3D INTENSE BEAM DYNAMICS SIMULATION BY USING MOMENTS METHOD

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The program for 3D simulation of the intense charge particle beam dynamics on the base of the Multi-Component Ion Beam code is described. Fast analysis and study of the averaged beam characteristicsis performed by the moments method.

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INTRODUCTION

Within the framework of the Multi-Component Ion Beam code (MCIB04) [1] the program for 3D simulation of the intense charge particle beam dynamics is created.

Fast analysis and study of the averaged beam characteristics, such as root-mean-square (RMS) dimensions, is performed by the moments method [2].

The main advantage of the moments method in comparison with macro particle one is fast calculation and therefore applicability for transport line optimization.

The model describing the charge density of the bunched beam is introduced. The external electromagnetic fields are assumed to be linear. The approach of effective linearization [2] of both longitudinal and transversal beam self fields gives possibility to get the closed system of the equations for second order moments.

The fitting procedure based on minimization of a quadratic functional at any point of the beam line by using either gradient or simplex-method is available [3].

BEAM MODEL

Let consider the train of bunches (Fig.1), moving with average velocity $\beta_0 c$ with distance between its center-of-mass $\lambda = \beta_0\lambda_0$. Here λ_0 is cyclotron RF field wave length.

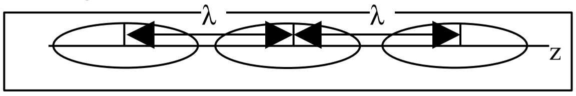

Fig.1.

The beam density may be defined as:

$$\rho(x,y,z-\beta_0 ct) = N\rho_{//}(z-\beta_0 ct)\rho_{\perp}(x,y)\,, \qquad (1)$$

where $N = \dfrac{I\lambda}{Ze\beta_0 c}$ – the number of particle at spatial period λ, I – beam current, Ze – ion charge.

Longitudinal $\rho_{//}$ and transverse densities $\rho_{\perp}$ are equal to:

$$\rho_{//}(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z}\sum_{n=-\infty}^{\infty}\exp\left(-\frac{(z-n\lambda)^2}{2\sigma_z^2}\right) \qquad (2.1)$$

$$\rho_{\perp}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}\right) \qquad (2.2)$$

According to formula (2.1) longitudinal density is periodical function $\rho_{//}(z) = \rho_{//}(z+\lambda)$ with a constant number of particles at period λ:

$$\int_{-\lambda/2}^{\lambda/2}\rho_{//}(z)dz = N \qquad (3)$$

In the case $\sigma_z \gtrsim \lambda$ this model describes the beam with constant density and for $\sigma_z \ll \lambda$ gives Gaussian beam. The dependencies on z of the longitudinal beam density for various values of ratio λ/σ_z are shown in Fig.2.

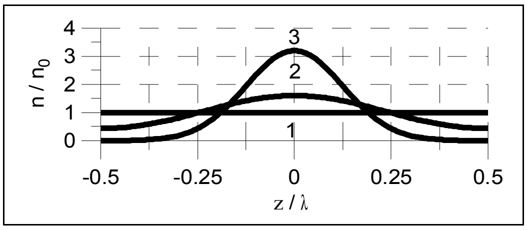

Fig.2. Longitudinal beam density
Curve 1 – $\lambda/\sigma_z = 1$; 2 – $\lambda/\sigma_z = 4$; 3 – $\lambda/\sigma_z = 8$

BEAM SELF FIELD

By using formulae (1, 2) the beam self field may be represent in the following form [4]:

$$E_x \cong 2\pi ZeN\rho_{//}(z-\beta_0 ct)\sigma_x\sigma_y\int_0^{\infty}\frac{x}{(\sigma_x^2+s)R(s)}\rho_{\perp}(T)ds$$

$$E_y \cong 2\pi ZeN\rho_{//}(z-\beta_0 ct)\sigma_x\sigma_y\int_0^{\infty}\frac{y}{(\sigma_y^2+s)R(s)}\rho_{\perp}(T)ds \qquad (4)$$

$$T = \frac{x^2}{\sigma_x^2+s}+\frac{y^2}{\sigma_y^2+s}\quad;\quad R(s) = \sqrt{(\sigma_x^2+s)(\sigma_y^2+s)}$$

$$E_z \cong 2ZeN\rho'_{//}(z-\beta_0 ct)\left(\ln\frac{b}{a}+\frac{1}{2}-\frac{x^2+y^2}{2a^2}\right)\quad,\quad x^2+y^2 \le a^2$$

Here $a = \sqrt{2(\sigma_x^2+\sigma_y^2)}$ – RMS radius of the beam, b – vacuum pipe radius and prime denotes derivative with respect to z.

MOMENTS EQUATIONS

Let us define the second order moments M of the beam distribution function f:

$$M = \overline{YY^T} = \frac{1}{N}\int YY^T f\,dy\,, \qquad (5)$$

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where superscript T denotes transpose vector or matrix, $Y^T = (x, y, x', y', z - \beta_0 ct, \delta) = (X^T, V^T, Y_{//}^T) = (Y_\perp^T, Y_{//}^T)$ – vector of phase space coordinates of the particle, $\delta = (\beta - \beta_0)/\beta_0$ – relative momentum spread. Integration in (4) is fulfilled over all phase space occupied by bunch particles (at one spatial period), prime denotes derivative with respect to longitudinal coordinate of the bunch center-of-mass.

The equations for transverse second order moments $M_\perp = \overline{Y_\perp Y_\perp^T}$ does not changed significantly in comparison with the case of non-bunched beam[2]:

$$M'_\perp = AM_\perp + M_\perp A^T \quad ; \quad A = \begin{pmatrix} 0 & E \\ b_{ext} + b_s & a_{ext} \end{pmatrix} \tag{6}$$

Here $M_\perp$, A are fourth order matrices, E is second order unit matrix, a_{ext} and b_{ext} are 2×2 matrices defined by external fields. Second order matrix b_s depends on RMS dimensions and is defined by beam self fields:

$$b_s = k_\perp \frac{Z}{A}\frac{I}{I_A}\frac{1}{\beta_0^3}\frac{1}{\sigma_x + \sigma_y}\begin{pmatrix} 1/\sigma_x & 0 \\ 0 & 1/\sigma_y \end{pmatrix}, \tag{7}$$

where A – ion mass, $I_A = mc^3/e$ – Alfven's current.

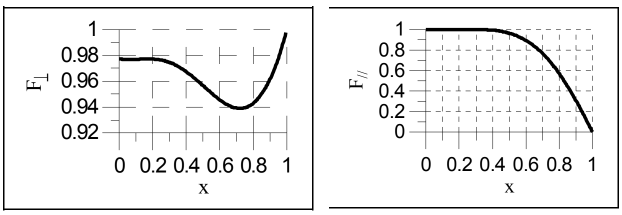
The bunching factor $k_\perp$ is connected with changing of the transverse beam self fields due to changing of the longitudinal density:

$$k_\perp = \lambda \int_{-\lambda/2}^{\lambda/2} \rho_{//}^2(z)dz = \sqrt{\frac{\overline{z_0^2}}{\overline{z^2}}} F_\perp\left(\frac{\overline{z^2}}{\overline{z_0^2}}\right) \tag{8}$$

Here $\sqrt{\overline{z^2}}$ is current longitudinal RMS dimension of the bunch:

$$\overline{z^2} = \int_{-\lambda/2}^{\lambda/2} z^2 \rho_{//}(z)dz \tag{9}$$

and $\sqrt{\overline{z_0^2}} = \lambda/\sqrt{3}$ its value for non-bunched beam. The plot of function $F_\perp(x)$ is shown in Fig.3,a.



a *b*

Fig.3.

As may be seen from Fig.3,a function $F_\perp(x)$ is approximately equal to unity with difference does not greater than 6%. In the program this function is represented as the sixth order polynomial.

The equations for the longitudinal second order moments $M_{//}$ has the following form:

$$M_{//} = \overline{Y_{//} Y_{//}^T} = \begin{pmatrix} \overline{z^2} & \overline{z\delta} \\ \overline{z\delta} & \overline{\delta^2} \end{pmatrix} \tag{10.1}$$

$$\left(\overline{z^2}\right)' = 2\,\overline{z\delta} \tag{10.2}$$

$$\left(\overline{z\delta}\right)' = \overline{\delta^2} + \frac{Ze}{Am\beta_0^2 c^2}\overline{zE_z} \tag{10.3}$$

$$\left(\overline{\delta^2}\right)' = \frac{Ze}{Am\beta_0^2 c^2}\frac{\overline{zE_z}}{\overline{z^2}}\overline{z\delta} \tag{10.4}$$

Computation of average $\overline{zE_z}$ in accordance with formulae (4, 5) results in:

$$\frac{Ze}{Am\beta_0^2 c^2}\overline{zE_z} = k_{//}\frac{Z}{A}\frac{I}{I_A}\frac{1}{\beta_0^3}\left(\ln\frac{b}{\sqrt{2(\sigma_x^2 + \sigma_y^2)}} + \frac{1}{4}\right) \tag{11}$$

The bunching factor of the longitudinal motion $k_{//}$ is defined by formula:

$$k_{//} = \lambda \int_{-\lambda/2}^{\lambda/2} [\rho_{//}^2(z) - \rho_{//}^2(\lambda/2)]dz = k_\perp F_{//}\left(\frac{\overline{z^2}}{\overline{z_0^2}}\right) \tag{12}$$

The plot of function $F_{//}(x)$ is shown in Fig.3,b. In the case $x \sim 1$ function $F_{//}$ is close to zero because of the longitudinal electric field of non-bunched beam is equal to zero. For the well bunched beam($x << 1$) due to small longitudinal density at point $z = \lambda/2$ formulae (11) and (12) become identical and function $F_{//}$ is close to unity. In the program function $F_{//}(x)$ is approximated by the fifth order polynomial for all values of x.

MCIB04 CODE MODIFICATION

The 3D moments equations were introduced into existing program library code MCIB04 [1]. The interface of the program is shown in Fig.4.

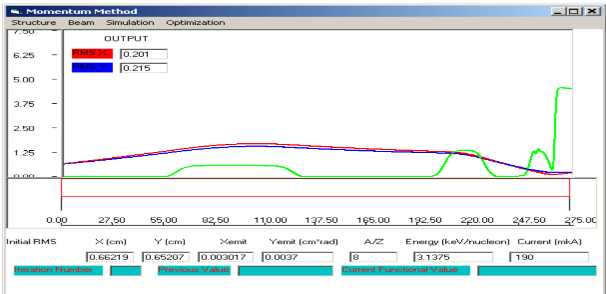


Fig.4. Interface of the program

Before launching of the program the files containing the beam-line lattice, initial beam parameters and (optionally) the longitudinal magnetic field distribution have to be created.

During working of the program the changes of the second order moments along the beam-line are computed. The plots of the longitudinal magnetic field distribution (green line in Fig.4) and RMS dimensions of the beam (red – x and blue – y) are given at monitor. The special windows are intended for values of the beam RMS dimensions at the exit of the channel (RMSX, RMSY) and initial parameters – RMS dimensions(X,Y), emittances (Xemit, Yemit), mass-to-charge ratio (A/Z), kinetic energy (Energy) and beam current (Current).

The fitting procedure based on minimization by using either gradient or simplex-method of a quadratic functional computed for every second order moments at any point of the beam line is available [3].

The dependencies on distance along the channel of the beam envelopes, emittances, momentum spread and other parameters are writing to the file and processing by the graphing program package.

BUNCHING SYSTEM COMPUTATION

The simulation of the bunching system of the DC350 cyclotron axial injection beam-line [5] was fulfilled by using created 3D version of MCIB04 code.

The bunching system consists of linear and sinusoidal bunchers. The linear buncher is placed at 275 cm and sinusoidal – at 80 cm from median plane of the cyclotron. In the simulation all bunchers were replaced by infinitesimal width gap with variable voltage.

The initial parameters of the beam are contained in Table.

^{48}Ca beam initial parameters

Injected beam	$^{48}Ca^{6+}$
Mass, A	48
Charge, Z	2…8
Injected current, μA	0…190
Ca beam current, μA	0…700
He beam current, μA	200
$^{48}Ca^{6+}$ kinetic energy, keV/u	3.1375
Diametr, mm	8
Emittance, π mm×mrad	142

The initial conditions for the moments were defined at the entrance of the linear buncher and were found by macro-particle simulation. Charge state distributions for ion beam and its self fields were taken into account in this simulation.

The beam focusing is provided by two solenoids. The longitudinal magnetic field of the cyclotron is considered also.

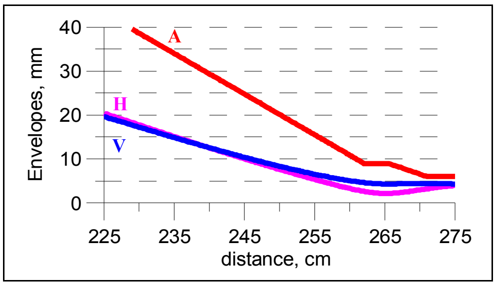


Fig.5. Apertutre (A), horizontal (H) and vertical (V) ^{48}-Ca^{6+} beam envelopes near inflector

The matching condition at the entrance of the spiral inflector corresponds to the steady state of the beam (without envelopes oscillation) in the uniform magnetic field with magnitude to be equal to the field in the cyclotron center. The amplitude of the voltage at linear buncher was found to provide the equality $k_{\perp} = 2$ at the entrance of sinusoidal buncher.

The beam envelopes near spiral inflector of the cyclotron are shown in Fig.5.

Let define the bunching efficiency as ratio of the number of particles within RF phase interval $|\Delta\varphi| \le 15^0$ to non-bunched beam one. This quantity shows the possible increasing of the number of particle captured into acceleration in the cyclotron due to the bunching system. The dependence of the bunching efficiency on the $^{48}Ca^{6+}$ beam current is shown in Fig.6.

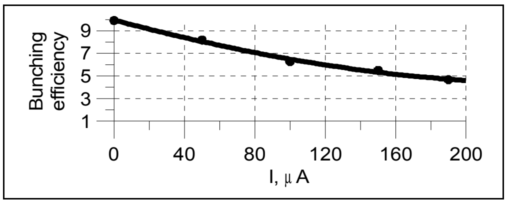


Fig.6. Bunching efficiency versus beam current

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3D-МОДЕЛИРОВАНИЕ ДИНАМИКИ СИЛЬНОТОЧНОГО ПУЧКА С ИСПОЛЬЗОВАНИЕМ МЕТОДА МОМЕНТОВ

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Описывается программа 3D-численного моделирования интенсивного пучка заряженных частиц, созданная на основе Multi-Component Ion Beam кода. Быстрый анализ и исследование средних характеристик пучка проводится методом моментов

3D-МОДЕЛЮВАННЯ ДИНАМІКИ ПОТУЖНОСТРУМОВОГО ПУЧКА З ВИКОРИСТАННЯМ МЕТОДУ МОМЕНТІВ

Н. Казарінов, В. Олександров, В. Шевцов

Дано опис програми 3D-чисельного моделювання інтенсивного пучку заряджених частинок, що створена на основі Multi-Component Ion Beam коду. Швидкий аналіз і дослідження середніх характеристик пучку проводиться методом моментів.