THERMONUCLEAR FUSION. COLLECTIVE PROCESSES

CYCLOTRON WAVE ABSORPTION IN LARGE ASPECT RATIO ELONGATED TOKAMAKS

N.I. Grishanov[1,2], N.A. Azarenkov[1]
[1]V.N. Karazin Kharkov National University, Kharkov, Ukraine;
[2]Ukrainian State Academy of Railway Transport, Kharkov, Ukraine

Transverse dielectric susceptibility elements are derived for radio frequency waves in a large aspect ratio toroidal plasma with elliptic magnetic surfaces by solving the Vlasov equation for untrapped, *t*-trapped and *d*-trapped particles. These dielectric characteristics are suitable for estimating the wave absorption by the fundamental cyclotron resonance damping in the frequency range of ion-cyclotron and electron cyclotron resonances.

PACS: 52.55.Fa, 52.50.Qt

INTRODUCTION

Tokamaks represent a promising route to controlled thermonuclear fusion. In order to achieve the fusion conditions in these devices an additional plasma heating must be employed. Effective schemes of heating and current drive in tokamak plasmas can be realised by the wave dissipation in the frequency range of ion-cyclotron (ICR) and/or electron-cyclotron (ECR) resonances. As is known, kinetic wave theory of high-temperature plasmas should be based on the solution of Vlasov-Maxwell's equations. However, this problem is not simple even in the scope of linear theory since to solve the wave equations it is necessary to use the suitable dielectric tensor valid in the given frequency range for a realistic plasma model. In this paper the transverse susceptibility elements are derived for radio frequency waves in a two-dimensional (2D) axisymmetric large aspect ratio tokamak with elliptic magnetic surfaces using an approach developed in Refs. [1, 2].

1. REDUCED VLASOV EQUATION

To describe a 2D axisymmetric tokamak with elliptic magnetic surfaces we use the quasi-toroidal coordinates (r,θ,ϕ) connected with cylindrical ones (ρ,ϕ,z) as $\rho = R_0 + r\cos\theta$, $z = -(b/a)r\sin\theta$, $\phi=\phi$. Here R_0 is the large torus radius, r is the small plasma radius, θ is the poloidal angle, ϕ is the toroidal angle; b and a – large and small semiaxis of the external elliptic tokamak cross-section. In this case, the stationary magnetic field components, $\mathbf{H}_0 = \{H_{0\rho}, H_{0\theta}, H_{0\phi}\}$, are

$$H_{0\phi}(r,\theta) = \frac{H_{\phi 0}R_0}{R_0 + r\cos\theta}, \quad H_{0\rho}(r,\theta) = \frac{H_{\theta 0}R_0\sin\theta}{R_0 + r\cos\theta},$$

$$H_{0z}(r,\theta) = \frac{b}{a}\frac{H_{\theta 0}R_0\cos\theta}{R_0 + r\cos\theta}.$$

To evaluate the transverse susceptibility elements for waves in such plasma we should resolve the Vlasov equation for the first, $l=\pm1$, harmonics of the perturbed distribution functions of ions and electrons:

$$f(t,\mathbf{r},\mathbf{v}) = \sum_s^{\pm 1}\sum_l^{\pm\infty} f_l^s(r,\vartheta,v,\mu)\exp(-i\omega t + in\phi - il\sigma),$$

using the coordinates (r,ϑ,ϕ) with the "straight" magnetic field lines and new variables in velocity space

$$v^2 = v_\parallel^2 + v_\perp^2, \qquad \mu = \frac{v_\perp^2}{v_\parallel^2 + v_\perp^2}\frac{1}{g(r,\vartheta)},$$

where $\vartheta = 2\,\mathrm{arctg}\left[\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\,\mathrm{tg}\left(\frac{\theta}{2}\right)\right]$, $\varepsilon = \frac{r}{R_0}$,

$$g(r,\vartheta) = \frac{H_0(r,\vartheta)}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}} = 1 - \varepsilon\cos\vartheta + \frac{\lambda}{2}\cos^2\vartheta,$$

$$\lambda = h_\theta^2\left(\frac{b^2}{a^2} - 1\right), \quad h_\theta = \frac{H_{\theta 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}, \quad h_\phi = \frac{H_{\phi 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}.$$

Here the index of particle species is omitted; by $s=\pm1$ for f_l^s we distinguish the particles with positive and negative parallel velocities, $v_\parallel = sv\sqrt{1-\mu\cdot g(r,\vartheta)}$, respectively to $\mathbf{H}_0$. In this case, the Vlasov equation for $f_{\pm1}^s$ harmonics can be rewritten as

$$\sqrt{1-\mu g(r,\vartheta)}\frac{\partial f_l^s}{\partial\vartheta} - inq\sqrt{1-\mu g(r,\vartheta)} f_l^s +$$

$$+ i\frac{sr}{h_\theta v}\left[\omega - l\Omega_{c0}g(r,\vartheta)\right]f_l^s + il\sqrt{1-\mu g(r,\vartheta)}\gamma(\vartheta)f_l^s =$$

$$= -\frac{serF_0}{Mh_\theta v_T^2}\sqrt{\mu g(r,\vartheta)}E_l, \qquad l=\pm1. \tag{1}$$

Here $F_0 = \frac{N_0}{\pi^{1.5}v_T^3}\exp\left(-\frac{v^2}{v_T^2}\right)$, $v_T^2 = \frac{2T_0}{M}$, $q = \varepsilon\frac{h_\phi}{h_\theta}$,

$$\gamma(\vartheta) = \frac{3\frac{a}{b}}{\cos^2\vartheta + \frac{b^2}{a^2}\sin^2\vartheta} - 3\frac{b}{a}\varepsilon\cos\vartheta(1-\varepsilon\cos\vartheta) +$$

$$+ \frac{\frac{b}{a}\left(1-\frac{a^2}{b_2}\right)^2\cos^2\vartheta\sin^2\vartheta}{\cos^2\vartheta + \frac{b^2}{a^2}\sin^2\vartheta} + \frac{b}{a}\left(1 - \frac{r}{q}\frac{dq}{dr}\right)\left(\cos^2\vartheta + \frac{b^2}{a^2}\sin^2\vartheta\right).$$

$E_l = E_n - ilE_b$ is the combination of the normal and binormal (respectively to $\mathbf{H}_0$) electric field projections; equilibrium distribution function F_0 is maxwellian with the particle density N_0, temperature T_0, charge e, mass M. Describing the wave-particle interaction in elongated tokamaks we should separate all particles (in the general case if $\lambda > \varepsilon$) on the three groups of untrapped, t-trapped and d-trapped particles. Such separation can be done in dependence of μ and ϑ by inequalities:

$0 \le \mu \le \mu_u \quad -\pi \le \vartheta \le \pi$ – untrapped particles,

$\mu_u \le \mu \le \mu_t \quad -\theta_t \le \vartheta \le \theta_t$ – t-trapped particles,

$\mu_t \le \mu \le \mu_d \quad -\theta_t \le \vartheta \le -\theta_d$ – d-trapped particles,

$\mu_t \le \mu \le \mu_d \quad \theta_d \le \vartheta \le \theta_t$ – d-trapped particles,

analyzing the condition $v_{\|}(\mu,\vartheta)=0$. Here $\mu_u = 1-\varepsilon-\frac{\lambda}{2}$, $\mu_t = 1+\varepsilon-\frac{\lambda}{2}$, $\mu_d = 1+\frac{\varepsilon^2}{2\lambda}$, and the stop points $\pm\theta_t$ and $\pm\theta_d$ for t- and d-trapped particles on the considered magnetic surface are

$$\pm\theta_t = \pm\arccos\left\{\frac{\varepsilon}{\lambda}\left[1-\sqrt{1+\frac{2\lambda(1-\mu)}{\varepsilon^2\mu}}\right]\right\},$$

$$\pm\theta_d = \pm\arccos\left\{\frac{\varepsilon}{\lambda}\left[1+\sqrt{1+\frac{2\lambda(1-\mu)}{\varepsilon^2\mu}}\right]\right\}.$$

To find the perturbed distribution functions of the untrapped $f^s_{l,u}$, t-trapped $f^s_{l,t}$ and d-trapped $f^s_{l,d}$ particles we should resolve Eq, (1) using the corresponding boundary conditions: the periodicity of $f^s_{l,u}$ on ϑ, and continuity of $f^s_{l,t}$ and $f^s_{l,d}$ at the stop points $\pm\theta_t$ and $\pm\theta_d$, respectively. Moreover, we use the new variables instead of poloidal angle ϑ by the first kind elliptic integrals:

$$w(\vartheta) = \int\limits_0^{\mathrm{arctg}\left(\sqrt{1+\beta}\,\mathrm{tg}\frac{\vartheta}{2}\right)} \frac{d\eta}{\sqrt{1-\kappa^2\sin^2\eta}}$$

– for untrapped particles;

$$w(\vartheta) = \int\limits_0^{\arcsin\left(\kappa\sqrt{\frac{(1+\beta)\sin^2\frac{\vartheta}{2}}{1+\beta\sin^2\frac{\vartheta}{2}}}\right)} \frac{d\eta}{\sqrt{1-\sin^2\eta/\kappa^2}}$$

– for t-trapped particles;

$$w(\vartheta) = \int\limits_0^{\mathrm{arctg}\sqrt{\frac{(1-\cos\theta_t)(\cos\theta_d-\cos\vartheta)}{(1-\cos\theta_d)(\cos\vartheta-\cos\theta_t)}}} \frac{d\eta}{\sqrt{1-\kappa^2\sin^2\eta}}$$

– for d-trapped particles. Here

$$\kappa^2 = \frac{2\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}}{1-\left(1-\frac{\lambda}{2}\right)\mu+\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}},$$

$$\beta = \frac{2\lambda}{\varepsilon-\lambda+\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}}.$$

In this case, the transit-time of untrapped particles and the bounce-periods of t-trapped and d-trapped particles are proportional to T_u, T_t and T_d, respectively:

$$T_u = T_d = \frac{2\sqrt{2}\kappa K(\kappa)}{\left(\varepsilon^2\mu^2+2\lambda\mu(1-\mu)\right)^{0.25}},$$

$$T_t = \frac{4\sqrt{2}K(1/\kappa)}{\left(\varepsilon^2\mu^2+2\lambda\mu(1-\mu)\right)^{0.25}},$$

where $K(\kappa) = \int\limits_0^{\pi/2}\frac{d\eta}{\sqrt{1-\kappa^2\sin^2\eta}}$.

2. TRANSVERSE SUSCEPTIBILITY

Knowing $f^s_{l,u}$, $f^s_{l,t}$ and $f^s_{l,d}$, we can calculate the contribution of u-, t- and d-particles to the 2D transverse current density components by

$$j_l(r,\vartheta) = \frac{\pi e}{2} g(r,\vartheta)^{3/2}\sum_s^{\pm 1}\int_0^\infty v^3 dv \times$$

$$\left\{\int\limits_0^{\mu_u}\frac{f^s_{l,u}\sqrt{\mu}\,d\mu}{\sqrt{1-\mu g(r,\vartheta)}} + \int\limits_{\mu_u}^{\mu_t}\frac{f^s_{l,t}\sqrt{\mu}\,d\mu}{\sqrt{1-\mu g(r,\vartheta)}} + \int\limits_{\mu_t}^{\mu_d}\frac{f^s_{l,d}\sqrt{\mu}\,d\mu}{\sqrt{1-\mu g(r,\vartheta)}}\right\}.$$

To evaluate the transverse susceptibility elements we use the Fourier-expansions of the perturbed current density and electric field components on angle ϑ:

$$\frac{j_l(\vartheta)}{g^{3/2}(r,\vartheta)} = \sum_m^{\pm\infty} j_l^{(m)}\exp(im\vartheta),$$

$$g^{1/2}(r,\vartheta)E_l(\theta) = \sum_{m'}^{\pm\infty} E_l^{(m')}\exp(im'\vartheta).$$

As a result, the m-th harmonic $j_l^{(m)}$ of the transverse current density can be calculated by

$$\frac{4\pi i}{\omega} j_l^{(m)} = \sum_m^{\pm\infty}\chi_l^{m,m'}E_l^{(m')} = \sum_{m'}^{\pm\infty}(\chi_{l,u}^{m,m'}+\chi_{l,t}^{m,m'}+\chi_{l,d}^{m,m'})E_l^{(m')}$$

Here $\chi_{l,u}^{m,m'}$, $\chi_{l,t}^{m,m'}$ and $\chi_{l,d}^{m,m'}$ denote the independent contribution of the untrapped, t-trapped and d-trapped particles of any kind (electrons or ions) to the transverse susceptibility elements $\chi_l^{m,m'}$:

$$\chi_{l,u}^{m,m'} = \frac{\omega_p^2 r}{4\pi^{2.5}\omega h_\theta v_T}\sum_{p=-\infty}^{\infty}\int\limits_0^{\mu_u}\frac{\mu\,\kappa^2 d\mu}{\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}}\times$$

$$\times\int\limits_{-\infty}^{+\infty}\frac{u^4 e^{-u^2}A_{p,l}^{m'}(u,\mu)A_{p,l}^{m}(u,\mu)}{\frac{rT_u}{2\pi v_T h_\theta}\left(l\Omega_{c0}\bar{g}_u-\omega\right)-\left[p-nq_t+l\frac{I(\pi)}{\pi}\right]u}du,$$

$$\chi_{l,t}^{m,m'} = \frac{\omega_p^2 r}{4\pi^{2.5}\omega h_\theta v_T}\sum_{p=1}^{\infty}\int\limits_{\mu_u}^{\mu_t}\frac{\mu\,d\mu}{\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}}\times$$

$$\times\int\limits_{-\infty}^{+\infty}\frac{u^4 e^{-u^2}B_{p,l}^{m'}(u,\mu)B_{p,l}^{m}(u,\mu)}{\frac{rT_t}{2\pi v_T h_\theta}\left(l\Omega_{c0}\bar{g}_t-\omega\right)-pu}du, \tag{2}$$

$$\chi_{l,d}^{m,m'} = \frac{\omega_p^2 r}{2\pi^{2.5}\omega h_\theta v_T}\sum_{p=1}^{\infty}\int\limits_{\mu_t}^{\mu_d}\frac{\mu\,\kappa^2 d\mu}{\sqrt{\varepsilon^2\mu^2+2\lambda\mu(1-\mu)}}\times$$

$$\times\int\limits_{-\infty}^{+\infty}\frac{u^4 e^{-u^2}D_{p,l}^{m'}(u,\mu)D_{p,l}^{m}(u,\mu)}{\frac{rT_d}{2\pi v_T h_\theta}\left(l\Omega_{c0}\bar{g}_d-\omega\right)-pu}du.$$

Here $A_{p,l}^m(u,\mu) = \int\limits_{-K(\kappa)}^{K(\kappa)}\exp\left(i\Phi_{l,u}^{p,m}(u,\mu,w)\right)dw$,

$$B_{p,l}^m(u,\mu) = \int\limits_{-2K(1/\kappa)}^{2K(1/\kappa)}\exp\left(i\Phi_{l,t}^{p,m}(u,\mu,w)\right)dw,$$

$$D_{p,l}^m(u,\mu) = \int\limits_{-K(\kappa)}^{K(\kappa)}\exp\left(i\Phi_{l,d}^{p,m}(u,\mu,w)\right)dw,$$

$$\Phi_{l,u}^{p,m}(u,\mu,w) = (p-nq)\frac{\pi w}{K(\kappa)}+(nq-m)\vartheta_u(w)-$$

$$-l\left[I(\vartheta_u(w))-\frac{w}{K(\kappa)}I(\pi)\right]+$$

$$+\frac{lr\Omega_{c0}\sqrt{2}\kappa/(uv_T h_\theta)}{\left[\varepsilon^2\mu^2+2\lambda\mu(1-\mu)\right]^{\frac{1}{4}}}\left[\int_0^w g_u(\tau)d\tau-\frac{w}{K(\kappa)}\int_0^{K(\kappa)} g_u(w)dw\right],$$

$$\Phi_{l,t}^{p,m}(u,\mu,w)=\frac{p\pi w}{K(\kappa^{-1})}+(nq-m)\vartheta_t(w)-lI(\vartheta_t(w))+$$

$$+\frac{lr\Omega_{c0}\sqrt{2}/(uv_T h_\theta)}{\left[\varepsilon^2\mu^2+2\lambda\mu(1-\mu)\right]^{\frac{1}{4}}}\left[\int_0^w g_t(\tau)d\tau-\frac{w}{K(1/\kappa)}\int_0^{K(1/\kappa)} g_t(w)dw\right],$$

$$\Phi_{l,d}^{p,m}(u,\mu,w)=\frac{p\pi w}{K(\kappa)}+(nq-m)\vartheta_d(w)-lI(\vartheta_d(w))+$$

$$+\frac{lr\Omega_{c0}\sqrt{2}\kappa/(uv_T h_\theta)}{\left[\varepsilon^2\mu^2+2\lambda\mu(1-\mu)\right]^{\frac{1}{4}}}\left[\int_0^w g_d(\tau)d\tau-\frac{w}{K(\kappa)}\int_0^{K(\kappa)} g_d(w)dw\right],$$

$$\bar{g}_u=\frac{1}{K(\kappa)}\int_0^{K(\kappa)} g_u(w)dw,\qquad I(\vartheta)=\int_0^{\vartheta}\gamma(\eta)d\eta,$$

$$\bar{g}_t=\frac{1}{K(1/\kappa)}\int_0^{K(1/\kappa)} g_t(w)dw,\qquad \bar{g}_d=\frac{1}{K(\kappa)}\int_0^{K(\kappa)} g_d(w)dw,$$

$$g_u(w)=g(r,\vartheta_u(w)),\quad g_t(w)=g(r,\vartheta_t(w)),$$

$$g_d(w)=g(r,\vartheta_d(w)),$$

$$\delta=\frac{\lambda-\varepsilon-\sqrt{\varepsilon^2+2\lambda(1-\mu)/\mu}}{\lambda+\varepsilon+\sqrt{\varepsilon^2+2\lambda(1-\mu)/\mu}},$$

$$\vartheta_u(w)=2\,\mathrm{arctg}\left(\frac{\mathrm{sn}(w,\kappa)}{\sqrt{1+\beta}\,\mathrm{cn}(w,\kappa)}\right),$$

$$\vartheta_t(w)=2\arcsin\left(\frac{\kappa^{-1}\,\mathrm{sn}(w,\kappa^{-1})}{\sqrt{1+\beta\mathrm{dn}^2(w,\kappa^{-1})}}\right),$$

$$\vartheta_d(w)=\arccos\left(\frac{d\,\mathrm{n}^2(w,\kappa)-\delta}{d\,\mathrm{n}^2(w,\kappa)+\delta}\right).$$

CONCLUSIONS

In conclusion, let us summarized the main results of the paper. As is well known [3], the collisionless wave dissipation in the frequency range of ICR and ECR can be realized under the conditions if the plasma particles interact effectively with transverse electric field components, $E_n \pm iE_b$. The specific features of the wave-particle interaction in tokomak geometry are due to that a) the resonance conditions for untrapped, *t*- and *d*-trapped particles are different and b) all the harmonics of $E_{\pm1}=E_n \pm iE_b$ contribute into the *m*-th harmonic of the transverse current density component, $j_{\pm1}^{(m)}$. The absorbed wave power under the HF plasma heating on the fundamental cyclotron harmonic, $P_{C,l}=0.5\,\mathrm{Re}\left(E_l\, j_{(l)}{}^{*}\right)$, can be estimated by the expression

$$Ð_{C,l}=\frac{\omega}{8\pi}\sum_m^{\pm\infty}\sum_{m'}^{\pm\infty}\left(\mathrm{Im}\,\chi_{l,u}^{m,m'}+\mathrm{Im}\,\chi_{l,t}^{m,m'}+\mathrm{Im}\,\chi_{l,d}^{m,m'}\right)\times$$
$$\times\left[\mathrm{Re}\,E_l^{(m)}\,\mathrm{Re}\,E_l^{(m')}+\mathrm{Im}\,E_l^{(m)}\,\mathrm{Im}\,E_l^{(m')}\right].$$

As was mentioned above, $l=1$ corresponds to wave power absorbed under the ICR plasma heating, when $\omega\sim\Omega_{c,i}$ and the left-hand polarized waves E_n+iE_b interact effectively with the resonant ions. The case $l=-1$ should be considered under the ECR plasma heating when $\omega\sim|\Omega_{c,e}|$ and the right-hand polarized waves E_n-iE_b interact with the electrons. Contribution of untrapped, *t*-trapped and *d*-trapped particles to the imaginary parts of the transverse susceptibility elements, $\mathrm{Im}\,\chi_{l,u}^{m,m'}$, $\mathrm{Im}\,\chi_{l,t}^{m,m'}$ and $\mathrm{Im}\,\chi_{l,d}^{m,m'}$, can be estimated by Eqs. (2) using the well known Landau residues method.

REFERENCES

1. F.M. Nekrasov et al. The perpendicular permittivity of magnetized toroidal plasmas with elliptic magnetic surfaces // *Czech. J. Phys.* 1997, v. 47, № 8, p. 801-819.
2. N.I. Grishanov et al. Longitudinal permittivity of a tokamak plasma with elliptic and circular magnetic surfaces // *Phys. Plasmas*. 1998, v. 5, № 3, p. 705-715.
3. T.D. Kaladze, A.I. Snout, K.N. Stepanov. On the cyclotron damping of fast magnetosonic waves in a tokamak // *Ukr. Nat. J.* 1983, v. 28, p. 995-1005.

Article received 25.03.2013.

ПОГЛОЩЕНИЕ ЦИКЛОТРОННЫХ ВОЛН В ВЫТЯНУТЫХ ТОКАМАКАХ С БОЛЬШИМ АСПЕКТНЫМ ОТНОШЕНИЕМ

Н.И. Гришанов, Н.А. Азаренков

Поперечные элементы диэлектрической восприимчивости получены для радиочастотных волн в тороидальной плазме с большим аспектным отношением и эллиптическим сечением магнитных поверхностей при решении уравнений Власова для пролетных, *t*-запертых и *d*-запертых частиц. Эти диэлектрические характеристики применимы для оценки циклотронного поглощения электромагнитных волн (например, во время нагрева плазмы) в диапазоне частот ионно-циклотронного или электронно-циклотронного резонансов.

ПОГЛИНАННЯ ЦИКЛОТРОННИХ ХВИЛЬ У ВИТЯГНУТИХ ТОКАМАКАХ З ВЕЛИКИМ АСПЕКТНИМ ВІДНОШЕННЯМ

М.І. Гришанов, М.О. Азарєнков

Поперечні елементи діелектричної сприйнятливості отримані для радіочастотних хвиль у тороїдальній плазмі з великим аспектним відношенням та еліптичним перерізом магнітних поверхонь через розв'язок рівнянь Власова для пролітних, *t*-запертих та *d*-запертих частинок. Ці діелектричні характеристики застосовані для оцінки циклотронного поглинання електромагнітних хвиль (наприклад, під час нагріву плазми) у діапазоні частот іонно-циклотронного або електронно-циклотронного резонансів.