THE QUANTUM-FIELD VACUUM MODEL OF DARK ENERGY

P.I. Fomin[1,2], *A.P. Fomina*[1*]

[1]*Bogolyubov Institute for Theoretical Physics NAS of Ukraine, 03680, Kiev, Ukraine;*

[2]*Institute of Applied Physics NAS of Ukraine, 40000, Sumy, Ukraine*

(Received January 23, 2013)

The gravitational mechanism of the radical reduction, comparing with the predictions of the local quantum field theory, of physical vacuum energy density is considered. It is shown that the account of self-gravitation of the quantum vacuum fluctuations decreases the total energy of fluctuations of the Planck size to zero, transforming them in topologically closed configurations. The total energy is defined as the positive self-energy of fluctuation plus the negative energy of its self-gravitation. By reasons of statistical physics, a hypothesis about the crystal-like ordering of the whole system of massless "Planck cells" is suggested. In this massless (or almost massless) crystal-like structure the subsystem of vacuum condensates with positive energy is integrated. The condition for the topological closure of Universe allows us to obtain an upper bound for positive energy density in the system close to the currently observed value.

PACS: 95.35.+d, 98.80.Es

1. INTRODUCTION

The existence of the physical vacuum is the most important consequence of modern quantum field theories: the quantum electrodynamics, the Weinberg-Salam-Glashow theory of electroweak interactions, and the quantum chromodynamics the theory of strong interactions. The physical vacuum can be regarded as a real relativistically invariant quantum medium (a kind of quantum fluid) filling out all the world space and realizing the lowest energy state of quantum fields. One can say that this is a remake of the classical "luminiferous ether." The real particles such as electrons, positrons, photons, hadrons etc.as well as all macroscopic bodies are quantum wave-like excitations of this medium endowed with certain quantum numbers ensuring their relative stability. The physical vacuum can be compared with an ocean with particles being oceanic waves. Waves may be created and destroyed, but the ocean is eternal.

All modern quantum field theories do not take into account gravitation, which is regarded as a superweak interaction, and describe local fields in the continuous Minkowski space without any parameters of non-locality such as fundamental length. A consequence of locality is the presence of ultraviolet divergences in quantum field theory. For instance, divergent are the integrals describing the field contribution to the electron mass and charge. There exists a phenomenological way of treating these divergences, the so-called renormalization theory, consisting in isolating all such infinities in all orders of perturbation theory and replacing them by finite experimental values of masses and charges. This operation, however, is not physically and mathematically consistent and closes the way to the theoretical explanation of the observed values of masses and charges. In local quantum field theories, the value of the vacuum energy density also diverges. This infinity is replaced by zero in the renormalization theory under the assumption that the energy of all observed particles are measured from the vacuum energy [1].

The situation,however, fundamentally changes under the account of gravitation. in general relativity, every energy density, described by the stress-energy tensor on the right-hand side of the Einstein equations, gravitates (i.e., curves the space-time metric). Therefore, one cannot simply neglect the vacuum energy density by subtracting its value.

The discovery of dark energy in cosmology and the estimate of its relatively low value (see the review paper [2] and references therein) raises the issue about the mechanism of radical reduction of the vacuum energy density with respect to the predictions of local quantum field theories. We will show here that one of such powerful mechanisms is self-gravity and gravitational inter-action of the quantum fluctuations of vacuum. Gravity makes homogeneous vacuum gravitationally unstable, in a way, similarly to the Jeans instability of a uniform distribution of gravitating cosmic gas. Breaking out into ling-living quasilocal massless cells of Planckian size, the space of vacuum becomes discrete and crystal-like. Quantum field theory in such a space ceases to be local, similarly to the quantum theory of quasiparticle in a

*Corresponding author E-mail address: afomina@ukr.net

solid body. A fundamental Planck length

$$l_P = \sqrt{\frac{G\hbar}{c^2}} = 1.6 \times 10^{-33}\, cm \qquad (1)$$

appears in the theory, and the "curse of divergences" is lifted from the quantum field theory. In such a scheme, now there is a prospect for the theoretical calculation of masses and charges of particles. For example, one can justify, in a certain approximation, the following formula for the electron mass [3]:

$$m_e = \frac{e^2}{l_P c^2} \exp\left(-A\frac{\hbar c}{e^2}\right) , \qquad (2)$$

which, for $A = 1/3$, gives $m_e \sim 10^{-27} g$. If one formally switches off gravity ($G \to 0$) and runs l_P to zero, one gets the original divergence.

Next, from the condition of topological closeness of the Universe, we obtain the upper bound for the vacuum energy density close to the currently observed value.

2. GRAVITATING FLUCTUATIONS OF THE VACUUM

In the relativistic domain, the energy and spatial dimensions of quantum fluctuations of the vacuum in quantum field theory without account of gravitation are regulated by the uncertainty relations:

$$\triangle E \triangle t \sim \hbar\,, \quad \triangle E \sim c\triangle p\,, \quad \triangle p \triangle q \sim \hbar\,. \qquad (3)$$

Here, Δt is the lifetime of a fluctuation with energy ΔE, momentum dispersion Δp, and size Δq. Taking into account gravity, which reduces the total energy of fluctuation according to the law

$$\Delta E_{tot} = \Delta E - \eta G \frac{(\Delta E/c^2)^2}{\Delta q} \geq 0\,, \qquad (4)$$

(here, η is a numerical coefficient taking into account the form of a fluctuation; it will be specified below), one can replace the first of relations (3) by

$$\Delta E_{tot} \Delta t \sim \hbar\,. \qquad (5)$$

For long-living fluctuations ($\Delta t \to \infty$), from (4) and (5) we obtain the condition

$$\Delta E_{tot} = \Delta E\left(1 - \frac{\eta G \Delta E}{c^4 \Delta q}\right) \sim \frac{\hbar}{c} \to 0\,. \qquad (6)$$

It follows from (6) that, under the condition

$$\Delta E = \frac{c^4 \Delta q}{\eta G} \qquad (7)$$

there exist nontrivial fluctuations living infinitely long. Further, taking into account the second and third relation in (3), we get the equation for the size Δq of such a fluctuation:

$$\Delta E \sim c\Delta p \sim \frac{c\hbar}{\Delta q} = \frac{c^4 \Delta q}{\eta G}\,, \qquad (8)$$

which gives

$$(\Delta q)^2 \sim \eta \frac{G\hbar}{c^3} = \eta {l_P}^2\,. \qquad (9)$$

From the viewpoint of general relativity, equation (6) means spatial topological closeness of such gravitating configurations (see, e.g., [4]). The property of longevity of such fluctuations ($\Delta t \to \infty$) means that here we deal with a spontaneous phase transition of the vacuum with breaking of the translational symmetry of the homogeneous system. A gravitational vacuum condensate is formed.

Thus, the self-gravity of the quantum vacuum fluctuations turns the space of vacuum into a discrete crystal-like structure [5] consisting of massless and charge-neutral "Planck cells" of size (1). In this case, the energy density of the zero-point oscillations of the vacuum radically reduces from formally infinite (for local quantum field theories) to finite and even close to zero. It is this vacuum energy density, reduced by self-gravity, that constitutes, according to our idea, the "dark energy" observed in astrophysics. It does not appear possible at present to calculate its value from the first principles because it should essentially depend on the unknown structure of the discrete crystal-like space of the vacuum formed by interacting massless "Planck cells." B virtue of their topological closeness, interaction between such cells can occur, for example, via quadrupole gravitational forces [6] and, as a whole, be similar to the van-der-Waals forces between electrically neutral atoms that form a crystal.

We have considered above the most extreme manifestation of the self-gravity of the vacuum in the form of gravitational self-closing of fluctuations of the Planck size. Somewhat lower, but also considerable reduction of energy is produced, of course, by the mutual gravity of spatially close fluctuations of small size.

A substantial contribution into the lowering of the vacuum energy also comes from the origin of the so-called vacuum condensates connected with the dynamical breaking of various continuous symmetries in quantum field theory (see, e.g., [7]). These condensates are formed by binding of fermion particle-antiparticle vacuum pairs via strong gauge interactions. In this case, the vacuum of fermionic fields sort of "bosonizes" and re-duces its energy also by the suppression of the Pauli principle.

3. ESTIMATE OF THE ENERGY DENSITY FROM THE CONDITION OF SPATIAL CLOSENESS OF THE UNIVERSE

The modern idea that the universe as a whole should have the topology of a closed three-dimensional sphere is connected, first and foremost, with the idea of quantum creation of our universe from the vacuum od some "mother universe" [8]. Initially, the daughter universe is born small and then starts quickly growing in volume and internal mass (energy), for example, in the inflationary regime [9]. The size and energy density in this case are, to a certain extent, connected by the condition of closeness and, therefore, one can estimate the density from the known size. The condition of topological closeness of a gravitating system, according to general relativity [4], is accompanied by the zero total energy of such a system by virtue of the exact compensation of the positive proper energy (mass) of matter by the negative contribution from its self-gravity. In a simplified version, this condition can be written, similarly to (4), as

$$\Delta E_{tot} = Mc^2\left(1-\eta\frac{GM}{c^2 a}\right) = 0\,. \qquad (10)$$

where a is the size (radius) of such a spherical "zero-system," and η is a numerical factor dependent on its shape. This equation has two roots; one of them is trivial, $M = 0$, and the other is non-trivial:

$$M = \frac{c^2}{\eta G}a\,. \qquad (11)$$

For a closed three-dimensional spherical universe, M is expressed through the mass density ρ as follows [4]:

$$M = 2\pi^2 a^3 \rho\,. \qquad (12)$$

Substituting (12) into (11), we find the connection between ρ and a arising from the simplified condition (10) of the closeness of the universe:

$$\rho(a) = \frac{c^2}{2\pi\eta G a^3}\,. \qquad (13)$$

Simplification in (10) and (13) consists in neglecting the term $\sim \dot{a}^2$, which is justified under the condition $\dot{a}^2 \ll c^2$. Taking into account this "energy of expansion" of the universe and introducing the Hubble parameter $H(t) \equiv \dot{a}/a$, we get the corresponding Fried-mann equation

$$H^2(t) + \frac{c^2}{a^2(t)} = \frac{8\pi G}{3}\rho(t)\,. \qquad (14)$$

Dividing by H^2 and introducing the standard notation $\rho_c = 3H^2/8\pi G$, $c^2/H^2 = {a_H}^2$, where a_H is the Hubble radius, we write (14) in the form

$$1 + \frac{a^2}{{a_H}^2} = \frac{\rho}{\rho_c}\,. \qquad (15)$$

Taking into account the known uncertainty in the determination of the modern value of the Hubble parameter

$$H_0 = (50...80)\frac{km}{sMpc}\,, \qquad (16)$$

we obtain the intervals for the corresponding uncertain-ties for ρ_c and a_H:

$$\rho_c = (0.5\ldots 1.3)\times 10^{-29}\frac{g}{cm^3}\,, \qquad (17)$$

$$a_H = (1.8\ldots 1.1)\times 10^{28}\,cm\,. \qquad (18)$$

We note that relation (15) describes the dependence of the total density of the proper mass (energy) of the universe on the critical density (17) and radius a. The fraction of dark energy in $\rho(a)$ depends on the whole physical history of the universe. Observations give that the fraction of dark energy ($\sim 72\%$) together with dark matter ($\sim 24\%$) dominate over the fraction of baryonic matter ($\sim 4\%$). The smallness of the fraction of baryons is determined by the the physics of baryogenesis connected with the breaking of the baryon and lepton symmetry in the early universe and cannot be calculated without the quantitative deciphering of this physics.

As regards the fraction of dark matter, here we only announce the following. In our next papers devoted to this issue, we will show, proceeding from rather general assumptions, that one can justify theoretically the observed ratio $\rho_{DM}/\rho_{DE} \approx 1/3$.

In conclusion, we stress that formula (15) is obtained from the requirement of spatial closeness of the universe necessary for the possibility of its spontaneous quantum creation from the vacuum of some mother universe. This is a very important requirement since it is only a closed universe that does not require external energy for its creation.

We note that the current analysis based on cosmological observations does not exclude the closed topology of the universe. It is clear from (15) that, for $a \gg a_H$, the closed model approximately imitates the flat one, and these models may not be distinguishable observationally. A crucial theoretical argument in favor of a closed universe is the possibility of its spontaneous quantum creation.

4. THE CONCLUSIONS

From the macroscopic viewpoint, the physical vacuum as a continuous medium can be ascribed a stress-energy tensor of the form [4]

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - p g_{\mu\nu}\,, \qquad (19)$$

where $g_{\mu\nu}$ is the metric tensor locally coinciding with the Minkowski tensor

$$\eta_{\mu\nu} = diag(1,-1,-1,-1)\,. \qquad (20)$$

The vacuum is, by definition, a Lorentz-invariant state of quantum fields. The last term in (19) possesses this property, which is not true for its first term contain-ing the four-velocity vector u_μ. Hence, it follows that the vacuum as a macroscopic medium should be characterized by negative pressure:

$$p = -\varepsilon\,, \qquad T_{\mu\nu} = \varepsilon g_{\mu\nu}\,. \qquad (21)$$

It is this property that allows one to identify dark energy with the vacuum energy since the Friedmann equation for $\ddot{a}$ and equation (21) imply the "antigravity" property of the cosmological vacuum [2].

Taking into account the discussion of the previous sections, one can propose the following working model of the gravitating physical vacuum. On Planckian spatial scales, the self-gravity of quantum fluctuations of the Planck sizes turns them into spatially closed massless "Planck cells" interacting with neighbors through quadrupole forces of the type of van-der-Waals forces. From the physical reasoning one can expect that this sufficiently rigid subsystem of vacuum forms a regular quantum-crystal lattice. This lattice is sort of filled with the mobile part of the gravitationally bound quantum fluctuations forming the "gravitational vacuum condensate" with positive energy as well as other vacuum condensates formed by bosonization of the fermionic degrees of freedom via strong gauge interactions. Vacuum condensates as coherent quantum-field subsystems similar to quantum fluids should be superfluous. In our future works, by laying this assumption as a basis for the "vacuum condensate" model of dark matter, we will try to show that this model leads to a sufficiently plausible estimate of the dark-matter density and spatial dimensions occupied by dark matter in the universe.

References

1. A.I. Akhiezer and V.B. Berestetskii. *Quantum Elec-trodynamics.* Interscience Publishers, New York, 1965.
2. A.D. Chernin. Cosmic vacuum // *Phys.Usp.*, 2001, p.1099 [*Usp. Fiz. Nauk*, 2001, N11, p.1153].
3. P.I. Fomin. Some questions of quantum electrodynamics at small distances // *Fiz. Elem. Chast. Atom. Yad.* 1976, N7, p.687.
4. L.D. Landau and E.M. Lifshits. *The Classical Theory of Fields.* Pergamon Press, Oxford, 2003.
5. P.I. Fomin. On the crystal-like nature of the physical vacuum at the Planck distances // *Problems of Physical Kinetics and Physics of Solid Body* [in Russian]. Naukova Dumka, Kiev, 1990, p.387.
6. P.I. Fomin and V.V. Kuzmichev. Gravitational fields of massive and massless axial-symmetric quadrupoles in general relativity // *Phys. Rev. D.* 1994, p.1854.
7. P.I. Fomin. *On the vacuum condensates and the problem of the nature of masses and innertia forces.* National Scientific Center, Kharkov Institute of Physics and Technology, 2008, p.367.
8. P.I. Fomin. *Gravitational instability of vacuum and the cosmological problem.* Preprint ITP-73-137P [in Russian], Kiev, 1973;// *Dokl. Akad. Nauk Ukr.SSR*, 1975, N9, p.831.
9. A.D. Linde. *Particle Physics and Inflationary Cosmology.* Harwood, Chur, Switzerland, 1990.

Received January 23, 2013.

КВАНТОВО-ПОЛЕВАЯ ВАКУУМНАЯ МОДЕЛЬ ТЕМНОЙ ЭНЕРГИИ

П.И. Фомин*, *А.П. Фомина

Рассмотрен гравитационный механизм радикального понижения, по сравнению с предсказаниями локальных квантовых теорий поля, величины плотности энергии физического вакуума. Показано, что учет самогравитации квантовых флуктуаций вакуума понижает полную энергию флуктуаций планковских размеров до нуля, превращая их в топологически замкнутые конфигурации. Полная энергия определяется как собственная положительная энергия флуктуации плюс отрицательная энергия ее самогравитации. Из статфизических соображений выдвигается гипотеза о кристаллоподобном упорядочении всей системы таких безмассовых "планковских ячеек". В эту безмассовую кристаллоподобную структуру как бы влита подсистема вакуумных конденсатов с положительной энергией. Использование условия топологической замкнутости Вселенной позволяет получить оценку сверху для положительной плотности энергии системы, близкую к наблюдаемым значениям.

КВАНТОВО-ПОЛЬОВА ВАКУУМНА МОДЕЛЬ ТЕМНОЇ ЕНЕРГІЇ

П.І. Фомін*, *А.П. Фоміна

Розглянуто гравітаційний механізм радикального зниження, у порівнянні з передбаченнями локальних квантових теорій поля, величини густини енергії фізичного вакууму. Показано, що врахування самогравітації квантових флуктуацій вакууму знижує повну енергію флуктуацій планківських розмірів до нуля, перетворюючи їх у топологічно замкнуті конфігурації. Повна енергія визначається як власна позитивна енергія флуктуації плюс від'ємна енергія її самогравітації. З статфізіческіх міркувань висувається гіпотеза про кристалоподібне впорядкування всієї системи таких безмасових "планківських комірок". У цю безмасову кристалоподібну структуру ніби влита підсистема вакуумних конденсатів з позитивною енергією. Використання умови топологічної замкнутості Всесвіту дозволяє отримати оцінку зверху для позитивної густини енергії системи, близьку до значень, що спостерігались.