beta_0 c$ crosses the plane $z = 0$ at $t = 0$, one can get in the right hand side of (14) the additional factor $2i\sin(\omega t_0)$.

If $\omega t_0 \ll 1$ and $\pi/2 - \theta \gg \beta/A$ then for the solution, which may be obtained from the approximate equations $C_E\widetilde{E}_\perp(0) - \widetilde{H}_\varphi(0) \approx 2i\omega t_0 B_E I_{z0}$ and $\widetilde{E}_\perp(0) + w_z\widetilde{H}_\varphi(0) \approx ik_\perp\beta_0(\omega t_0)^2 I_{z0}$, one has $|\widetilde{H}^r_\varphi(0-)/\widetilde{H}_\varphi(0) - 1| \ll 1$. If $\beta^3 A^{-5/2-\kappa} \ll \omega t_0 \ll 1$ then the main contribution to radiation gives the particle motion in the half-space $z < 0$ and the relationships $\widetilde{H}^r_\varphi(0-) \approx ik_\perp\beta_0 {w_z}^{-1}(\omega t_0)^2 I_{z0}$ and $|\widetilde{E}_\perp(0)/\widetilde{H}_\varphi(0)| \ll 1$ are held. But if $\omega t_0 \ll \beta^3 A^{-5/2-\kappa}$ then the main part of transition radiation is generated due to the particle motion in the half-space $z > 0$, and there are held the relationships $\widetilde{E}_\perp(0)/\widetilde{H}_\varphi(0) \approx -w_z$ and

$$\widetilde{H}^r_\varphi(0-) \approx 2i\exp[-i\pi(\kappa+1)/3] \times$$
$$\times \beta^3 A^{-5/2-\kappa} k_\perp(F_\lambda - F_\tau)F_a(\Psi_{\tau 1} w_z)^{-1}\omega t_0 I_{z0}.$$

7. EFFICIENCY OF ENERGY TRANSFORMATION

If the particle transits from the free half-space into the plasma then before the transition the particle is attracted to the plasma medium polarized by the particle (in the case of ideally-conducting medium the attraction corresponds to interaction with mirror image having opposite sign of charge). So, for such motion direction, the transition radiation is accompanied with particle acceleration and increase of its kinetic energy, and the energy source for the acceleration and radiation is the potential energy of interaction of the free particle with the plasma medium polarized by it (and in the case of ideally conducting medium the radiated energy is formally infinite). But to become free the particle has to come out of some medium or accelerator. Such going out is also accompanied with transition radiation, and the particle decelerates, attracts to the medium polarized by it and gives its kinetic energy to increase of potential energy and to the generation of radiation. For equal velocities of transition through the boundary the radiation amplitudes in the cases of going in and going out at low frequencies (when $\omega \ll \omega_e$) are approximately equal. But the efficiency of energy transformation may be considerable in the case when the particle losses the considerable part of its kinetic energy during deceleration and this energy goes mainly on radiation. If the particle bunch goes out of accelerator exit device ended with antenna then for considerable bunch deceleration during its interaction with its, conditionally saying, mirror image in antenna the charge of the bunch has to be considerably large (and for such charge, in particular, the force, pushing the bunch apart, is greater than the force, decelerating the bunch). To be the bunch comparatively compact during its going out of the pulsed direct action accelerator (for generation of wide-range radiation) it is necessary the accelerating field sufficiently homogeneous with respect to the bunch dimension, and for its forming the electrode dimension has to be greater than the bunch dimension, and the field strength has to be greater than one pushing the bunch apart, and so, the charge at the electrode has to be greater than the bunch charge. In such a case, the radiation generated during the quick displacement of such charge to the electrode is considerably more powerful and not lesser wide-range than one generated during deceleration of the accelerated bunch in the antenna region.

So, using only the charge displacement in the conductors, which assemble the power supply system of the pulsed direct action accelerator, without beam in vacuum, it may be generated much more powerful wide-range radiation than one, which may be generated by beam with antenna.

8. CONCLUSIONS

So, it is presented the solving construction of the problem of transition radiation for the case of normal incidence of the particle on the locally isotropic plasma with the sharp boundary and the mixed type of electron scattering from it at the frequencies much greater than collision frequency, but much less than plasma frequency. When the particle motion is uniform, the radiation amplitude is close to one, which may be obtained in the case of crossing the boundary of ideally conducting medium. But if the particle goes out of the medium on the short time then the difference of the medium from ideally conducting one may lead to considerable difference in the radiation amplitude.

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ВЛИЯНИЕ СКИН-ЭФФЕКТА НА ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ

В. И. Мирошниченко , ***В. Н. Остроушко***

Рассмотрено переходное излучение для нормального падения заряженной частицы на границу плазменной среды в условиях аномального скин-эффекта, на частотах, значительно меньших, чем плазменная. Задача решена в предположении, что отражение электронов от границы является частично зеркальным, частично диффузным. Получена спектральная плотность излученной энергии для случаев равномерного движения частицы и движения с двукратным пересечением границы.

ВПЛИВ СКІН-ЕФЕКТУ НА ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ

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Розглянуто перехідне випромінювання для нормального падіння зарядженої частинки на межу плазмового середовища в умовах аномального скін-ефекту, на частотах, значно менших від плазмової. Задачу розв'язано в припущенні, що відбиття електронів від межі є частково дзеркальним, частково дифузним. Отримано спектральну густину випроміненої енергії для випадків рівномірного руху частинки та руху з дворазовим перетинанням межі.