# SPECIFICS OF DEVELOPMENT OF INSTABILITY IN A BEAM WITH A CURRENT BELOW BURSIAN LIMIT

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A principal possibility of excitation is specified for Bursian instability in flat drift space at beam current which is lower than limited by space charge at the account of initiation of charge accumulation mechanism in the system excitable by wave of density. Times of development of such instability are much greater than electron transit time through the drift space.

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## INTRODUCTION

It is known that at injection into diode space of current higher than some critical value (limited by space charge or so called above-critical current), the virtual cathode oscillating in space and time appears in transit time space [1], reflecting part of electrons back to plane of injection. In case of absence of electrons space charge compensation by ionic background, such effect called Bursian instability [2]. Significant, that state with virtual cathode has hysteresis. Virtual cathode appears [3] at values of Pierce parameter square $\alpha^2 > \alpha_{c1}^2 = \omega_p^2 L^2 / v_0^2 = 16/9$, and disappears at its decreasing to values $\alpha^2 < \alpha_{c2}^2 = 8/9$. Here $\omega_p$ – plasma frequency, $L$ – length of system and $v_0$ – initial velocity of electrons. These ratios are correct for wide non-relativistic mono-energetic beam. As the square of Pierce parameter is proportional to current in diode $\alpha^2 \sim I$, virtual cathode exists after first appearance till double decreasing of current through the diode.

This raises the question of possibility of occurrence of virtual cathode at current of injection lower than limited by space charge. Principally, it is possible at presence of some kind of excess charge accumulation mechanism in system. The influence of thermal straggling of electrons velocities in beam on development of instability at limited by space charge currents is considered in [4]. Therewith it was noticed that development of preliminary slow stage of instability was connected with the growth of spatial scale and amplitude of fluctuations of charge density. It is precisely this fact that explains small scales of fluctuations determine possibility of development of instability at currents lower but extremely close to critical. It is could be supposed that more efficient accumulation of charge in system is possible at presence of sufficiently large-scale fluctuations. In this case threshold of development of instability by current should be also lowered.

## SIMULATION MODEL AND RESULTS

The goal of this work is an investigation by the method of numerical simulation of Bursian instability initiation possibility in wide high-current electron beam at current lower than limited by space charge.

In capacity of simulation object the area of drift limited in direction of beam spreading by grounded planes which are transparent for electrons is considered. The drift area in transverse direction is not limited. Electrical field is self-consistent and potential. The constant injection of electrons is performed to the system with average energy of ≈20 keV. Initial pulses of particles are follow to Gaussian distribution. The magnitude of relative straggling of pulses $\Delta p / p_0$ were examined over the range 0 to $1 \cdot 10^{-2}$, where $p_0$ – average value of initial pulses of particles, $\Delta p$ – thermal straggling of pulses.

System is described by equations for self-consistent potential $\varphi$ and electric-field *E:*

$$\Delta\varphi = -4\pi\rho, \qquad \varphi\big|_{\Gamma} = 0,$$

$$E = -grad\,\varphi,$$

and also equations of electrons motion:

$$\frac{dp}{dt} = qE, \qquad p\big|_{x=0} = f(p_0),$$

where $f(p_0)$ – statistical function of distribution by pulses, $p$ – pulse of electron and $q$ – charge. Equation for coordinates $x$ particles with allowance made for relativistic correction be of the form:

$$\frac{dx}{dt} = \frac{pc}{\sqrt{p^2 + m^2 c^2}},$$

where $m$ – mass of particle, $c$ – velocity of light.

Solution was performed by the way of simulation by particle-in-cell method (PIC). The calculation model is described in details in [5].

Fixing density of input current for diode space below level corresponding to Pierce parameter of $\alpha < 4/3$, but enough close to this value, ascertain, system is in stable condition during unrestricted time.

Then, single wave of density with shape, like represented in Fig. 1 was launched for initiation of instability in electron beam (density of particles $n_e$ is normalized by averaged value for non-excitable beam). Dimension of wave was from 0.15 to 0.2 of the overall system length. Altogether such wave does not bring into diode excessive charge but essentially disturbs electron flow.

On Fig. 3,a,b it is visible, that during passage of the diode gap $\tau$ it essentially changes the shape and is diffused, leaving behind large-scale perturbations of density.

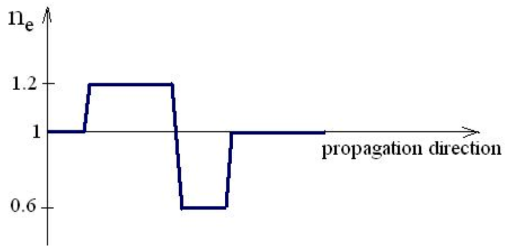


*Fig. 1. Charge density wave shape*

What is more significant is that during this time excess of electrons is accumulated in system. This section of graph in Fig. 2 is a part of curve corresponding to time interval from $20\cdot\tau$ to $21\cdot\tau$. On insets of Fig. 3 a,b the pictures of phase space of electron ensemble are represented. It is obvious that there are "delayed" electrons in area $x > 0.5\cdot L$. Exactly they are responsible for accumulation of charge during passing wave of density through diode.

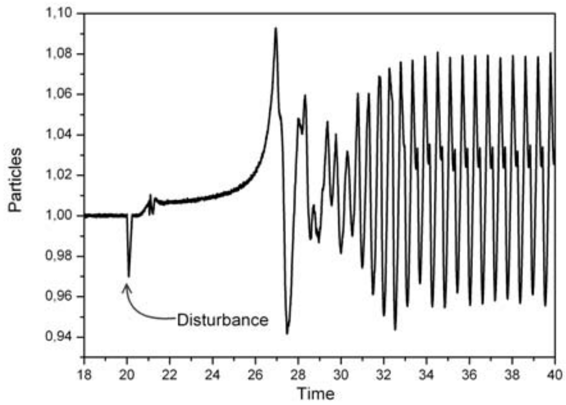


*Fig. 2. Dependency of total number of particles in system from time*

After wave of density will leave system excessive charge is concentrated in area of minimum potential (see Fig. 3,c) and continue to increase slowly. The rate of rise non-linearly depends on the magnitude of disturbance and correspondingly from value of excessive charge accumulated at initial stage. Dependency of instability development time from relative excess of charge in system after time interval $\tau$ from appearance of wave front is shown in Fig. 4.

This time can exceed by an order of magnitude specific times of development of Bursian instability of above-critical current.

Scope of this work includes investigation of influence of energy straggling of electron beam for development of instability caused by current-density wave. The range of numerical experiments was implemented with different relative straggling of pulses $\Delta p/p_0$ (from 0 to $10^{-2}$). Model times of virtual cathode initiation at different values of width of relative straggling of pulses for value of excessive accumulated charge of ≈0,7 % are represented in Table. As it seen from the represented results, at increasing of energy dispersion of beam a time of development of instability is reduced. Virtual cathode formed without disturbing influence after the scenario for «warm» beam, described in [4] on condition that $\Delta p/p_0 \geq 1\cdot 10^{-2}$.

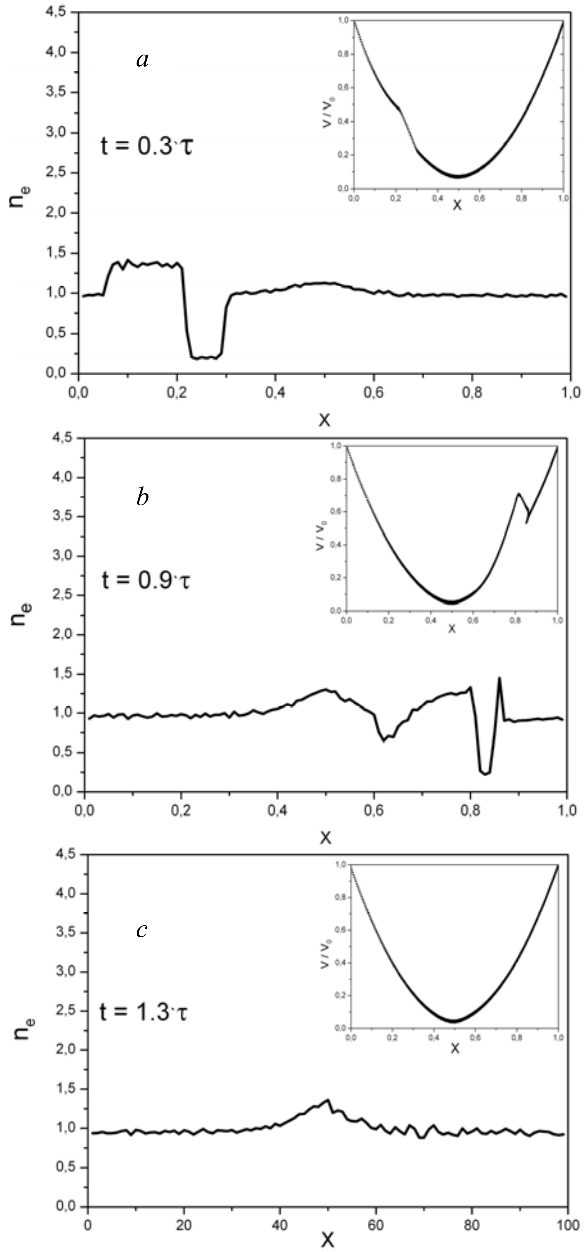


*Fig. 3. Density of electrons and snapshots of phase space of particles in different moments of time from beginning of disturbance*

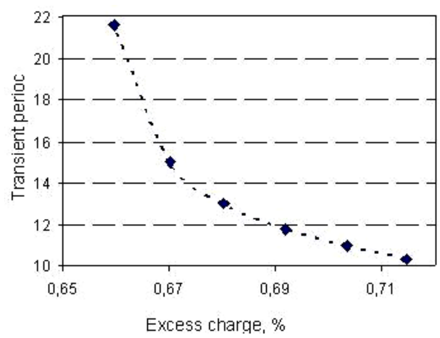


*Fig. 4. Dependence of VC setting time from excessive charge in system*

In addition to this, a sensitivity of solution relative to parameters of numeric model was investigated. The time of development of instability at different combinations of following parameters was determined: method of calculation of charge density (CIC, NGP), order of accuracy for movement equations (1-st, 2-nd order), type of injection of particles (equipartition, random distribution), size of time step. In all cases the deviation from specified in Table values was not exceed 10 %. It allows to assert that obtained results really represent properties of physical model of system.

| $\Delta p/p_0$ | $t/\tau$ |
|---|---|
| 0 | 11.8 |
| $1\cdot10^{-5}$ | 11.8 |
| $1\cdot10^{-4}$ | 11.79 |
| $1\cdot10^{-3}$ | 10.43 |
| $2\cdot10^{-3}$ | 8.21 |
| $5\cdot10^{-3}$ | 3.4 |

The result of research is setting of principal possibility of excitation of Bursian instability at current of beam lower than limited by space charge, at the account of initiation of charge accumulation mechanism in system, excitable by wave of density. At this, times of such instability development significantly exceed electron transit time through drift space.

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## ОСОБЕННОСТИ РАЗВИТИЯ НЕУСТОЙЧИВОСТИ ПУЧКА С ТОКОМ НИЖЕ БУРСИАНОВСКОГО ПРЕДЕЛА


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Установлена принципиальная возможность возбуждения бурсиановской неустойчивости в плоском дрейфовом промежутке при токе пучка ниже предельного, ограниченного пространственным зарядом, за счет инициирования механизма накопления заряда в системе, возбуждаемой волной плотности. Времена развития такой неустойчивости значительно превосходят пролетное время электрона через дрейфовый промежуток.


## ОСОБЛИВОСТІ РОЗВИТКУ НЕСТІЙКОСТІ ПУЧКА З СТРУМОМ НИЖЧЕ БУРСІАНОВСКОЇ ГРАНИЦІ


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Встановлена принципова можливість збудження бурсіанівської нестійкості в плоскому дрейфовому проміжку при струмі електронного пучка меншім граничного, обмеженого просторовим зарядом, за рахунок ініціювання механізму накопичення заряду в системі, яка збуджується хвилею густини. Період розвитку такої нестійкості значно перевищує час перельоту електрона через дрейфовий проміжок.