

# DYNAMICAL DESCRIPTION OF FUSION-QUASIFISSION OF HEAVY NUCLEI AS OPEN QUANTUM SYSTEM

*K. V. Pavlii\**

*National Science Center "Kharkov Institute of Physics and Technology", 61108, Kharkov, Ukraine*

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The description of open quantum dissipative systems was analyzed on the basis of equations for the density matrix and the nonstationary Schrodinger equation with linear and nonlinear (non-Hermitian) Hamiltonians. Questions of capture, the first stage of the fusion reaction, and the nucleon transmission of interacting nuclei were considered. The quantum-dynamical description of fusion-quasifission of nuclei was proposed as open quantum system based on the calculations for emission of nucleons from the dinuclear system (DNS) and the transmission of nucleons at allowed  $[\pm n, \pm p]$  and  $[\pm n, \pm p \pm (n + p)]$  transitions.

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## 1. INTRODUCTION

The study of quantum dynamical laws in fusion-quasifission of heavy nuclei will enable to more accurately plan experimental studies of synthesis of SHE and correctly identify the processes that occur in nuclei capture dynamics, fragmentation of nuclei and compound nucleus, transitions of nucleons from one nucleus to another, emission of nucleons in different stages of the formation of SHE, etc. Influence of closed quantum systems on one another and/or of the environment (reservoir) on the dynamics of particles may be taken into account by using equations for the density matrix and/or stochastic Schrodinger equations.

## 2. QUANTUM DISSIPATIVE SYSTEMS

### 2.1. Density matrix-based equations

The approach which is based on the interaction of the system (subsystem) with the reservoir [1, 2, 3] is a way to describe open quantum systems. It is necessary to make a reduction, i.e. to sum in the degrees of freedom of the reservoir in order to calculate the behavior of subsystem + reservoir. So we obtain a description of the system by means of the reduced density matrix. So,  $S_1 + S_2$  (where  $S_1$  - subsystem of concern,  $S_2$  - reservoir) is a closed system describing by vector of state  $|\psi\rangle$  or density matrix  $\hat{R} = |\psi\rangle\langle\psi|$ . Then we obtain the only description of subsystem of concern  $S_1$  by summing the density matrix in degrees of freedom  $S_2$  (get partial trace of the full density matrix),  $\hat{\rho} = Tr_{S_2}\hat{R}$ . The equation for the density matrix with dissipation was proposed by Lindblad [4]:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] - \sum_i \left( \hat{L}_i^\dagger \hat{L}_i \hat{\rho} - 2\hat{L}_i \hat{\rho} \hat{L}_i^\dagger + \hat{\rho} \hat{L}_i^\dagger \hat{L}_i \right), \quad (1)$$

where  $\hat{H}$  - Hamiltonian of the system,  $\hat{L}_i$  - operators describing the dissipation.

Description of the quantum system without a specific model of reservoir is presented in references [2, 3]. One approach is as follows: each measurement is described by the reduction of von Neumann, and continuous measurement is presented as a series of instantaneous measurements [2].

The theory of continuous measurements, which is based on the restricted path integrals [2, 5], effectively represents the dissipation of the quantum system and describes the dynamics of open systems as a result of continuous measurements. This approach enables to construct a general theory of dissipation, then, a description of its dynamics is reduced to the Schrodinger equation with a complex Hamiltonian [2]:

$$|\dot{\psi}\rangle = -\frac{i}{\hbar} \hat{H}_{[a]} |\psi\rangle, \quad (2)$$

where  $\hat{H}_{[a]} = \hat{H} + \lambda a(t) \hat{B} - i\hbar\chi(\hat{A} - a(t))^2$ . Reservoir characteristics are contained in  $a(t)$  term up to  $\chi$  constant. This approach yields Lindblad-type equations [5]. In case of discrete measurements total density matrix is defined by the equation [2]:

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] - \frac{\chi}{2} [\hat{A}, [\hat{A}, \hat{\rho}]] - \frac{\lambda^2}{8\chi\hbar^2} [\hat{B}, [\hat{B}, \hat{\rho}]] - \frac{i\lambda}{2\hbar} [\hat{B}, [\hat{A}, \hat{\rho}]_+], \quad (3)$$

where  $[\cdot, \cdot]_+$  - anticommutator. This equation describes the dissipative system, and  $\lambda$  is proportional to the friction coefficient.

Consequently, the same equation for the density matrix corresponds to the various forms of stochastic equations for the state vector. In addition, one of the forms of such equations corresponds to the Schrodinger equation with a complex Hamiltonian.

\*Corresponding author E-mail address: kvint@kipt.kharkov.ua

## 2.2. Dissipative Schrodinger equations

The Schrodinger equation, including the dissipative terms in the Hamiltonian, may be used for the description of open quantum systems. In references [7, 8], the attenuation factor was introduced to describe the dissipation at the microscopic level, and Hamiltonian was represented as follows:

$$\hat{H}(t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} e^{-\gamma t} + V(t)e^{\gamma t}. \quad (4)$$

In reference [6], an average value of the momentum was received on the basis of the Heisenberg-Langevin equations for the momentum operator  $\hat{p}$  and coordinate  $\hat{x}$ , thus the Schrodinger equation takes the following form:

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U + U_R + U_L \right) \psi, \quad (5)$$

where  $U$  - interaction potential,  $U_R(x, t)$  - potential for accidental exposure, depending only on  $x$ ,  $U_L$  - dissipative potential which is equal to the following:

$$U_L = \frac{\hbar f}{2im} \ln \left( \frac{\psi}{\psi^*} \right) + W(t), \quad (6)$$

where  $W(t) = -(\hbar f/2im) \int \psi^* \ln(\psi/\psi^*) \psi dx$ ,  $\psi^*$  - complex conjugate of the wave function,  $f$  - friction coefficient,  $m$  - mass of a particle. Equation (6), excluding accidental exposure, satisfies the continuity equation for the probability density  $\psi^* \psi$ . Hamiltonian of the system can be written as  $\hat{H}_0 + \gamma \hat{W}$ , where  $\hat{W}$  dissipative term.

A series of dissipative terms [8], in which the ground state of the Schrodinger equation without dissipative potential is reserved and the law of energy dissipation is similar to the classic  $dE/dt = -f\dot{x}^2$  was proposed to be used for nonstationary Schrodinger equation with nonlinear (non-Hermitian) Hamiltonian. The linear (Hermitian) Hamiltonians [8] should also be extracted.

Papers [7, 8] which offer the challenge for the study of irreversible processes in quantum theory and papers [8, 11] where different approaches of quantum dissipative theory to the nuclear research are represented should be allocated among the papers dedicated to the problem of quantum equations of motion with dissipative terms.

## 3. PROCESSES IN FUSION-QUASIFISSION OF HEAVY NUCLEI

Two approaches - the model of nucleons collectivization (MNC) and the dinuclear system concept (DNSC) - should be distinguished to describe the fusion-quasifission of heavy nuclei.

### 3.1. Fusion models

The nucleon structure of colliding nuclei is considered in MNC [9]. The nucleons are moving from one

core to another after the dissipation of kinetic energy due to the overlap of nuclear surfaces. This leads to a sequential process of nucleons collectivization and, eventually, to fusion.

DNSC [10] can be interpreted as follows:

1. The dynamics of the capture process is considered from the moment of contact to the minimum of the interaction potential, where "excited" DNS is formed after dissipation of the kinetic energy. DNS fragments retain its individuality; it is reflected by considering the binding energies of the nuclei in the potential energy of DNS.
2. The compound nucleus is formed due to transitions of nucleons from one nucleus to another one in the direction of DNS minimal potential energy. This raises the competition between the channels of complete fusion and quasifission (DNS decay). Internal fusion barrier that reduces the fusion cross section was found.
3. Emission of nucleons and  $\gamma$ -quantum which removes the excitation takes place after the formation of the compound nucleus.

As it follows from DNSC that the fusion process may be conventionally divided into three parts, then the evaporation residue cross section, leading to the formation of SHE, is written as [10]:

$$\sigma_{ER}(E_{cm}) = \sum_{L=0} \sigma_c(E_{cm}, L) \cdot P_C(E_{cm}, L) \cdot W_s(E_{cm}, L), \quad (7)$$

where  $\sigma_c$  - partial capture cross section of the nucleus-target of the incident nucleus (first stage);  $P_C$  - probability of the compound nucleus formation after capture (second stage);  $W_s$  - survival of the compound nucleus (third stage).

On the basis of DNSC, the paper [11] proposes a quantum-mechanical description of the initial stage of fusion (capture), in which the capture is presented as a process for settlement of a part of the initial Gaussian packet in a "pocket" of the nucleus-nucleus potential. Thus, the process is described as an open quantum system by using the equation for the reduced density matrix  $\rho(t, R, z)$  of Lindblad type [4]:

$$\frac{d}{dt} \rho(t, R, z) = \left[ i \frac{\hbar}{\mu} \frac{\partial^2}{\partial R \partial z} - iz \frac{\partial V}{\partial R} - i \frac{z^3}{24} \frac{\partial^3 V}{\partial R^3} - \lambda_p z \frac{\partial}{\partial z} - D_{PP} \frac{z^2}{\hbar^2} - \frac{i}{\hbar} \left( z D_{RP} \frac{\partial}{\partial R} + \frac{\partial}{\partial R} z D_{RP} \right) \right] \rho(t, R, z), \quad (8)$$

where  $D_{PP}$ ,  $D_{RP}$  and  $\lambda_p$  - momentum diffusion coefficient, mixed diffusion coefficient and friction coefficient, respectively. The probability of capture is determined as follows:

$$P(t = \tau, E_{cm}, L, \Omega_P, \Omega_T) = \frac{\int_0^0 \rho(t = \tau, R) dR}{\int_0^{\infty} \rho(t = 0, R) dR}. \quad (9)$$

The total capture probability  $P_{cap}$  was obtained by averaging over all possible orientations  $\Omega_P$  and  $\Omega_T$  of deformed interacting nuclei. The statistical approach is generally used in the calculation of probabilities of compound nucleus formation from DNS and survival rates.

### 3.2. Transitions of nucleons

Nucleon exchange between interacting nuclei has a determining influence on the probability of fusion. Nucleus-nucleus potential, binding energy, potential energy of DNS, barriers of quasifission and fusion are changed within successive transitions of nucleons.

Sequential method of coupled channels [12] has been successfully used to describe the collective excitations in the near-barrier fusion processes. The semi-empirical model [13] has shown that the intermediate neutron transfer with  $Q > 0$  leads to a considerable increase of sub-barrier fusion. The most successful description of neutron transfers is proposed in paper [14], where the semi-classical model was used with independent description of the neutron wave function evolution by means of nonstationary S Schrodinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_3} \Delta_{r_3} \psi + V_n(r_3; r_1(t), r_2(t)) \psi(r_3, t), \quad (10)$$

where  $\psi(r_3, t)$  - wave function of an external neutron in the field of heavy nuclei whose centers are moving along trajectories  $r_1(t)$  and  $r_2(t)$ . Dissipative forces are determined as follows:  $F_1 = -\gamma \dot{r}_1$ ,  $F_2 = -\gamma \dot{r}_2$ , where  $\gamma$  - phenomenological friction coefficient.  $V_n$  - interaction potential between the neutron and nuclei was selected as the Woods-Saxon potential. The nonstationary Schrodinger equation with the Hamiltonian was used in the quantum model with three-dimensional movement of the neutron as follows:

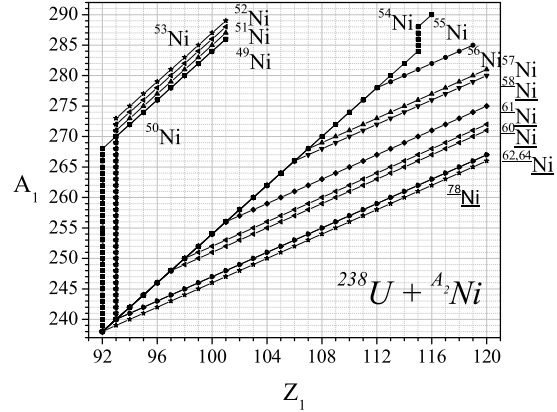
$$H = -\frac{\hbar^2}{2\mu} \Delta_r - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial R^2} + V_{12}(R) + V_n(r_3; r_1, r_2), \quad (11)$$

where  $r_1 = Rn\eta_1 - r\eta_2$ ,  $r_2 = -Rn(1 - \eta_1) - r\eta_2$ ,  $n$  - unit vector in the direction of the internuclear axis. The calculations in [14] have shown that an increase of barrier penetration was observed for certain combinations of nuclei; this is due to the transition of the neutron from upper-energy state to the underlying free one and to lowering the energy of originally occupied states in the near-barrier region.

### 4. RATIONALE FOR DYNAMICAL DESCRIPTION OF NUCLEAR FUSION

In the description of the processes occurring in fusion of heavy nuclei it is necessary to consider the following factors which have a major influence on the formation of SHE: transitions of nucleons (protons and neutrons) in the formation of compound nucleus, emission of nucleons from DNS fragments, as well as characteristics of the nucleus-nucleus potential and potential energy at the dynamics of nuclei passage along the interaction potential. It means that the process of heavy nuclei fusion-quasifission should be considered in view of the dynamic concept of DNS (DC DNS). In paper [15] the dynamics of the transitions of nucleons in DNS

fragments and its influence on the nucleus-nucleus potential and the potential energy at  $[\pm n, \pm p]$ ,  $[\pm n, \pm p \pm (n + p)]$  and  $[\pm n, \pm p \pm (n + p) \pm \alpha]$  allowed transitions were analyzed. Fig.1 shows the dependence of the number of nucleons in the heavy DNS fragment on the number of protons in the same fragment, the initial nucleus -  ${}^{238}_{92}\text{U}$  for different Ni isotopes. The dynamics of transitions were obtained in the minimum nucleus-nucleus potential.



**Fig.1.** Dependence of the number of nucleons in the heavy DNS fragment on the number of protons in the same fragment at  $[\pm n, \pm p \pm (n + p)]$  allowed transition for the reaction:  ${}^{238}_{92}\text{U} + {}^{A_1}_{28}\text{Ni}$

Emission of nucleons from DNS and the nucleon transitions change the isotopic composition of the fragments, and therefore, the potential energy of the system and the nucleus-nucleus potential.

In the description of multiparticle theory of decay, a neutron and a proton may be considered as existing clusters. As for  $\alpha$ - and heavy cluster decay, where such conditions do not exist, there is a problem in the formation of such clusters from nucleons, i.e. the calculation of probabilities of its formation. The calculations in [16] show a low probability of formation of such systems. Therefore, it is appropriate to consider only the neutron and proton emission from DNS fragments. Emission process from DNS fragments in quasi-stationary state may be described by multiparticle Schrodinger equation ( $J$  - spin;  $M$  - projection of the spin on the axis;  $\sigma$  - other quantum numbers, which include charge  $Z_i$  and atomic weight  $A_i$  of DNS fragment) [16]:

$$H_{A_i} \psi_{\sigma}^{JM}(\xi) = \bar{E}_{\sigma}^J \psi_{\sigma}^{JM}(\xi), \quad (12)$$

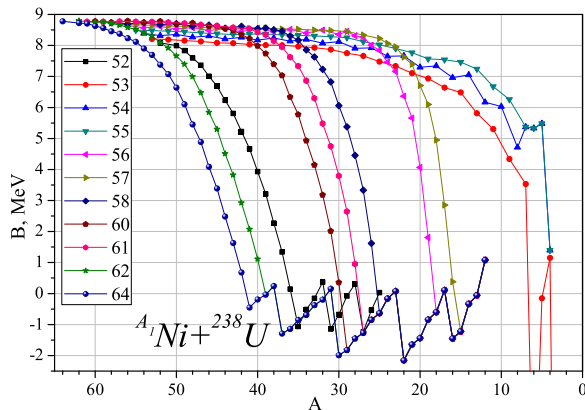
where  $\Psi_{\sigma}^{JM}$  - wave function of the fragment isolated state;  $H_{A_i}$  - many-particle Hamiltonian of the one of DNS fragments with  $A_i$  number of nucleons. Moreover, the number of nucleons is determined by the dynamics of transition, and the complex energy may be represented as  $\bar{E}_{\sigma}^J = E_{\sigma}^J - (i\Gamma_{\sigma}^J/2)$ ,  $E_{\sigma}^J = B_i$  - binding energy of the fragment  $A_i$ ,  $\Gamma_{\sigma}^J$  - total width, which is determined by the sum of partial widths of fragment decay (emissions) in all open channels. This pa-

per considers only neutron and proton emissions, thus the DNS fragment may pass to either  $(A_i - 1, Z_i)$  or  $(A_i - 1, Z_i - 1)$  states. Moreover, proton emission is typical for neutron-deficient nuclei and neutron one - for proton-deficient nuclei. The energy of the relative motion, the emission energy,  $Q_{c.em}$  of child nucleus and nucleon is determined as follows:

$$Q_{c.em} = B(A_i) - B(A_i - 1) \approx \frac{\mu_{em} v_{c.em}^2}{2}, \quad (13)$$

where  $B(A_i)$  - the binding energy of one of the DNS fragments, from which the emission takes place (by hypothesis, it should be negative),  $B(A_i - 1)$  - binding energy of the same fragment after emission,  $\mu_{em}$  - reduced mass,  $v_{c.em}$  - speed of relative movement. All known half-lives of nuclei are usually equal to  $> 10^{-6} s$  which is significantly higher than the lifetime of DNS ( $\approx 10^{-20} s$ ), thus the emission may be considered to be instantaneous. Moreover, a higher value of the negative binding energy of fragment is observed with increasing asymmetry in the DNS fragments, which is formed by transitions of nucleons. As several dozens of nucleon transitions take place during the DNS lifetime, the process of nucleon emission with negative binding energy of the fragment may be considered as an adiabatic one in comparison with the process of single-particle nucleon transition.

Calculation of nucleon transitions were made for  ${}^{A_1}Fe + {}^{238}U$ ,  ${}^{A_1}Ni + {}^{238}U$ ,  ${}^{A_1}Zn + {}^{238}U$  reactions with provision for the emission of nucleons from DNS fragments. Fig. 2 shows the dependence of changes in binding energy of light DNS fragment for  ${}^{A_1}Ni + {}^{238}U$  reaction at  $[\pm n, \pm p \pm (n + p)]$  allowed transitions. The binding energy of the heavy DNS fragment at transitions and emissions is reduced by not more than 15 percents of the initial value.



**Fig. 2.** Dependence of the binding energy of light DNS fragment

It follows from the analysis of the influence of the number of neutrons in the heavy DNS fragment on the dynamics of transitions of nucleons with provision for the emission that light nucleus should be neutron-deficient one in order to extend the chain of transmission pairs  $(n + p)$  and to decrease the emission of neutrons, as well as that heavy nuclei

should be neutron-rich one for the greater content of neutrons in the compound nucleus.

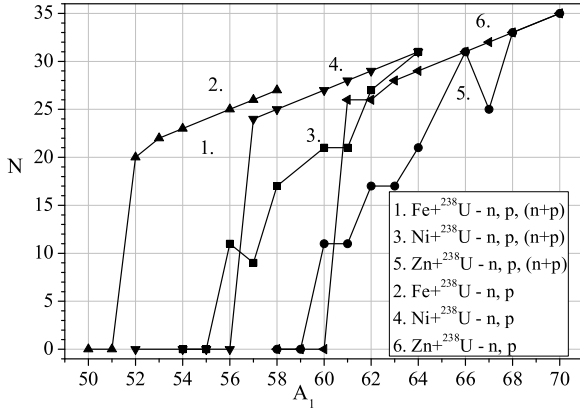
For  ${}^{A_1}Fe + {}^{238}U$  reaction the full range of transitions may be divided into five sections where the emission of neutrons takes place. Moreover, the more original content of neutrons is in the light fragment, the greater number of neutrons is emitted from the fragment. Table shows the emission energy  $Q_{c.em}$  for each neutron and the number of neutrons emitted in each section. The last column shows the total number of neutrons emitted for each Fe isotope and the sum of emission energies of all neutrons.

*Emission parameters for the reactions  ${}^AFe + {}^{238}U$*

	1	2	3	4	5	Sum.n
${}^AFe$	$Q_{c.em_i}$ , MeV					$Q_C$ , MeV
${}^{58}Fe$	11.2 6.4 11.15 6.1 11	6.84 12.2 6.51 12.14 6.11 12.04 5.6	12.85 5.78 12.81 5.13 12.69	4.9 13.57 4.01 13.7	6.48 18.57 5.32 19.49 46.05	26/ 288.64
${}^{57}Fe$	0	12.14 6.11 12.04 5.6	12.85 5.78 12.81 5.13 12.69	4.9 13.57 4.01 13.7	6.48 18.57 5.32 19.46 46.05	18/ 217.24
${}^{56}Fe$	0	12.2 6.51 12.14 6.11 12.04 5.6	12.85 5.78 12.81 5.13 12.69	4.9 13.57 4.01 13.7	6.48 18.57 5.32 19.49 46.05	20/ 235.95
${}^{54}Fe$	0	12.04 5.6	12.85 5.78 12.81 5.13 12.69	4.9 13.57 4.01 13.7	6.48 18.57 5.32 19.49 46.05	16/ 189.99
${}^{53}Fe$	0	0	0	13.57 4.01 13.7	6.48 18.57 5.32 19.49 46.05	8/ 127.09
${}^{52}Fe$	0	0	0	0	0	0
${}^{51}Fe$	0	0	0	0	0	0
${}^{50}Fe$	Three protons are emitted at the end of transition					21.77

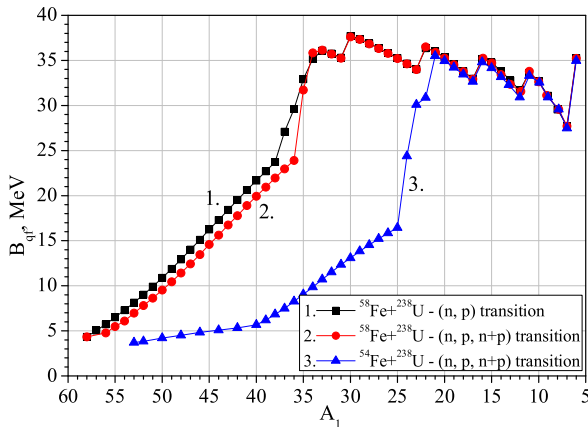
For  ${}^{A_1}Ni + {}^{238}U$  and  ${}^{A_1}Zn + {}^{238}U$  reactions the emission ranges may be divided into six or seven sections, respectively. Picture of emission from light fragment is fully consistent with  ${}^{A_1}Fe + {}^{238}U$  reaction with the repetition of the energy spectrum values. It follows that the light DNS fragment is responsible for the emission and the final results after transitions are the same according to the number of neutrons and protons in the light

fragments regardless of the source channel. Fig. 3 shows the dependence of number of neutrons emitted from the mass number of a light nucleus, from which it follows that the number of emitted neutrons increases with increasing mass number of light fragment for  $[\pm n, \pm p]$  allowed transitions, and for  $[\pm n, \pm p \pm (n+p)]$ . Neutron emission is not observed for neutron-deficient DNS fragments in the above-mentioned reactions, thus it leads to a higher content of neutrons in the compound nucleus. Proton emission (3p) from light DNS fragment at the final stage is observed in  $^{50}\text{Fe}+^{238}\text{U}$ ,  $^{53}\text{Ni}+^{238}\text{U}$  and  $^{57}\text{Zn}+^{238}\text{U}$  reactions at  $[\pm n, \pm p \pm (n+p)]$  allowed transitions. That is, the proton emission promotes formation of a compound nucleus without the fusion barrier.



**Fig. 3.** Dependence of the number of emitted neutrons on the mass number of light DNS fragments. Reactions are shown in the figure

The depth of quasifission barrier ( $B_{qf}$ ) is increased by  $0.8...0.9 \text{ MeV/n}$  (proton) within the transition of protons from light DNS fragment to heavy one. Neutron emission from light fragments reduces the quasifission barrier by  $0.6...0.7 \text{ MeV/n}$  (neutron).



**Fig. 4.** Dependence of the quasifission barrier, within transition of nucleons, on mass number in a light fragment

Fig. 4 shows that for reaction 3, where (n+p) transitions occur, the growth of quasifission barrier up to

$A_1 = 40$  is slower than in case of transfer of protons only  $A_1 > 40$ . Saw-tooth behavior of curves is characterized by the emission of neutrons from the light fragment. The quasifission barrier decreases with neutron-deficient light fragments [16], when neutrons only from light nuclei are transfer to heavy one. This leads to the disappearance of quasifission barrier and the probability of fusion becomes practically zero. In such a case quasifission barriers  $B_{qf}$  are reduced by  $0.1...0.3 \text{ MeV/n}$  (neutron). In transmissions of nucleons there is a shift of minimum value of potential energy in the opposite direction of nuclei DNS touch configuration, this is correct at both neutron and proton transitions. Maximum (Coulomb barrier) for proton transitions is shifted to touch configuration of fragments overrunning the contact, and its value decreases for neutron transitions.

## 5. DESCRIPTION OF DC DNS AS OPEN QUANTUM SYSTEMS

On the basis of the above results it is possible to offer a dynamic description of the fusion-quasifission of heavy nuclei with provision for the transitions and emission of nucleons in motion of DNS along the nucleus-nucleus potential.

**5.1.** The dynamic process of DNS movement can be described by classical Newtonian equations [17]:

$$\begin{cases} \mu(t) \frac{d\dot{R}(t)}{dt} + k_R \left( \frac{\partial V_n(R)}{\partial R} \right)^2 \dot{R}(t) = -\frac{\partial V_{nn}(R,L)}{\partial R} \\ \mu(t) \frac{dL(t)}{dt} + k_\theta \left( \frac{\partial V_n(R)}{\partial R} \right)^2 L(t) = 0 \end{cases}, \quad (14)$$

where  $k_R = 1 \times 10^{-23} \text{ s/MeV}$  and  $k_\theta = 0.01 \times 10^{-23} \text{ s/MeV}$  are radial and tangential friction coefficients respectively,  $R$  – distance between the centers of nuclei,  $\mu$  – reduced mass,  $L$  – system angular momentum,  $V_{nn} = V_{Coul} + V_n + V_{rot}$  – nucleus-nuclear potential,  $V_{Coul}$  – Coulomb potential,  $V_n$  – nuclear potential,  $V_{rot}$  – centrifugal potential. This system was numerically solved on the range of  $R$  from the contact point of interacting nuclei  $R_{cont} = 1.28(A_1^{1/3} + A_2^{1/3})$ . The excitation energy  $E_{ki}^*(R, A_i, Z_i)$  – the main value, which is necessary to determine the energy levels, is defined in solving this system, where  $i=1, 2$  – number of the nucleus,  $k$  – step in time or  $R$ .

**5.2.** Nonstationary Schrodinger equation (see Section 1), excluding fluctuations, with dissipative Hamiltonians may be used in order to calculate the probability of SHE formation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2\mu(t)} \frac{\partial^2}{\partial R^2} e^{-\gamma t} + U e^{\gamma t} \right) \psi, \quad (15)$$

or

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2\mu(t)} \frac{\partial^2}{\partial R^2} + U + U_L \right) \psi. \quad (16)$$

For (16):  $U_L = \frac{\hbar\gamma}{2i\mu(t)} \ln \left( \frac{\psi}{\psi^*} \right) + W(t)$ ,  $W(t) = -(\hbar\gamma/2i\mu(t)) \int \psi^* \ln(\psi/\psi^*) \psi dR$ , which is more gen-

eral. The interaction potential  $U$  depends on  $R$  - distance between the centers of nuclei,  $A_1(t)$ ,  $A_2(t)$  and  $Z_1(t)$ ,  $Z_2(t)$ , which are determined by the transitions and emission of nucleons, as well as deformation and relative orientation of nuclei.

**5.3.** The allowed transitions for nucleons of interacting nuclei are determined by the stationary Schrodinger equations. The potential energy for a neutron DNS sub-system is chosen as the sum of the Woods-Saxon potential, and for the proton - the sum of the optical potentials is added with the sum of Coulomb potentials of DNS fragments. The transition is considered to be allowed if the overall energy level is formed with provision for the excitation energy  $E_{ki}^*(R, A_i, Z_i)$  of each subsystem and the relative distance between the centers of nuclei  $R$ . The barrier of transition decreases with decreasing  $R$ , and increase of DNS energy levels is observed with increasing  $E_{ki}^*(R, A_i, Z_i)$ . It should be expected that the nuclei approach and the increase of excitation energy will lead first to allowed neutron transitions, and then to the  $(n + p)$  transitions.

If the sum of transition time is less than the time of DNS movement along the nucleus-nucleus potential, then  $R$  may be written as follows:  $R = (1.28A_1^{1/3} + 1.28A_2^{1/3}) - 2 \cdot 1.28 \cdot (\Delta A^{1/3})$ . This expression may be used, when the overlap of the nuclei volumes is not large, and within the fusion the overlap is less than 7 percents. Thus the following nonstationary Schrodinger equation may be applied to calculate the transitions of nucleons:

$$i\hbar \frac{\partial \psi_{n,p}}{\partial t} = \left( \frac{\hbar^2}{2m_{n,p}} \cdot 1.76A^{5/3} \frac{\partial^2}{\partial A^2} + V_{n,p} \right) \psi_{n,p}, \quad (17)$$

where  $\psi_{n,p}$  - probability of a neutron or a proton transition,  $m_{n,p}$  - mass of a neutron or a proton,  $V_{n,p}$  - potential of neutron or proton interaction with DNS,  $A^{5/3} \frac{\partial^2}{\partial A^2} = n^{5/3} \frac{\partial^2}{\partial n^2}$  - for allowed neutron transitions or  $A^{5/3} \frac{\partial^2}{\partial A^2} = p^{5/3} \frac{\partial^2}{\partial p^2}$  - for proton transitions. If the transformation time of the nucleons is comparable with the time of DNS movement along the nucleus-nucleus potential, then the following Schrodinger equations (15) or (16) with a dissipative Hamiltonian should be applied:

$$i\hbar \frac{\partial \psi_{n,p}}{\partial t} = \left( -\frac{\hbar^2}{2m_{n,p}} \frac{\partial^2}{\partial R^2} e^{-\gamma_{n,p}t} + V_{n,p} e^{\gamma_{n,p}t} \right) \psi_{n,p},$$

or

$$i\hbar \frac{\partial \psi_{n,p}}{\partial t} = \left( -\frac{\hbar^2}{2m_{n,p}} \frac{\partial^2}{\partial R^2} + V_{n,p} + U_{n,p(L)} \right) \psi_{n,p}. \quad (18)$$

**5.4.** The tunneling of nucleons from one nucleus to another should be considered in addition to its transitions. Tunneling is a universal phenomenon in the quantum mechanics and lies in the fact that a particle intersects the barrier, the height of which is greater than the energy of such particle regardless of the laws of classical mechanics. Such quantum system with two potential wells and the barrier between them is formed due to the interaction of heavy nuclei. The wave function at the initial time can be given as

a Gaussian packet with the initial velocity, which depends on the excitation energy, in the following form:

$$\psi_0(R) = A \exp \left[ -\alpha(R - R_0)^2/2 + i(R - R_0)V_0 \right], \quad (19)$$

where  $R_0$  - coordinate of the center of package,  $\alpha$  - constant.  $V_0(E^*)$  value characterizes the initial velocity of the package center;  $A$  - dimensionless normalization constant. The potential energy is given as in **5.3**. The nonstationary Schrodinger equation or equation (16) with a dissipative Hamiltonian should be applied in order to find the probability density, which depends on  $R$  coordinate. The detailed conclusions should be obtained from the calculation of the average values of coordinate  $\langle R \rangle$  and velocity  $\langle V \rangle$ , as functions of time, as well as within the study of the Fourier spectra of these dynamic variables. The coordinate dependence of the probability density at certain times should be given for the considered mode of motion.

Emission of neutrons and protons from DNS fragments may be described by the stationary Schrodinger equations for neutron and proton subsystems (12), where the energy levels for neutron subsystems become greater than zero, and for proton - exceeds the Coulomb potential barrier.

## 6. CONCLUSIONS

The quantum-dynamic approach to the description of fusion-quasifission processes is proposed with reference to the analysis of the inelastic interaction of heavy nuclei. This description is as follows: the processes of neutrons and protons transitions, the emission of nucleons from DNS fragments and tunneling of protons and neutrons through the potential barrier formed by the potentials of subsystems should be considered in DNS movement along the nucleus-nucleus potential with provision for the deformation and the relative orientation of nuclei.

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## ДИНАМИЧЕСКОЕ ОПИСАНИЕ СЛИЯНИЯ-КВАЗИДЕЛЕНИЯ ТЯЖЕЛЫХ ЯДЕР КАК ОТКРЫТОЙ КВАНТОВОЙ СИСТЕМЫ

*К.В. Павлий*

Проведен анализ описания открытых квантовых диссипативных систем на основе уравнений для матрицы плотности и нестационарного уравнения Шредингера с линейными и нелинейными (неэрмитовыми) гамильтонианами. Рассмотрены вопросы захвата, первой стадии реакции слияния, и передач нуклонов взаимодействующих ядер. На основании проведенных расчетов по эмиссии нуклонов из двойной ядерной системы (ДЯС) и передач нуклонов при разрешенных  $[\pm n, \pm p]$  и  $[\pm n, \pm p \pm (n + p)]$  переходах, предложено квантово-динамическое описание слияния-квазиделения ядер, как открытой квантовой системы.

## ДИНАМІЧНИЙ ОПИС ЗЛИТТЯ-КВАЗІДІЛЕННЯ ВАЖКИХ ЯДЕР ЯК ВІДКРИТОЇ КВАНТОВОЇ СИСТЕМИ

*К.В. Павлій*

Проведено аналіз опису відкритих квантових дисипативних систем на основі рівнянь для матриці густини і нестационарного рівняння Шредингера з лінійними і нелінійними (неермитовими) гамильтоніанами. Розглянуто питання захоплення, першої стадії реакції злиття, та передач нуклонів взаємодіючих ядер. На підставі проведених розрахунків р емісії нуклонів з подвійної ядерної системи (ПЯС) та передач нуклонів при дозволених  $[\pm n, \pm p]$  та  $[\pm n, \pm p \pm (n + p)]$  переходах, запропановано квантово-динамічний опис злиття-квазіділення ядер, як відкритої квантової системи.